Magnetophonon Shakeup in a Wigner Crystal: Applications to Tunneling Spectroscopy in the Quantum Hall Regime

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(Received 17 June 1993)

We calculate the tunnel current between two parallel two-dimensional electron systems in a strong perpendicular magnetic field. We model the strongly correlated electron systems by Wigner crystals, and describe their low-energy dynamics in terms of magnetophonons. A tunneling electron shakes up magnetophonons, which results in a conductance peak that is displaced away from zero voltage and broadened compared with the case of no magnetic field. Our results are in very good quantitative agreement with recent experiments.

PACS numbers: 73.40.Gk, 73.20.Dx

The Coulomb blockade effect has received a great deal of attention over the past few years. In this effect the transport through a small metallic grain or quantum dot is suppressed at small bias voltages [1]. The charging energy due to one single electron, $e^2/2C$, becomes large compared with other relevant energies such as k_BT once the grain or dot, and consequently the capacitance C, becomes small enough.

In this paper we consider a Coulomb blockade effect of a somewhat different origin. In a recent experiment Eisenstein, Pfeiffer, and West [2] studied the tunneling between two parallel two-dimensional (2D) electron systems confined in quantum wells (QW's) separated by a rather thick barrier. The 2D electron systems were furthermore subject to a strong perpendicular magnetic field so that only the lowest Landau level was (partially) filled. They found that (i) the small-bias conductance was strongly suppressed at low temperatures, (ii) the tunnel current showed a broad maximum at voltage $V_{\rm peak} \sim$ 6–7 mV, and (iii) as a function of temperature, the small-bias conductance was activated with activation temperature $T_A^{\rm exp}$ =5–10 K [3]. This suggests that there is an energy cost $eV_{\rm peak}$ of the order of $e^2/4\pi\varepsilon\langle a\rangle$, for moving an electron from one electron system to the other in a strong magnetic field. Here ε is the dielectric constant of the material and $\langle a\rangle$ is the average interelectron spacing. At zero magnetic field the same system shows, in contrast, a conductance that is sharply peaked at zero bias, which indicates that the energy cost vanishes for B =0. Thus, the strong magnetic field causes a Coulomb blockade. In contrast to experiments with small metal grains or quantum dots, the "smallness" is in this case set, not by geometric dimensions, but by the magnetic field. It quenches the kinetic energy so that the 2D electron systems are strongly correlated.

In the following, we present a model calculation of the tunnel current in the system studied in the experiment [2]. Our model, which contains *no adjustable parameters*, is the simplest one that we believe can capture most of the essential physics. The two-dimensional electron systems are likely to be in compressible liquid states. Since the kinetic energy of the electrons is quenched the Coulomb interaction causes strong correlations between them. An electron tunneling from one such liquid to the other induces sudden changes in the potential felt by the other electrons. This consequently shakes up a number of collective excitations in the electron liquids. The displacement of the maximum current away from zero bias reflects the energy carried away by these excitations. The major rationalization in the design of our model is that we approximate the strongly correlated electron liquids by Wigner crystals (WC's), which is a reasonable model for the short-range correlations, but overestimates the long-range structural order in an electron liquid. A Wigner crystal has a gapless mode of low-energy excitations, magnetophonons, that are mainly transverse [4,5], whereas the mostly longitudinal magnetoplasmons only appear above the cyclotron energy. We find that the conductance peak is broadened and displaced away from zero bias due to shakeup of magnetophonons. Our results are in very good agreement with the experiment. We find a conductance gap and a maximum in the tunnel current at about the same voltage as in the experiment. Also our results in the case of a finite temperature are in general agreement with experiment.

Except for this work a few other theories have addressed the same problem. Efros and Pikus [6] studied an essentially classical model of a two-dimensional electron liquid using Monte Carlo methods. He, Platzman, and Halperin [7] calculated the one-electron spectral function in a finite system by means of exact diagonalization. Moreover, they presented analytical results for the low-voltage behavior of the current. Our theory is complementary to theirs. The use of a Wigner crystal means that we may not be able to describe certain quantum-mechanical effects and long-range correlations. On the other hand, our model is conceptually and calculationally simple. Furthermore, we get results also in the thermodynamic limit.

We assume that the 2D electron systems are ordered as triangular Wigner crystals; the lattice parameter a_0 is set by the electron density (see Fig. 1). Since the dis-

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tance separating the two electron systems is rather large compared with the interelectron spacing in each of the QW's in the experiment, we neglect interactions as well as mutual correlations between the WC's. We consider a process in which an electron tunnels from a lattice position in one Wigner crystal into an interstitial position of the other. It does not cost any extra kinetic energy to localize an electron to an area $2\pi l_c^2$ in a magnetic field [$l_c = (\hbar/eB)^{1/2}$ is the magnetic length]. Therefore, localized wave packets (guiding center states) form a good basis set for the single-particle states. Thus, the immediate effect of a tunneling event on the electron systems can be described as a local perturbation, not only in the Wigner crystals, but also in the real liquid systems. We describe the low-energy dynamics of all the other electrons in terms of magnetophonons and obtain the model Hamiltonian

$$\begin{aligned} H &= H_0 + H_T^+ + H_T^- \\ &= \Big[\varepsilon_L + \sum_\alpha (M_{\alpha L} a_\alpha + M_{\alpha L}^* a_\alpha^\dagger)\Big] c_L^\dagger c_L \\ &\quad + \Big[\varepsilon_R + \sum_\alpha (M_{\alpha R} a_\alpha + M_{\alpha R}^* a_\alpha^\dagger)\Big] c_R^\dagger c_R + \sum_\alpha \hbar\omega_\alpha a_\alpha^\dagger a_\alpha + T_{LR} c_R^\dagger c_L + T_{LR}^* c_L^\dagger c_R. \end{aligned} \tag{1}$$

Here $c_{L(R)}$ and $c_{L(R)}^\dagger$ are destruction and creation operators for an electron in the lowest Landau level in the left (right) QW. Considering tunneling from left to right, ε_L is the energy of an electron in a lattice position and ε_R is the energy of an interstitial electron. The operators a_α ($a_\alpha^\dagger$) destroy (create) a magnetophonon (MP) in mode α; α is a composite index denoting in which QW the magnetophonon is, as well as its wave vector. We have calculated the MP frequencies ω_α following the method of Bonsall and Maradudin [4]. This means that we treat the electrons as classical point particles. Such an approximation, of course, works best for small filling factors. It deserves to be pointed out that, even at the highest filling factors ($\nu \approx 0.8$) considered in this paper, the quantum-mechanical overlap between an *interstitial* electron and each of the nearest lattice electrons is only $\approx 3\%$. A natural improvement of the calculation of the magnetophonon frequencies would be to use the theory of Côté and MacDonald [5]. Their calculation can also give us some idea about how well the MP's approximate the collective modes of the real system. In the theory of Ref. [5] the electrons could move itinerantly so the calculated modes should in a sense be "liquidlike." Still the dispersion relation of the MP's was qualitatively the same as in the harmonic approximation [4], and even the quantitative differences were not very large for filling factors $\nu \lesssim 0.5$. We conclude that it is reasonable to assume that the MP's approximate the collective modes of the real system fairly well as long as the wavelength does not exceed a few lattice spacings.

To quantize the MP's one must perform a canonical transformation [8]. This leaves the frequencies unchanged compared with the classical calculation but the eigenmodes α become linear combinations of classical magnetophonon plane waves with wave vectors $\mathbf{q}$ and $-\mathbf{q}$. The matrix element $M_{\alpha L(R)}$ gives the interaction between magnetophonon α and an electron in the left (right) QW. A matrix element is obtained from the interaction energy, linearized in the displacements, between the Wigner crystal and the Coulomb potential due to an added interstitial, or removed lattice electron. The last two terms in Eq. (1) allow electrons to tunnel between the wells; T_{LR} is the calculated tunneling matrix element [9].

Since the barrier separating the quantum wells is thick, we can, assuming that the chemical potentials differ by an amount eV between the two wells, get an expression for the tunnel current as a function of the bias voltage V from the Fermi golden rule,

$$I(V) = \frac{e}{\hbar^2}\int_{-\infty}^{\infty} dt [e^{ieVt/\hbar}\langle H_T^-(t) H_T^+(0)\rangle - e^{-ieVt/\hbar}\langle H_T^+(t) H_T^-(0)\rangle], \tag{2}$$

where $H_T^\pm(t) = e^{iH_0 t/\hbar} H_T^\pm e^{-iH_0 t/\hbar}$. The Hamiltonian H_0 in Eq. (1) is an example of an exactly solvable independent-boson model [10]. In writing down the Hamiltonian (1) we have assumed that the tunneling electron remains at rest when exciting MP's. As a consequence the interactions with different magnetophonons are independent, which is essential for the solution of

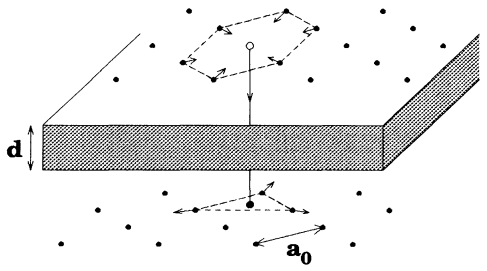

FIG. 1. Schematic illustration of the tunneling process and the Wigner crystals. An electron tunnels from a lattice position on one side of the barrier to an interstitial position on the other side. This induces relaxation processes carrying away energy in the form of magnetophonons.

the model. This no-recoil assumption is reasonable in a strong magnetic field since, as stated earlier, localized wave packets form a good basis for the single-particle states in the lowest Landau level. The correlation functions in Eq. (2) are [10]

$$\langle H_T^-(t)H_T^+(0)\rangle = \langle H_T^+(t)H_T^-(0)\rangle = \nu(1-\nu)|T_{LR}|^2 C(t), \tag{3}$$

where

$$C(t) = \exp\left(-\sum_\alpha \frac{|M_{\alpha L} - M_{\alpha R}|^2}{(\hbar\omega_\alpha)^2}[(1+N_\alpha)(1-e^{-i\omega_\alpha t}) + N_\alpha(1-e^{i\omega_\alpha t})]\right). \tag{4}$$

Here $(M_{\alpha L} - M_{\alpha R})$ is the sudden change in the magnetophonon coupling due to the tunneling event. To get the expressions in Eqs. (3) and (4) we consider a region of size $2\pi l_c^2$, and assume that the relaxed electron energies ($\bar{\varepsilon}_L$ and $\bar{\varepsilon}_R$ in the notation of Ref. [10]) are equal [11]. Physically this means that the electron systems can relax completely after a tunneling event and the threshold voltage is zero. The symbol N_α in Eq. (4) denotes the thermal occupation of mode α, $N_\alpha = [\exp(\hbar\omega_\alpha/k_BT) - 1]^{-1}$.

To calculate the current we must find the Fourier transform $C(\omega)$ of the correlation function. At zero temperature $C(\omega)$ is determined by the integral equation [12]

$$\omega C(\omega) = \int_0^{\omega_{\max}} \frac{d\Omega}{2\pi} g(\Omega) C(\omega - \Omega) \tag{5}$$

and the additional conditions $C(\omega) = 0$ for $\omega < 0$ and $\int_0^\infty d\omega\, C(\omega)/2\pi = 1$. The function $g(\Omega)$ is a measure of the coupling strength to the magnetophonons,

$$g(\Omega) = 2\pi \sum_\alpha \frac{|M_{\alpha L} - M_{\alpha R}|^2}{\hbar^2\Omega}\delta(\Omega - \omega_\alpha), \tag{6}$$

and $\omega_{\max}$ is the maximum magnetophonon frequency. In the cases that we consider, we find that $\hbar\omega_{\max}$ is typically 1–2 meV. At a finite temperature $C(\omega)$ can be calculated by noticing that $C(t)$ can be factorized into two parts associated with emission and absorption of MP's. The Fourier transforms of the two factors can be calculated independently by the integral-equation method, and $C(\omega)$ is then found by a frequency convolution. Assuming that tunneling events at different places of the two-dimensional system are independent, we obtain the final expression for the tunnel current,

$$I(V) = \frac{S}{2\pi l_c^2}\frac{e}{\hbar^2}\nu(1-\nu)|T_{LR}|^2[C(eV/\hbar) - C(-eV/\hbar)], \tag{7}$$

where S is the total area of the system.

We first examine the current $I(V)$ at zero temperature, displayed in Fig. 2. The current is strongly suppressed at small bias voltages, and it exhibits a broad maximum at a finite voltage V_{peak}, just as in the experiment. The peak voltage lies in the range 8–10 mV, i.e., a little higher than in the experiment [2], but still well below the voltage corresponding to the cyclotron energy. The peak voltage increases, at least to begin with, with increasing magnetic field. An increased magnetic field lowers the magnetophonon frequencies and thereby increases the zero-point motion of the WC to which the tunneling electron couples. We see, however, that the peak does not shift much at larger magnetic fields (between 13 T and 16 T).

When the electron density in the 2D systems is reduced while the magnetic field is kept constant, the peak voltage varies as $V_{\text{peak}} \sim 1/a_0$, where a_0 is the lattice parameter. A lower electron density leads to a weaker Coulomb repulsion between the tunneling electron and the electrons residing in the 2D electron systems, and the gap in the tunnel current is consequently reduced. Our findings here are in agreement with what was reported in Ref. [2]. Moreover, as the electron density is reduced the current-voltage characteristics behave more and more classically. By this we mean (i) the I-V curves become more sharply peaked when the density is lowered and (ii) the peak voltage shows almost no dependence on magnetic field for small filling factors (typically $\nu \lesssim 1/3$). Our results thus show a crossover from a regime at "high" filling factors where the magnetic field plays a role in determining the peak voltage, to a regime where the peak

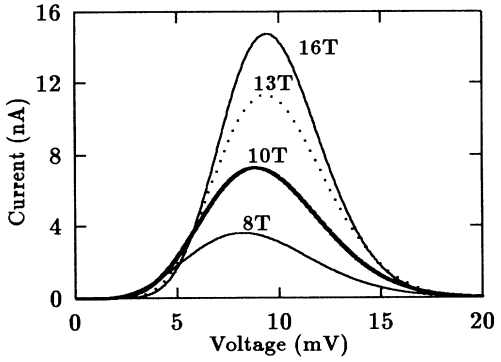

FIG. 2. Calculated current-voltage characteristics at zero temperature at four different magnetic fields. The parameter values have been taken from the experiment [2]; i.e., the lattice parameter a_0 =270 Å ($n \approx 1.6\times10^{11}$ cm^{-2}), the quantum well widths L =200 Å, the barrier thickness d =175 Å, and the sample area S =0.0625 mm^2. The barrier height was 250 meV.

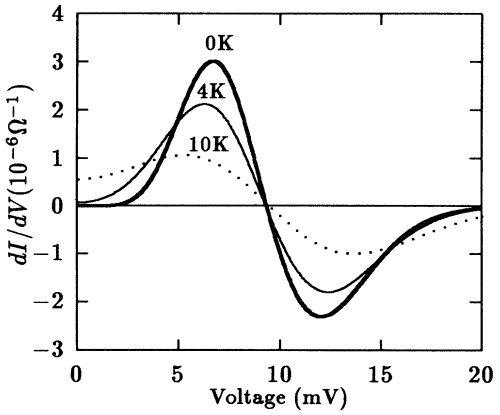

FIG. 3. The differential conductance dI/dV as a function of voltage. In these calculations the magnetic field was B =13 T, while the temperature was varied as indicated next to the curves. The rest of the parameter values are the same as in Fig. 2.

voltage is set by the interelectron spacing alone.

The low-voltage behavior of the current in our calculation is approximately given by a power-law singularity characteristic of x-ray edge problems [12]. If the function g is constant up to a certain frequency, the current is determined by $I \sim V^{g/2\pi-1}$ up to the corresponding voltage. In this calculation $g(\Omega)$ varies rather slowly over most of the range $0 < \Omega < \omega_{\max}$. For very low frequencies (less than a few percent of $\omega_{\max}$) g exhibits a $\Omega^{-1/3}$ singularity; however, this does not influence the results very much. Our calculated I-V curves show approximate power-law behavior for eV from $\approx 0.1\hbar\omega_{\max}$ up to $\approx 0.5\hbar\omega_{\max}$ (i.e., typically $V \sim 1$ mV), where effects of the Brillouin zone edge become important. He, Platzman, and Halperin [7] found, using the theory presented in Ref. [13], the limiting behavior $I \sim e^{-V_0/V}$. Their results should be more reliable than ours at this end of the spectrum since the Wigner-crystal model cannot give a completely correct description of the long-range correlations that are involved in the slow relaxation processes.

The effects of quantum fluctuations on the "ordinary" Coulomb blockade have also been treated by independent-boson models [14,15]. The I-V curves in that case look very different from what we find here. In the ordinary Coulomb blockade electrons tunnel between continuous energy bands, and in the absence of electron-electron interactions the I-V curve is linear. The shakeup of bosons leads to a pseudogap at small voltages and an offset at higher voltages. Here, on the other hand, we consider tunneling between states (the Landau levels) that are discrete in energy in the absence of shakeup. Therefore, our I-V curves exhibit pseudogaps followed by *peaks*.

At finite temperatures thermally excited magnetophonons assist tunneling and thereby reduce, and eventually eliminate, the Coulomb gap. In Fig. 3 we display results for the differential conductance dI/dV as a function of voltage at three different temperatures. We see that, just as in the experiment, the Coulomb gap vanishes at a temperature of $\sim$10 K. The differential conductance at zero voltage as a function of temperature behaves as $(dI/dV)_{V=0} \sim e^{-T_A/T}$, where T_A is an activation temperature. We get $T_A^{\rm th} \approx 8$ K at B =8 T, and $T_A^{\rm th} \approx 13$ K at B =13 T, which are $\approx$20% larger than what were found in the experiment.

In conclusion, we have calculated the tunnel current between two two-dimensional electron systems in a strong perpendicular magnetic field. We have done this by modeling the strongly correlated electron systems by Wigner crystals. As an electron tunnels, low-energy magnetophonon excitations are shaken up. Consequently, the differential conductance acquires a gap and shows a broadened peak at a finite voltage. Our results are in very good agreement with experiment.

We thank Karsten Flensberg for many stimulating discussions. Discussions with A.-P. Jauho and H. Smith are also gratefully acknowledged.

Note added.—After the submission of this paper two other Letters related to this work appeared [16,17].

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