Probing Bose-Einstein Condensed Atoms with Short Laser Pulses

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We propose a method of probing a system of cooled atoms in a trap using short laser pulses. Above the critical temperature for Bose-Einstein condensation such a system scatters very weakly. Coherent scattering occurs primarily in the forward direction. Below the critical temperature, the number of scattered photons increases dramatically and the scattered light is emitted in the solid angle determined by the size of the condensate.

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Recently there has been a great deal of interest in experimental realization of the Bose-Einstein condensate (BEC) [1–3] in various systems of trapped and cooled atoms [4]. Apart from the fundamental question of the possibility of achieving sufficiently low temperatures and high densities to obtain BEC, there is another question concerning detection and observation of the condensate.

Obviously, the latter goal may be realized by scattering light on the system of cooled atoms. Until now, quantum optics of BEC has not been studied in great detail [5,6]. Politzer considered the problem of scattering of weak light on the condensate at $T \simeq 0$. In an infinite trap, atomic and photonic degrees of freedom mix, giving rise to a gap in the excitation spectrum. Because of this gap the resonant light will be strongly reflected back from the sharp boundary of the condensate. Note, however, that these conclusions would be significantly modified in more realistic conditions, i.e., in a trap of finite size and no sharp boundaries [7].

The aim of this Letter is to investigate another limiting case of scattering of short but intense laser pulses on the system of cooled atoms in a trap. In particular, we consider laser pulses of the area of multiples of 2π. The area of the pulse is defined as an integral of the slowly varying envelope of the electric field multiplied by the atomic dipole moment. When the area is $2\pi K$, the atom cycles K times between the excited and ground state. We show that above the critical temperature for the Bose-Einstein condensation, the coherent scattering from such a system of atoms is very weak and takes place primarily in the forward direction due to the phase matching effects. Below the critical temperature the number of scattered photons increases dramatically and the coherent scattering occurs in a solid angle determined by the size of the condensate. Even below T_c, sufficiently short $2\pi K$ pulses leave the system relatively intact, providing a nondemolishing tool for observing the BEC.

We focus our attention on the range of parameters describing a magneto-optical trap developed at JILA [1,8]. The potential for the atomic center-of-mass motion in the ground electronic state can be well described by the harmonic oscillator potential of the frequency $\omega_t \simeq (2\pi) \times 10$ Hz. Note that this potential does not extend to infinity but rather forms a finite barrier of a height of the order of $(2\pi)2 \times 10^4$ Hz or more. Nevertheless, several thousands of degenerated harmonic oscillator energy levels can be formed within the trap. By exploiting the evaporative cooling technique, the trap can store about $N = 10^8$ cesium atoms, which will interact with the resonant light of frequency $\sim (2\pi)4.0 \times 10^{14}$ Hz. A typical photon recoil energy will then be $\simeq (2\pi)2$ kHz, whereas the natural linewidth (HWHM) $\gamma \simeq (2\pi)2.5$ MHz. Characteristic lengths in this system are the size of the ground state wave function, $a \simeq 10^{-5}$ m, and the resonant wavelength, $\lambda \simeq 800$ nm. Both of these are, of course, much larger than the size of the electronic wave function, i.e., Bohr radius $a_0 \simeq 5 \times 10^{-11}$ m. It is worth stressing that both ω_t and $a = 1/\sqrt{2M\omega_t}$, where M is the atomic mass, can be controlled in the experiment. In general, atoms in excited electronic states move in different potentials from that characterizing the ground state. Although the shape of those potentials plays a very important role in the case of scattering of a weak, cw laser light [7], it will turn out to be not as essential in the scattering of short laser pulses. We will assume only that the excited state potentials are smooth and vary on a similar scale to that of the ground state potential, so that the corresponding level spacing is of the order of 10 Hz or even less.

The Hamiltonian governing the evolution of the atoms in the trap takes the following second quantized form in the rotating wave approximation (RWA) and in atomic units:

$$\begin{aligned}\mathcal{H} = &\sum_n E_n^g g_n^\dagger g_n + \sum_m (E_m^e + \omega_0)\mathbf{e}_m^\dagger \mathbf{e}_m \\ &+ \sum_\mu \int d^3\mathbf{k}\; ck\; a_{\mathbf{k}\mu}^\dagger a_{\mathbf{k}\mu} \\ &+ \sum_{n,m}\sum_\mu \int d^3\mathbf{k}\; \varrho(k)[\eta_{nm}(\mathbf{k}) g_n^\dagger a_{\mathbf{k}\mu}^\dagger \mathbf{e}_m \cdot \boldsymbol{\epsilon}_{\mathbf{k}\mu} + \text{H.c.}]\,,\end{aligned} \tag{1}$$

where g_n, $g_n^\dagger$ denote atomic annihilation and creation operators for the nth state of the ground state potential. For a rotationally invariant potential, n is actually a triple index (n_x, n_y, n_z). The corresponding energy is

$E_n^g = \omega_t(n_x + n_y + n_z)$. $\mathbf{e}_m$, $\mathbf{e}_m^\dagger$ denote atomic annihilation and creation operators in the excited state potential. The corresponding energies are $E_m^e + \omega_0$, i.e., are shifted by the electronic transition frequency. We consider here the case of the transition from an s state to a p state and therefore $\mathbf{e}_m$'s and $\mathbf{e}_m^\dagger$'s have a corresponding vector character. This is not the case of the transition in cesium ($6S_{1/2}F = 4$ to $6P_{3/2}F = 5$), but the character of the transition is not essential for our conclusions. $a_{\mathbf{k}\mu}$ and $a_{\mathbf{k}\mu}^\dagger$ denote annihilation and creation operators for photons of the momentum $\mathbf{k}$ and linear polarization $\boldsymbol{\epsilon}_{\mathbf{k}\mu}$ ($\mu = 1, 2$). All the introduced operators fulfill standard bosonic commutation relations. The coupling $\varrho(k)$ is a slowly varying function of k related to the natural linewidth $\gamma = (8\pi^2 k_0^2/3c)|\varrho(k_0)|^2$, with $k_0 = \omega_0/c$. Finally, $\eta_{nm}(\mathbf{k})$ are matrix elements for the transition from the nth state of the ground state potential to the mth state of the excited state potential,

$$\eta_{nm}(\mathbf{k}) = \langle n|e^{-i\mathbf{k}\cdot\mathbf{R}}|m\rangle. \tag{2}$$

The above Hamiltonian includes that part of the strong resonant atomic interactions due to electronic dipole-dipole forces and exchange of transverse photons [9]. We neglect, however, other forces that may play a crucial role in atomic collisions.

Suppose the system is driven by a short coherent laser pulse. If such a pulse is strong enough and short enough, we may neglect spontaneous emission effects and substitute the electric field operator entering the interaction Hamiltonian in Eq. (1) by a c-number. The pulses we intend to use should have duration ≤ 300 ps or shorter, i.e., width $\gamma_L \simeq 3 \times 10^9$–$10^{11}$ Hz. The first estimate shows that indeed $\gamma_L \gg \gamma$, i.e., the spontaneous emission may be legitimately neglected during the time of interaction of the pulse with the atoms. Note, however, that this estimate might be misleading, since the atoms will respond collectively and the effective spontaneous emission rate might thus be greatly enhanced [10]. We will therefore have to check our assumption self-consistently in the following, to assure that the total number of emitted photons N_{tot} is much smaller than N.

Assuming that the spontaneous emission is slow enough we substitute the electric field operator multiplied by the absolute value of the electronic transition dipole moment by

$$d\boldsymbol{\mathcal{E}}^{(+)} \to \frac{\Omega}{2}\sum_\mu \int d^3\mathbf{k}\, \boldsymbol{\varrho}(\mathbf{k},\mu)e^{i\mathbf{k}\cdot\mathbf{R}-ickt}, \tag{3}$$

where Ω is a peak Rabi frequency of the laser pulse. The function $\boldsymbol{\varrho}(\mathbf{k},\mu)$ describes a $(\mathbf{k},\mu)$-dependent envelope of the pulse. We assume that the pulse has the form of a plane wave packet moving in the $\mathbf{k}_L$ direction with the central frequency ω_L and the linear polarization $\boldsymbol{\epsilon}_L$, so that

$$d\boldsymbol{\mathcal{E}}^{(+)} \to \frac{\Omega}{2}\boldsymbol{\epsilon}_L\mathcal{F}\left[\gamma_L(t - \mathbf{k}_L\cdot\mathbf{R}/\omega_L)\right]e^{i\mathbf{k}_L\cdot\mathbf{R}-i\omega_L t}. \tag{4}$$

Here, $\mathcal{F}(\gamma_L t)$ is the temporal envelope of the pulse chosen to be real and assumed to have a bell shape and a maximum at $t = 0$ equal to 1.

Note that to obtain Eq. (4), the $(\mathbf{k},\mu)$-dependent envelope $\boldsymbol{\varrho}(\mathbf{k},\mu)$ must change on a scale of momenta of the order of $\gamma_L/c \simeq 10$–$300\ \text{m}^{-1}$. On the other hand, the characteristic scale on which the matrix elements $\eta_{nm}(\mathbf{k})$ change δk is of the order of $1/a \simeq 10^5\ \text{m}^{-1}$ for low n. As we go to higher n's, δk scales as $1/\sqrt{n}$, so it becomes $10^3\ \text{m}^{-1}$ for the highest energy levels that are still available in the trap. Since $\delta k \gg \gamma_L/c$ for all γ_L in question, we may thus safely substitute $\mathbf{k}$ by $\mathbf{k}_L$ inside $\eta_{nm}(\mathbf{k}_L)$. With this substitution, using Eq. (3) and Eq. (4), the Hamiltonian (1) becomes

$$\mathcal{H} = \sum_n E_n^g g_n^\dagger g_n + \sum_m (E_m^e + \omega_0)\mathbf{e}_m^\dagger\mathbf{e}_m + \frac{\Omega}{2}\mathcal{F}(\gamma_L t)\left(\exp(i\omega_L t)\sum_n g_n^\dagger\boldsymbol{\epsilon}_L\cdot\mathbf{f}_n + \text{H.c.}\right), \tag{5}$$

where we have introduced a new notation for annihilation and creation of wave packets of excited states created from the nth state of the ground state potential,

$$\mathbf{f}_n = \sum_m \eta_{nm}(\mathbf{k}_L)\mathbf{e}_m. \tag{6}$$

Note that these annihilation and creation operators describe independent wave packets, i.e., fulfill the standard bosonic commutation relations $[f_n^\alpha, f_{n'}^{\alpha'\dagger}] = \delta_{nn'}\delta_{\alpha\alpha'}$, with $\alpha, \alpha' = x, y, z$. Moreover, since we assume that the excited state potentials are flat, the energy E_m^e will not vary much for the states in question; for each of the wave packets $\mathbf{f}_n$, $\mathbf{f}_n^\dagger$, it can be approximated by $E_n^g + \omega_0 + k_L^2/2M$. This assumption is equivalent to the statement that the atoms in the excited state potential will not move within the duration of the laser pulse.

The Heisenberg equations are now linear. Thus at the resonance, $\omega_L \approx \omega_0 + k_L^2/2M$, and in the rotating frame $g_n \to e^{-iE_n^g t}g_n$, $\mathbf{f}_n \to e^{-iE_n^e t}f_n$ they take the form

$$\begin{aligned}\dot{g}_n &= -i\frac{\Omega}{2}\mathcal{F}(\gamma_L t)\boldsymbol{\epsilon}_L\cdot\mathbf{f}_n,\\ \boldsymbol{\epsilon}_L\cdot\dot{\mathbf{f}}_n &= -i\frac{\Omega}{2}\mathcal{F}(\gamma_L t)g_n.\end{aligned} \tag{7}$$

These equations may be easily solved analytically for any pulse envelope

$$\begin{aligned}g_n(t) &= g_n(-\infty)\cos[A(t)] - i\boldsymbol{\epsilon}_L\cdot\mathbf{f}_n(-\infty)\sin[A(t)],\\ \boldsymbol{\epsilon}_L\cdot\mathbf{f}_n(t) &= -ig_n(-\infty)\sin[A(t)] + \boldsymbol{\epsilon}_L\cdot\mathbf{f}_n(-\infty)\cos[A(t)],\end{aligned} \tag{8}$$

with $A(t) = (\Omega/2)\int_{-\infty}^t \mathcal{F}(\gamma_L t')dt'$ and other components

of $\mathbf{f}_n$ intact. The physical picture of the discussed process is now the following: Each of the nth levels of the ground state oscillator (when populated) creates an independent wave packet $\mathbf{f}_n$. The population oscillates coherently between the ground state and nth wave packet. The system behaves as if it consists of a set of independent two-level atoms coherently driven by the laser pulse. If the area of the pulse is a multiple of 2π, the system will be left in the same state after the pulse is gone as it was before it came. Obviously, as n increases, the approximations we have made become worse, but they should hold very well for the lowest available 10^4 states of the ground state potential.

Of course, in reality the atoms will scatter photons since γ is nonzero. The resonance fluorescence (RF) from a single atom driven by a short pulse has been studied by Rzążewski and Florjańczyk [11]. They have shown the RF spectrum in such a case consists of $2K-1$ peaks, provided the pulse area is $2\pi K$. Physically, multiple splitting results from the temporal interference effects, as photons emitted during the interaction with the pulse interfere with each other. These results were then generalized to include nonzero detunings, dissipation, and various pulse shapes (hyperbolic secant, exponential pulses, chirped pulses, etc.) [12]. The total number of photons emitted in such a process is typically of the order of γ/γ_L.

To calculate the properties of the scattered light from the system of trapped atoms we come back to the full Hamiltonian (1) and evaluate perturbatively the spectrum of emitted photons as well as their total number. We assume that initially the ground state energy levels were populated according to the Bose-Einstein distribution so that the mean number of atoms in the nth state was $N_n = \mathrm{z}e^{-\beta E_n}/(1-\mathrm{z}e^{-\beta E_n})$, where $\beta = 1/kT$, z is the fugacity, and $\sum_n N_n = N$. Below critical temperature T_c, $\mathrm{z}=1$ and N_0 becomes extensive [13,14].

As in Ref. [11] we define the spectrum $C(\mathbf{k},\mu)$ as a total number of scattered photons of the momentum $\mathbf{k}$ and polarization μ,

$$C(\mathbf{k},\mu) = \lim_{t\to\infty} \langle a^\dagger_{\mathbf{k}\mu}(t) a_{\mathbf{k}\mu}(t)\rangle. \tag{9}$$

After tedious, but elementary, calculations we obtain an analytic expression for the spectrum that consists of coherent and incoherent parts,

$$C(\mathbf{k},\mu) = C_{\mathrm{coh}}(\mathbf{k},\mu) + C_{\mathrm{in}}(\mathbf{k},\mu), \tag{10}$$

where the coherent part is proportional to the modulus squared of the Fourier transform of the mean atomic polarization, and take the following form,

$$C_{\mathrm{coh}}(\mathbf{k},\mu) = S_{\mathrm{coh}}(\varpi)\left|\sum_n N_n \eta_{nn}(\mathbf{k}-\mathbf{k}_L)\right|^2, \tag{11}$$

with $\varpi = (ck-\omega_L)/\gamma_L$, whereas the incoherent part can be expressed as

$$C_{\mathrm{in}}(\mathbf{k},\mu) = S_{\mathrm{coh}}(\varpi)\sum_n \delta N_n^2 |\eta_{nn}(\mathbf{k}-\mathbf{k}_L)|^2 + S_{\mathrm{coh}}(\varpi)\sum_n\sum_{m\neq n} N_n(N_m+1)|\eta_{nm}(\mathbf{k}-\mathbf{k}_L)|^2 + NS_{\mathrm{in}}(\varpi), \tag{12}$$

where we have defined the corresponding single-atom spectra as

$$S_{\mathrm{coh,in}}(x) = \frac{3}{8\pi^2}\frac{\gamma}{\gamma_L^2}\frac{c}{k_0^2}(\boldsymbol{\epsilon}_{\mathbf{k}\mu}\cdot\boldsymbol{\epsilon}_L)^2 W_{\mathrm{coh,in}}(x), \tag{13}$$

with the normalized form factors $W's$ given below,

$$W_{\mathrm{coh}}(x) = \gamma_L^2\left|\int_{-\infty}^{\infty} e^{-ix\gamma_L t'}\cos[A(t')]\sin[A(t')]dt'\right|^2,$$
$$W_{\mathrm{in}}(x) = \gamma_L^2\left|\int_{-\infty}^{\infty} e^{-ix\gamma_L t'}\sin^2[A(t')]dt'\right|^2. \tag{14}$$

Here $S_{\mathrm{coh,in}}(\varpi)$ are the single-atom spectra which can be found in Refs. [11,12]. For a hyperbolic secant pulse $1/\cosh(\gamma_L t)$ of the area 2π, for instance,

$$W_{\mathrm{coh}}(x) = \pi^2 x^2/\cosh^2(\pi x/2),$$
$$W_{\mathrm{in}}(x) = \pi^2 x^2/\sinh^2(\pi x/2). \tag{15}$$

The incoherent spectrum (12) consists of three parts coming from quantum dispersion of the occupation numbers $\delta N_n^2 = N_n(N_n+1)$, processes of creation of the nth wave packet accompanied by annihilation of the mth one for $n\neq m$, and finally from the single-atom incoherent spectrum. The total number of emitted photons is obtained by integrating the spectrum, $N_{\mathrm{tot}} = \sum_\mu \int d^3\mathbf{k}\, C(\mathbf{k},\mu)$.

For $T > T_c$ we obtain

$$C_{\mathrm{coh}}(\mathbf{k},\mu) = S_{\mathrm{coh}}(\varpi)\left|\sum_{m=1}^{\infty}\frac{\mathrm{z}^m}{(1-e^{-m\beta\omega_t})^3}\exp\left(-\frac{1}{2}a^2(\mathbf{k}-\mathbf{k}_L)^2\coth(m\beta\omega_t/2)\right)\right|^2. \tag{16}$$

Note that the range of possible scattering angles is determined by the size of the trap and a temperature-dependent factor that results from destructive interference of different n terms in the sum entering Eq. (11). As T grows, $\mathrm{z}\to 0$ and only the first term in the sum in Eq. (16) remains relevant. We obtain then

$$C_{\rm coh}(\mathbf{k},\mu) = S_{\rm coh}(\varpi) N^2 \exp\left[-2a^2(\mathbf{k}-\mathbf{k}_L)^2/\beta\omega_t\right]. \quad (17)$$

For $T \simeq 10\ \mu$K, $\beta\omega_t \simeq 5\times 10^{-5}$ and the scattering will occur practically in the forward direction and will cover only a tiny solid angle with half angle $\leq 1.0\times 10^{-4}$. The total number of coherently scattered photons becomes then

$$N_{\rm tot} = \frac{3\pi^{3/2}}{16}\frac{\gamma}{\gamma_L} N^2 \left(\frac{\beta\omega_t}{2a^2k_L^2}\right)^{5/2}\left(\frac{\omega_L}{\gamma_L}\right)^3. \quad (18)$$

For $N = 10^8$, $N_{\rm tot}$ will be of the order of $2\%N$ for a 100 ps pulse and $(2\times 10^{-4})\%N$ for a 10 ps one. As we see, our theory is self-consistent in this high temperature limit, since $N_{\rm tot} \ll N$.

As T decreases, more and more terms in the sum over m in Eq. (16) contribute and more and more photons are emitted. The critical value of $\beta\omega_t$ for BEC may be estimated to be $(\beta\omega_t)_c \simeq (1.202/N)^{1/3} \simeq 2\times 10^{-3}$ [13,14]. Up to a numerical factor of the order of 1, the estimate (18) may still be used and $N_{\rm tot} > N$ for a 100 ps pulse, and $N_{\rm tot} = 4\%N$ for a 10 ps one. As we see, our theory is beyond the limits of its validity for 100 ps pulses. However, we call to attention the very strong dependence on a in Eq. (18), for flatter potentials with relatively larger a, the validity of the theory can be extended easily into nanosecond pulses. Note also that even at T_c most of the coherently scattered photons will be emitted in the forward direction and thus no significant scattering can be observed.

Similar conclusions hold also for the incoherent part of the spectrum, but the analysis in that case is much more complicated [7] and we will not discuss it here. The number of incoherently scattered photons is always of the order of a few percent of N or less, although they may be emitted in the full solid angle.

The situation dramatically changes when $T < T_c$. Then the spectrum will contain a new term arising from the condensate. Assuming that on the mean N_0 atoms form a coherent packet occupying the lowest energy state with $n = (0,0,0)$, we obtain for the coherent part

$$C_{\rm BEC}(\mathbf{k},\mu) = S_{\rm coh}(\varpi) N_0^2 \exp\left[-a^2(\mathbf{k}-\mathbf{k}_L)^2\right]. \quad (19)$$

As we would expect, the coherent scattering now covers a much larger solid angle with half angle $\sim 1.0\times 10^{-2}$. In a distance of 1 m from the trap the scattered photons will be about 1 cm off the optical axis. The total number of such photons also grows dramatically as N_0 grows and T becomes smaller. We obtain

$$N_{\rm tot} = \frac{\gamma}{\gamma_L} N_0^2 \frac{1}{2a^2k_L^2}. \quad (20)$$

The validity of the theory requires that $N_{\rm tot} \ll N_0$. For a 100 ps pulse this condition holds provided $N_0 \leq 10^7$. For $N = 10^8$, N_0 will in fact reach this value just below the critical temperature $[N_0/N = 1-(T/T_c)^3]$. About 10^4 photons will be scattered coherently into the solid angle $4\pi/ak_L$ as $1 - T/T_c$ becomes $\simeq 3\times 10^{-3}$. For 10 ps pulses, our theory is valid even if all the atoms were in the condensate.

To summarize, we have demonstrated that by scattering of short laser pulses of area $2\pi K$ we may detect the onset of the Bose-Einstein condensation. In the regime of validity of our theory, $2\pi K$ pulses leave the system of trapped atoms virtually unperturbed. This is particularly true for $T \geq T_c$. As T becomes smaller than T_c, angular distributions of scattered photons as well as their numbers change dramatically. Scattering of short laser pulses on systems of trapped atoms thus provides an alternative way of detecting the actual state of the system, i.e., its temperature, degree of condensation, etc.

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