Tau Polarimetry with Inclusive Decays

Eric Braaten
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208
(Received 7 April 1993)

The spin asymmetry parameter A_τ characterizing the angular distribution of the total hadron momentum in the decay of a polarized tau can be calculated rigorously using perturbative QCD and the operator product expansion. Perturbative QCD corrections to the free quark result $A_\tau = 1/3$ can be expressed as a power series in $\alpha_s(M_\tau)$ and nonperturbative QCD corrections can be expanded systematically in powers of $1/M_\tau^2$. The QCD prediction is $A_\tau = 0.41 \pm 0.02$. A measurement in agreement with this prediction would provide strong support for the precise determination of α_s from tau decay data.

PACS numbers: 13.35.+s,12.38.Bx,12.38.Lg,13.88.+e

The spin dependence of processes involving elementary particles contains a wealth of information about their fundamental interactions. Unfortunately this information is not easily accessible to experiment. It requires either the use of polarized beams and targets, or the measurement of the polarization of final state particles. The tau lepton is one of the few elementary particles whose spin can be effectively analyzed by its own decay. It is well known that several of the exclusive decay modes of the tau can be used to analyze its spin [1,2]. In this Letter, I point out that inclusive decays into hadrons can also be used for this purpose. The polarization of a sample of taus that decay into hadrons can be determined from the angular distribution of the total momentum of the hadrons. The asymmetry parameter that characterizes the angular distribution can be computed systematically using perturbative QCD and the operator product expansion. Measurements of these spin-dependent observables could provide dramatic support for the applicability of perturbative QCD to the inclusive hadronic decays of the tau.

It is convenient to normalize the inclusive decay rate of the tau lepton into a neutrino plus hadrons to the electronic decay rate by defining the ratio

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} . \tag{1}$$

For a sample of taus with polarization P, the angle θ between the total momentum of the hadrons in the tau rest frame and the spin quantization axis has a distribution proportional to $1 + A_\tau P \cos\theta$, where A_τ is an asymmetry parameter. This angular distribution can be used to separate R_τ into "forward" and "backward" components R_F and R_B:

$$\frac{dR_\tau}{d\cos\theta} = R_F \frac{1 + P\cos\theta}{2} + R_B \frac{1 - P\cos\theta}{2} . \tag{2}$$

The asymmetry parameter A_τ is then

$$A_\tau = \frac{R_F - R_B}{R_F + R_B} . \tag{3}$$

A naive estimate of the asymmetry parameter can be obtained by considering the decay of the tau into hadrons at the quark level, where it proceeds through the processes $\tau^- \to \nu_\tau d\bar{u}$ and $\tau^- \to \nu_\tau s\bar{u}$. The momentum of the $d\bar{u}$ and $s\bar{u}$ pairs can be identified with the total momentum of the hadrons. Ignoring the QCD interactions that bind the quarks into color singlet hadrons, the angular distribution of the total hadron momentum is

$$\frac{dR_\tau}{d\cos\theta} \approx \frac{3}{2}\left(|V_{ud}|^2 + |V_{us}|^2\right)\left(1 + \frac{1}{3}P\cos\theta\right) . \tag{4}$$

The squares of the Kobayashi-Maskawa matrix elements add up to 1 to high accuracy, so they will be omitted below. From (4), the naive estimates for the ratio (1) and the asymmetry parameter (3) are $R_\tau = 3$ and $A_\tau = 1/3$.

In the case of R_τ, the QCD corrections to the naive result can be computed systematically using perturbative QCD and the operator product expansion [3–5]. The perturbative corrections can be expanded as a power series in $\alpha_s(M_\tau)$ [6] and the nonperturbative corrections can be organized systematically into an expansion in powers of $1/M_\tau^2$. A thorough analysis of the QCD and electroweak corrections to the ratio R_τ has recently been carried out [7]. The methods that were used to calculate the ratio R_τ can also be used for a rigorous calculation of the asymmetry parameter A_τ. The starting point is an expression for the angular distribution of the total hadron momentum as an integral over the invariant mass s of the hadrons:

$$\frac{dR_\tau}{d\cos\theta} = 6\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \times \left[\text{Im}\Pi^{(1)}(s+i\epsilon)\left(1 + P\cos\theta + 2\frac{s}{M_\tau^2}(1 - P\cos\theta)\right) + \text{Im}\Pi^{(0)}(s+i\epsilon)(1 + P\cos\theta)\right] , \tag{5}$$

0031-9007/93/71(9)/1316(4)$06.00

where $\Pi^{(J)}(s)$, $J=0,1$ are the transverse and longitudinal correlators for the quark current that couples to the virtual W boson. The notation is the same as in Ref. [7]. The correlators $\Pi^{(J)}(s)$ are analytic functions of s except along the positive real s axis. This allows the integral in (5) to be expressed as a contour integral in the complex s plane. The contour can be deformed so that it runs counterclockwise around the circle $|s|=M_\tau^2$. The resulting expressions for the forward and backward components of R_τ defined in (2) are

$$R_F = -12\pi^2 \frac{1}{2\pi i}\int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 [\Pi^{(1)}(s) + \Pi^{(0)}(s)] \,, \tag{6}$$

$$R_B = -12\pi^2 \frac{1}{2\pi i}\int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left(2\frac{s}{M_\tau^2}\Pi^{(1)}(s)\right) . \tag{7}$$

The contour integral expressions (6) and (7) reveal that the polarization asymmetry A_τ, like the ratio R_τ, is completely determined by correlation functions at the distance scale $1/M_\tau$. This implies that the nonperturbative long distance effects of QCD can be expressed in terms of matrix elements of local operators. These matrix elements appear when the operator product expansion is used to expand the correlators $\Pi^{(J)}(s)$ in (6) and (7) in powers of $1/s$. Evaluating the contour integrals, the QCD corrections to the naive predictions $R_F=2$ and $R_B=1$ are obtained as systematic expansions in powers of $1/M_\tau^2$. There is also an important electroweak correction consisting of a multiplicative short distance factor $S_{\rm EW}=1.019$ [8]. The resulting expressions for the forward and backward components of R_τ have the form

$$R_F = 2\,S_{\rm EW}(1 + \delta_F^{(0)} + \delta_F^{(2)} + \delta_F^{(4)} + \delta_F^{(6)} + \cdots) \,, \tag{8}$$

$$R_B = S_{\rm EW}(1 + \delta_B^{(0)} + \delta_B^{(2)} + \delta_B^{(4)} + \delta_B^{(6)} + \cdots) \,, \tag{9}$$

where the fractional corrections $\delta_F^{(n)}$ and $\delta_B^{(n)}$ are proportional to $1/M_\tau^n$ with coefficients that depend logarithmically on M_τ. For $R_\tau = R_F + R_B$, the fractional corrections to the free quark value $3S_{\rm EW}$ are $(2\delta_F^{(n)}+\delta_B^{(n)})/3$.

The fractional corrections $\delta_F^{(n)}$ and $\delta_B^{(n)}$ can be calculated straightforwardly using the operator product expansions for the correlators $\Pi^{(J)}(s)$ that are collected in Ref. [7]. The dimension-0 corrections, which represent the purely perturbative effects from the interactions of massless quarks and gluons, are

$$\delta_F^{(0)} = \frac{\alpha_s}{\pi} + 5.765\left(\frac{\alpha_s}{\pi}\right)^2 + 34.48\left(\frac{\alpha_s}{\pi}\right)^3 + (d_4+165.1)\left(\frac{\alpha_s}{\pi}\right)^4 , \tag{10}$$

$$\delta_B^{(0)} = \frac{\alpha_s}{\pi} + 4.077\left(\frac{\alpha_s}{\pi}\right)^2 + 10.13\left(\frac{\alpha_s}{\pi}\right)^3 + (d_4-96.1)\left(\frac{\alpha_s}{\pi}\right)^4 , \tag{11}$$

where $\alpha_s=\alpha_s(M_\tau)$ is the running coupling constant of QCD in the modified minimal subtraction ($\overline{\rm MS}$) scheme, evaluated at the scale M_τ. The coefficient d_4 in the α_s^4 term in (10) and (11) is the fourth coefficient in the perturbative expansion of $-2\pi^2 s(d/ds)\Pi^{(1)}(s)$ in powers of α_s/π and has not been calculated. The previous coefficients are 1, 1, 1.64, and 6.37 [9]. We assign a very conservative error to this unknown coefficient: $d_4=0\pm100$. The corresponding coefficient in the fractional correction to R_τ is then 78.0 ± 100. The dimension-2 corrections are perturbative corrections due to the running quark masses. The only correction that is numerically significant comes from the strange quark mass $m_s=m_s(M_\tau)$:

$$\delta_F^{(2)} = -9\,\sin^2\theta_C\left(1+\frac{16}{3}\frac{\alpha_s}{\pi}\right)\frac{m_s^2}{M_\tau^2} , \tag{12}$$

$$\delta_B^{(2)} = -6\,\sin^2\theta_C\left(1+\frac{16}{3}\frac{\alpha_s}{\pi}\right)\frac{m_s^2}{M_\tau^2} , \tag{13}$$

where θ_C is the Cabbibo mixing angle: $\sin^2\theta_C=0.049$. For the running strange quark mass in the $\overline{\rm MS}$ scheme evaluated at the scale M_τ, we take the value $m_s(M_\tau) = 0.17\pm0.02$ GeV. The first nonperturbative corrections appear at dimension 4 in the form of scale invariant matrix elements called the gluon condensate and quark condensates:

$$\delta_F^{(4)} = 2\pi^2\left(1-\frac{11}{18}\frac{\alpha_s}{\pi}\right)\frac{\langle(\alpha_s/\pi)GG\rangle}{M_\tau^4} + 48\pi^2\frac{\langle m\bar\psi\psi\rangle}{M_\tau^4} - \frac{72}{7}\sin^2\theta_C\frac{\pi}{\alpha_s}\frac{m_s^4}{M_\tau^4} , \tag{14}$$

$$\delta_B^{(4)} = -4\pi^2\left(1-\frac{11}{18}\frac{\alpha_s}{\pi}\right)\frac{\langle(\alpha_s/\pi)GG\rangle}{M_\tau^4} . \tag{15}$$

Note that the contribution of the gluon condensate $\langle(\alpha_s/\pi)GG\rangle$ cancels to order α_s in the fractional correction $(2\delta_F^{(4)}+\delta_B^{(4)})/3$ to the ratio R_τ. This results in a suppression of the gluon condensate contribution to R_τ by 2 orders of magnitude. We take the value of the gluon condensate to be $\langle(\alpha_s/\pi)GG\rangle=(2\pm1)\times10^{-2}$ GeV^4 [10]. The matrix element $\langle m\bar\psi\psi\rangle$ in (14) is a weighted average of the quark condensates:

$$\langle m\bar\psi\psi\rangle = \frac{\langle m_u\bar uu\rangle + \cos^2\theta_C\langle m_d\bar dd\rangle + \sin^2\theta_C\langle m_s\bar ss\rangle}{2} . \tag{16}$$

Its value is $\langle m\bar{\psi}\psi\rangle = (-8\pm 1)\times 10^{-5}$ GeV^4. The inverse power of $\alpha_s(M_\tau)$ multiplying the $m_s(M_\tau)^4$ term in (14) was first understood by Broadhurst and Generalis [11]. At dimension 6, there are too many unknown matrix elements for a completely systematic treatment. Within the vacuum saturation approximation, these corrections are

$$\delta_F^{(6)} = \frac{256\pi^3}{27}\frac{\rho\alpha_s\langle\bar{\psi}\psi\rangle^2}{M_\tau^6}\,, \tag{17}$$

$$\delta_B^{(6)} = -\frac{2048\pi^3}{27}\frac{\rho\alpha_s\langle\bar{\psi}\psi\rangle^2}{M_\tau^6}\,, \tag{18}$$

with $\rho = 1$. It has been found empirically that this approximation underestimates the size of the dimension-6 correction, and it is better to treat $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$ as an effective scale-invariant operator of dimension 6, independent of $\langle\bar{\psi}\psi\rangle$. The best estimate for this parameter is $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2 = (4\pm 2)\times 10^{-4}$ GeV^6. The dimension-8 and higher corrections are assumed to be completely negligible.

Inserting the fractional corrections given above into (8) and (9), we obtain predictions for the ratio $R_\tau = R_F + R_B$ and the asymmetry parameter A_τ defined in (3) as a function of $\alpha_s(M_\tau)$ and the five parameters d_4, $m_s(M_\tau)$, $\langle(\alpha_s/\pi)GG\rangle$, $\langle m\bar{\psi}\psi\rangle$, and $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$. Alternatively, given a value for R_τ, we can predict both $\alpha_s(M_\tau)$ and A_τ. The predictions are shown in Table I. The uncertainty in $\alpha_s(M_\tau)$ is dominated by the assumed error of ± 100 in the coefficient d_4. For $R_\tau = 3.60$, the uncertainty in $\alpha_s(M_\tau)$ is 3.6%. After d_4, the next largest errors are 1.1% from $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$ and 0.4% from $m_s(M_\tau)$. The uncertainty in A_τ is dominated by the gluon condensate, and is 5.4% for $R_\tau = 3.60$. The next largest errors are 1.8% from $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$ and 0.7% from d_4.

There are two independent ways of measuring the ratio R_τ experimentally. Using the universality of electron and muon couplings, it can be expressed in terms of the electronic branching fraction B_e of the tau: $R_\tau = 1/B_e - 1.973$. Alternatively, using the universality of electron and tau couplings as well, it can be expressed in terms of the masses and lifetimes of the μ and τ: $R_\tau = (\tau_\mu/\tau_\tau)(M_\mu/M_\tau)^5 - 1.973$. The present world average for the electronic branching fraction is $B_e = (17.78\pm 0.15)\%$ [12], and it gives the ratio $R_\tau = 3.651\pm 0.047$. The present world average for the tau lifetime is $\tau_\tau = (2.96\pm 0.03)\times 10^{-13}$ s [12]. Combined with the recent precise measurement of the tau mass [13], it gives the ratio $R_\tau = 3.545\pm 0.056$. Forming the weighted average of the two independent determinations of R_τ, we get $R_\tau = 3.607\pm 0.036$. From Table I, this determines the running coupling constant at the scale M_τ to be $\alpha_s(M_\tau) = 0.319\pm 0.017$. We have added in quadrature the error from Table I and the error due to the experimental uncertainty in R_τ. Using the renormalization group to evolve the running coupling constant up to the Z^0 mass, we obtain $\alpha_s(M_Z) = 0.1176\pm 0.0021$. We evolved from M_τ up to the b quark threshold $M_b \approx 5$ GeV assuming 4 massless flavors of quarks and then up to M_Z assuming 5 massless flavors, demanding continuity of $\alpha_s(\mu)$ across the b quark threshold. The QCD prediction for the asymmetry parameter is $A_\tau = 0.413\pm 0.022$. A measurement of A_τ consistent with this prediction would provide dramatic support for the precise determination of α_s from τ decay.

Note that the precision of the predictions in Table I is about 5%, which is remarkably small for observables that involve the strong interactions. The fundamental assumption underlying the error analysis is that the operator product expansion can be applied to QCD correlation functions at the tau mass scale. Thus a measurement of A_τ would also provide a test of this crucial assumption.

To measure the asymmetry parameter A_τ, one must accumulate an unbiased sample of polarized tau decays. If the tau sample has polarization P, then the average value of the cosine of the angle θ between the spin quantization axis and the total hadron momentum in the rest frame of the decaying τ is

$$\langle\cos\theta\rangle = \tfrac{1}{3}\,A_\tau\,P\,. \tag{19}$$

At low energy e^+e^- machines (such as a tau/charm or B factory), one would have to use a golden decay mode of the τ^+, such as $\nu_\tau\pi^+$ or $\nu_\tau\rho^+$, to identify the recoiling τ^- and to measure its polarization. Unfortunately, the decay products of the τ^+ and τ^- are often overlapping in space, so assembling an unbiased sample of tau decays presents a severe loss of statistics.

At a Z^0 factory (such as SLC or LEP), it is relatively easy to obtain an unbiased sample of τ decays, because the high energy τ^+ and τ^- from the decay of the Z^0 are produced back to back with well-separated decay products. In addition, the taus are automatically produced with helicity polarization $P = -2(1-4\sin^2\theta_W) \approx -0.14$, so there is no need to measure their polarization. To determine the parameter A_τ, one must measure the invariant mass s of the hadrons produced in the decay of

TABLE I. QCD predictions for $\alpha_s(M_\tau)$ and A_τ as a function of the ratio R_τ. The errors due to variations of d_4, $m_s(M_\tau)$, $\langle(\alpha_s/\pi)GG\rangle$, $\langle m\bar{\psi}\psi\rangle$, and $\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$ have been added in quadrature.

R_τ	$\alpha_s(M_\tau)$	A_τ
3.50	0.287 ± 0.009	0.407 ± 0.023
3.52	0.294 ± 0.009	0.408 ± 0.023
3.54	0.301 ± 0.010	0.409 ± 0.023
3.56	0.308 ± 0.010	0.411 ± 0.023
3.58	0.315 ± 0.011	0.412 ± 0.023
3.60	0.321 ± 0.012	0.413 ± 0.022
3.62	0.328 ± 0.012	0.414 ± 0.022
3.64	0.334 ± 0.013	0.415 ± 0.022
3.66	0.340 ± 0.013	0.417 ± 0.022
3.70	0.352 ± 0.015	0.419 ± 0.022

the tau and also their energy fraction $z = E_H/E_\tau$, where E_H is the total energy of the hadrons in the rest frame of the Z^0 and $E_\tau = M_Z/2$ is the energy of the decaying tau. Choosing the spin quantization axis to lie along the direction of the tau momentum, the angle θ is given by [2]

$$\cos\theta = \frac{(2z-1)M_\tau^2 - s}{M_\tau^2 - s} . \tag{20}$$

Inserting this into (19), we obtain

$$\left\langle \frac{1-z}{1-s/M_\tau^2} \right\rangle = \frac{1}{2}\left(1 - \frac{1}{3}A_\tau P\right) . \tag{21}$$

If the average on the left side of (21) is taken over a sample of N tau decays, the error scales like $1/\sqrt{N}$. Thus by accumulating a large enough tau sample, it should be possible to measure A_τ with very high precision. The QCD prediction $A_\tau = 0.41 \pm 0.02$ can be clearly distinguished from the free quark value of 1/3 if the error on the quantity on the left side of (21) can be reduced to 1 part in 1000. If it could be measured with even higher precision, it could be used to measure the gluon condensate $\langle(\alpha_s/\pi)GG\rangle$, since the value of this matrix element is the dominant error in A_τ. Alternatively, if we accept the QCD prediction for A_τ, then (21) could be used to measure the polarization P with a precision of 5%, which corresponds to a 0.4% determination of $\sin^2\theta_W$.

By the same arguments that were used for R_τ and A_τ, the hadronic energy distribution dR_τ/dz can also be calculated rigorously from QCD for values of z sufficiently close to 1. Perturbative corrections can be expanded as a power series in $\alpha_s(\sqrt{z}M_\tau)$, and nonperturbative corrections can be expanded systematically in powers of $1/zM_\tau^2$. The QCD prediction for the hadronic energy distribution will be presented elsewhere [14].

In this Letter, we have introduced a new observable involving hadronic decays of the tau lepton that can be calculated rigorously from QCD. The parameter A_τ that governs the angular distribution of the total hadron momentum in the decay of a polarized tau can be calculated with the same degree of reliability as the ratio R_τ. The QCD prediction for A_τ differs from the free quark value by about 20%, which is much larger than the errors in the QCD prediction which are at the 5% level. Thus hadronic decays of the tau lepton continue to provide a remarkable laboratory in which QCD can be tested at low energies with unprecedented precision.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-91-ER40684.

[1] Y.-S. Tsai, Phys. Rev. D **4**, 2821 (1971).
[2] K. Hagiwara, A.D. Martin, and D. Zeppenfeld, Phys. Lett. B **235**, 198 (1990); B.K. Bullock, K. Hagiwara, and A.D. Martin, Durham Report No. DTP/92/36, 1992 (to be published).
[3] K. Schilcher and M.D. Tran, Phys. Rev. D **29**, 570 (1970).
[4] E. Braaten, Phys. Rev. Lett. **60**, 1606 (1988).
[5] S. Narison and A. Pich, Phys. Lett. B **211**, 183 (1988).
[6] E. Braaten, Phys. Rev. D **39**, 1458 (1989).
[7] E. Braaten, S. Narison, and A. Pich, Nucl. Phys. **B373**, 581 (1992).
[8] W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988); E. Braaten and C.S. Li, Phys. Rev. D **42**, 3888 (1990).
[9] S.G. Gorishny, A.L. Kataev, and S.A. Larin, Phys. Lett. B **259**, 144 (1991); L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. **66**, 560 (1991).
[10] S. Narison, *QCD Spectral Functions*, Lecture Notes in Physics Vol. 20 (World Scientific, Singapore, 1989).
[11] D.J. Broadhurst and S.C. Generalis, Open University Report No. OUT-4102-12, 1984 (unpublished); S.C. Generalis, J. Phys. G **15**, L225 (1989).
[12] W.J. Marciano, in *The Fermilab Meeting*, Proceedings of the 7th Meeting of the APS Division of Particles and Fields, 1992, edited by C. H. Albright *et al.* (World Scientific, Singapore, 1993).
[13] J.Z. Bai *et al.*, Phys. Rev. Lett. **69**, 3021 (1992).
[14] Eric Braaten, Northwestern report (to be published).