Proving the Low Energy Theorem of Hidden Local Symmetry

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Based on the Ward-Takahashi identity for the Becchi-Rouet-Stora symmetry, we prove to all orders of the loop expansion the low energy theorem of hidden local symmetry for the vector mesons (a version of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation) in the $U(N)_L \times U(N)_R / U(N)_V$ nonlinear chiral Lagrangian.

Hidden local symmetry (HLS) is a natural framework for describing the vector mesons in a manner consistent with the chiral symmetry of QCD [1]. The HLS Lagrangian yields at tree level a successful phenomenology for the pions and the ρ mesons. By choosing a parameter $a = 2$ in this HLS Lagrangian, we have [2] (1) universality of the ρ meson coupling [3] $g_{\rho\pi\pi} = g$ (g is the HLS gauge coupling), (2) Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [4] (version II) $m_\rho^2 = 2f_\pi^2 g_{\rho\pi\pi}^2$, and (3) ρ meson dominance of the electromagnetic form factor of the pion $g_{\gamma\pi\pi} = 0$ [3].

Most remarkably, we further obtain an a-independent relation [5] $g_\rho / g_{\rho\pi\pi} = 2f_\pi^2$, with g_ρ being the strength of ρ-γ mixing. This is actually another version of the celebrated KSRF relation (KSRF I) and is a decisive test of the HLS in the hadron physics. Since it follows from the symmetry structure alone, it was conjectured [5] to be a low energy theorem valid *at zero momenta* for *any* Lagrangian possessing the HLS and was further proved [6] at tree level. Although this is a relation among off-shell quantities, it is argued [7] that the consequence of HLS is physical when the ρ meson can be regarded as light.

If it is indeed a genuine low energy theorem surviving the loop corrections, analogs of such a relation would also be useful for strongly coupled Higgs models or models of the dynamical electroweak symmetry breaking. Actually this HLS Lagrangian can be straightforwardly applied to such models [8]. Quite recently, it was shown in Landau gauge [9] that the above "low energy theorem" as well as the $a = 2$ tree-level results at zero momenta is not altered by the one-loop corrections, thus strongly suggesting that it may indeed be a true low energy theorem. These results are actually the relations coming from $O(p^2)$ operators or dimension-2 operators (counting only dimensions of the gauge fields and the derivatives).

In this paper we shall prove that the above low energy theorem of HLS actually holds *at any loop order*, based on the Ward-Takahashi (WT) identity for the Becchi-Rouet-Stora (BRS) symmetry. By restricting ourselves to the dimension-2 operators mentioned above, we can prove it by mathematical induction technically in quite the same way as the renormalizability proof for gauge

theories [10] and two-dimensional nonlinear sigma models [11]. The proof of our statement is, however, by no means trivial: Indeed, as a gauge theory, the present system is a novel one of nonrenormalizable type. As a nonlinear sigma model also, the present system has *propagating gauge bosons*, which generate nonvanishing (in fact, divergent) loop corrections also to the dimension-2 parts in the Lagrangian, even if the dimensional regularization is used. This is in sharp contrast to the usual nonlinear sigma model without propagating gauge bosons, in which the L-loop corrections contribute only to the dimension-$2(L + 1)$ terms in the action [12–14].

Let us start with the $G_{\text{global}} \times H_{\text{local}}$ "linear model," with $G = U(N)_L \times U(N)_R$ and $H = U(N)_V$. The Lagrangian is given by [2]

$$\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{\text{kin}}(V_\mu),$$

$$\mathcal{L}_{\substack{A\\V}} = -\frac{f_\pi^2}{4}\text{tr}\left[\left(D_\mu\xi_L \cdot \xi_L^\dagger \mp D_\mu\xi_R \cdot \xi_R^\dagger\right)^2\right], \quad (1)$$

where a is a constant and $\mathcal{L}_{\text{kin}}(V_\mu)$ denotes the kinetic term of the hidden gauge boson (vector meson). Here $\xi_L(x)$ and $\xi_R(x)$ are two $U(N)$-matrix valued variables, which transform as $\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g_{L,R}^\dagger$, where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in G_{\text{global}}$. These variables are parametrized as $\xi_{L,R}(x) \equiv \exp[i\phi_{L,R}^a(x)T_a]$, where T_a denotes the $U(N)$ generator. The covariant derivatives of $\xi_{L,R}(x)$ are defined by

$$D_\mu\xi_L \equiv \partial_\mu\xi_L - iV_\mu\xi_L + i\xi_L\mathcal{V}_{L\mu}, \quad (2)$$

and similarly with replacement $L \leftrightarrow R$, $\mathcal{V}_{L\mu} \leftrightarrow \mathcal{V}_{R\mu}$, where V_μ ($\equiv V_\mu^a T_a$) is the hidden gauge boson field and $\mathcal{V}_{L\mu}$ and $\mathcal{V}_{R\mu}$ denote the external gauge fields gauging the G_{global} symmetry.

Now we consider the loop effects of the model. We assume that there exists a gauge invariant regularization (for example, the dimensional regularization). Let us take a covariant gauge condition for the HLS, and introduce the corresponding gauge-fixing (GF) and Faddeev-Popov (FP) terms:

$$\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} = B^a\partial^\mu V_\mu^a + \frac{1}{2}\alpha B^a B^a + i\bar{C}^a\partial^\mu D_\mu C^a, \quad (3)$$

where B^a is the Nakanishi-Lautrap (NL) field and C^a ($\bar{C}^a$) the FP ghost (antighost) field. In this paper we do not consider the radiative corrections due to the external gauge fields $\mathcal{V}^i_\mu \equiv (\mathcal{V}^a_{L\mu}, \mathcal{V}^a_{R\mu})$, so that we need not introduce the gauge-fixing terms for $\mathcal{V}_\mu$. Then, the corresponding ghost fields $\mathcal{C}^i \equiv (\mathcal{C}^a_L, \mathcal{C}^a_R)$ are nonpropagating.

The infinitesimal form of the $G_{\text{global}} \times H_{\text{local}}$ transformation of the Nambu-Goldstone (NG) field $\phi^i \equiv (\phi^a_L, \phi^a_R)$ is given by $\delta\phi^i = \theta^a W^i_a(\phi) + \vartheta^j \mathcal{W}^i_j(\phi) \left[\equiv \theta^A W^i_A(\phi)\right]$, where A denotes a set (a, i) of labels of H_{local} and G_{global}. Accordingly, the BRS transformation of the NG fields ϕ^i, the gauge fields $V^A_\mu \equiv (V^a_\mu, \mathcal{V}^i_\mu)$, and the FP ghost fields $C^A \equiv (C^a, \mathcal{C}^i)$ are respectively given by

$$
\begin{aligned}
\delta_B \phi^i &= C^A \hat{W}_A \phi^i \quad \left(\hat{W}_A \equiv W^i_A(\phi)\frac{\partial}{\partial\phi^i}\right), \\
\delta_B V^A_\mu &= \partial_\mu C^A + V^B_\mu C^C f_{BC}{}^A, \\
\delta_B C^A &= -\frac{1}{2} C^B C^C f_{BC}{}^A.
\end{aligned} \tag{4}
$$

We define the dimension of the fields as $\dim[\phi^a_{L,R}] = 0$, $\dim[V^A_\mu] = 1$. It is also convenient to assign the following dimensions to the FP ghosts: $\dim[C^A] = 0$, $\dim[\bar{C}^a] = 2$. Then the BRS transformation does not change the dimension. According to the above dimension counting, we may divide the Lagrangian Eq. (1) plus Eq. (3) into two parts: (1) dimension-2 part $\mathcal{L}_A + a\mathcal{L}_V$ and (2) dimension-4 part $\mathcal{L}_{\text{kin}}(V_\mu) + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}$. (We count the dimension of the fields and derivatives only, hereafter.)

Now, we consider the quantum correction to this system at any loop order, and prove the following proposition.

Proposition.—As far as the dimension-2 operators are concerned, all the quantum corrections, including the finite parts as well as the divergent parts, can be absorbed into the original dimension-2 Lagrangian $\mathcal{L}_A + a\mathcal{L}_V$ by a suitable redefinition (renormalization) of the parameters a, f^2_π, and the fields ϕ^i, V^a_μ.

This implies that the tree-level dimension-2 Lagrangian, with the parameters and fields substituted by the "renormalized" ones, already describes the exact action at any loop order, and therefore that all the low energy theorems derived from it receive no quantum corrections at all.

We prove the above proposition in the same way as the renormalizability proof for gauge theories [10] and two-dimensional nonlinear sigma models [11]. We can write down the WT identity for the effective action Γ. The NL fields B^a and the FP antighost fields $\bar{C}^a$ can be eliminated from Γ by using their equations of motion as usual. Then the tree-level action $S = \Gamma_{\text{tree}}$ reads

$$
\begin{aligned}
S[\Phi, \mathbf{K}; a] &= S_2[\phi, V] + S_4[\Phi, \mathbf{K}], \\
S_2[\phi, V] &= \int d^4x \left[a_\perp \mathcal{L}_A(\phi, V) + a_\| \mathcal{L}_V(\phi, V)\right], \\
S_4[\Phi, \mathbf{K}] &= \int d^4x \left[\mathcal{L}_{\text{kin}}(V_\mu) + \mathbf{K} \cdot \delta_B \Phi\right],
\end{aligned} \tag{5}
$$

where $\Phi \equiv (\phi^i, V^A_\mu, C^A)$ are the field variables and $\mathbf{K} \equiv (K_i, K^\mu_A, L_A)$ $[K^\mu_A \equiv (K^\mu_a, \mathcal{K}^\mu_i), L_A \equiv (L_a, \mathcal{L}_i)]$ denote the BRS source fields. We have rewritten a and f^2_π as $af^2_\pi \to a_\| f^2_\pi$ and $f^2_\pi \to a_\perp f^2_\pi$, so that the renormalization of a and f^2_π corresponds to that of $a \equiv (a_\|, a_\perp)$. According to the dimension assignment of the fields, the dimension of the above BRS source fields $\mathbf{K}$ is given by $\dim[K_i] = \dim[L_A] = 4$ and $\dim[K^\mu_A] = 3$.

The WT identity for the effective action Γ is given by

$$
\Gamma * \Gamma = 0, \tag{6}
$$

where the $*$ operation is defined by

$$
F * G = (-)^\Phi \frac{\overleftarrow{\delta} F}{\delta\Phi}\frac{\delta G}{\delta\mathbf{K}} - (-)^\Phi \frac{\overleftarrow{\delta} F}{\delta\mathbf{K}}\frac{\delta G}{\delta\Phi} \tag{7}
$$

for arbitrary functionals $F[\Phi, \mathbf{K}]$ and $G[\Phi, \mathbf{K}]$. [Here the symbols δ and $\overleftarrow{\delta}$ denote the derivatives from the left and right, respectively, and $(-)^\Phi$ denotes $+1$ or -1 when Φ is bosonic or fermionic, respectively.]

The effective action is calculated in the loop expansion: $\Gamma = S + \hbar\Gamma^{(1)} + \hbar^2\Gamma^{(2)} + \cdots$. The $\hbar^n$ term $\Gamma^{(n)}$ contains contributions not only from the genuine n-loop diagrams but also from the lower loop diagrams including the counterterms. We can expand the nth term $\Gamma^{(n)}$ according to the dimension

$$
\Gamma^{(n)} = \Gamma^{(n)}_0[\phi] + \Gamma^{(n)}_2[\phi, V] + \Gamma^{(n)}_4[\Phi, \mathbf{K}] + \cdots. \tag{8}
$$

Here again we are counting the dimension only of the fields and derivatives. The first dimension-0 term $\Gamma^{(n)}_0$ can contain only the dimensionless field ϕ^i without derivatives. The two dimensions of the second term $\Gamma^{(n)}_2$ is supplied by derivative and/or the gauge field V^A_μ. The BRS source field $\mathbf{K}$ carries dimension 4 or 3, and hence it can appear only in $\Gamma^{(n)}_4$ and beyond: the dimension-4 term $\Gamma^{(n)}_4$ is at most linear in $\mathbf{K}$, while the dimension-6 term $\Gamma^{(n)}_6$ can contain a quadratic term in K^μ_a, the BRS source of the hidden gauge boson V^a_μ. To calculate $\Gamma^{(n)}$, we need to use the "bare" action, $(S_0)_n = S\left[(\Phi_0)_n, (\mathbf{K}_0)_n; (a_0)_n\right]$, where the nth-loop-order bare fields $(\Phi_0)_n$, $(\mathbf{K}_0)_n$, and parameters $(a_0)_n$ are given by

$$
\begin{aligned}
(\Phi_0)_n &= \Phi + \hbar\delta\Phi^{(1)} + \cdots + \hbar^n\delta\Phi^{(n)}, \\
(\mathbf{K}_0)_n &= \mathbf{K} + \hbar\delta\mathbf{K}^{(1)} + \cdots + \hbar^n\delta\mathbf{K}^{(n)}, \\
(a_0)_n &= a + \hbar\delta a^{(1)} + \cdots + \hbar^n\delta a^{(n)}.
\end{aligned} \tag{9}
$$

Let us now prove the following by mathematical induction with respect to the loop expansion parameter n: (1) $\Gamma^{(n)}_0(\phi) = 0$. (2) By choosing suitably the nth-order counterterms $\delta\Phi^{(n)}$, $\delta\mathbf{K}^{(n)}$, and $\delta a^{(n)}$, $\Gamma^{(n)}_2[\phi, A]$ and the $\mathbf{K}$-linear terms in $\Gamma^{(n)}_4[\Phi, \mathbf{K}]$ can be made to vanish; $\Gamma^{(n)}_2[\phi, V] = \Gamma^{(n)}_4[\Phi, \mathbf{K}]|_{\mathbf{K}\text{-linear}} = 0$. (3) The field reparametrization (renormalization) $(\Phi, \mathbf{K}) \to [(\Phi_0)_n, (\mathbf{K}_0)_n]$ is a "canonical" transformation which

leaves the $*$ operation invariant.

Suppose that the above statements are satisfied for the $(n-1)$th-loop-order effective action $\Gamma^{(n-1)}$. We calculate, for the moment, the nth-loop effective action $\Gamma^{(n)}$ using the $(n-1)$th-loop-level bare action $(S_0)_{n-1}$, i.e., without nth-loop counterterms. We expand the $\hbar^n$ terms in the WT identity $S * \Gamma^{(n)} = -\frac{1}{2}\sum_{l=1}^{n-1}\Gamma^{(l)} * \Gamma^{(n-l)}$ according to the dimensions as in Eq. (8). Then using the above induction assumption, we find $S_4 * \Gamma_0^{(n)} + S_2 * \Gamma_2^{(n)} = 0$ (dim 0), $S_4 * \Gamma_2^{(n)} + S_2 * \Gamma_4^{(n)} = 0$ (dim 2), and $S_4 * \Gamma_4^{(n)} + S_2 * \Gamma_6^{(n)} = 0$ (dim 4). These three renormalization equations give enough information to determine possible forms of $\Gamma_0^{(n)}$, $\Gamma_2^{(n)}$, and $\Gamma_4^{(n)}|_{\text{K-linear}}$ (the K-linear term in $\Gamma_4^{(n)}$) which we are interested in.

First, the dimension-0 part of the renormalization equation reads $\delta_B \Gamma_0^{(n)} = 0$. Since there are no invariants containing no derivatives, we can immediately conclude $\Gamma_0^{(n)} = 0$, and hence our statement (1) follows.

Next, we solve the dimension-2 and the dimension-4 parts of the above renormalization equations. It is convenient to define the BRS-like transformation δ'_Γ on the fields Φ by $\delta'_\Gamma \equiv (\delta\Gamma_4^{(n)}/\delta\mathbf{K})\frac{\delta}{\delta\Phi}$. Then these equations read

$$\delta_B\Gamma_2^{(n)} + \delta'_\Gamma S_2 = 0, \tag{10}$$

$$\delta_B\Gamma_4^{(n)} + \delta'_\Gamma S_4 + \frac{\delta\Gamma_6^{(n)}}{\delta\mathbf{K}}\frac{\delta S_2}{\delta\Phi} = 0. \tag{11}$$

A tedious but straightforward analysis [15] of the K-linear term in Eq. (11) determines the general form of the $\Gamma_4^{(n)}|_{\text{K-linear}}$ and $\Gamma_6^{(n)}|_{\text{K-quadratic}}$ terms: the solution for $\Gamma_4^{(n)}|_{\text{K-linear}}$ or equivalently δ'_Γ is given by

$$\delta'_\Gamma C^a = \beta\delta_B C^a, \tag{12}$$

$$\delta'_\Gamma \phi^i = \{C^a([\hat{W}_a, \hat{F}] + \beta\hat{W}_a) + C^j[\hat{\mathcal{W}}_j, \hat{F}]\}\phi^i, \tag{13}$$

$$\delta'_\Gamma V_\mu^a = \alpha\partial_\mu C^a + \beta\delta_B V_\mu^a + \gamma\delta_B(V_\mu^a - \tilde{V}_\mu^a), \tag{14}$$

where α, β, and γ are constants, $\tilde{V}_\mu \equiv \xi_L \mathcal{V}_{L\mu}\xi_L^\dagger - i\partial_\mu\xi_L \cdot \xi_L^\dagger + (L \leftrightarrow R)$ and $\hat{F} \equiv F^i(\phi)\partial/\partial\phi^i$, with $F^i(\phi)$ being a certain dimension-0 function. Note that $\delta'_\Gamma \mathcal{V}_\mu^i = \delta'_\Gamma C^i = 0$, since the external G_{global}-gauge fields $\mathcal{V}_\mu^i$ and their ghosts C^i are not quantized and hence their BRS source fields $\mathcal{K}_i^\mu$ and $\mathcal{L}_i$ appear only in the tree action.

Using δ'_Γ thus obtained, we next solve the above WT identity Eq. (10) and easily find

$$\Gamma_2^{(n)} = A_{2\text{GI}}[\phi, V] - \left(\hat{F}S_2 + \alpha V_\mu^a\frac{\delta}{\delta V_\mu^a}S_2\right), \tag{15}$$

where $A_{2\text{GI}}$ is a dimension-2 gauge-invariant function of ϕ^i and V_μ^A.

The solutions are combined into a simple form

$$\Gamma_2^{(n)} + \Gamma_4^{(n)}\Big|_{\text{K-linear}} = A_{2\text{GI}}[\phi, V] - S * Y \tag{16}$$

up to irrelevant terms (dimension-6 or K-independent dimension-4 terms), where the functional Y is given by

$$Y = \int d^4x\big[K_i F^i(\phi) + \alpha K_a^\mu V_\mu^a$$
$$+ \beta L_a C^a + \gamma f_{abc}K_a^\mu K_{b\mu}C^c\big]. \tag{17}$$

Now, we prove our statements (2) and (3) in the above. We have calculated the above effective action $\Gamma^{(n)}$ without using nth-loop-level counterterms $\delta\Phi^{(n)}$, $\delta\mathbf{K}^{(n)}$, and $\delta a^{(n)}$. If we include those, we have the additional contribution given by

$$\Delta\Gamma^{(n)} = \delta\Phi^{(n)}\frac{\delta S}{\delta\Phi} + \delta\mathbf{K}^{(n)}\frac{\delta S}{\delta\mathbf{K}} + \delta a^{(n)}\frac{\partial S}{\partial a}, \tag{18}$$

where $S[\Phi, \mathbf{K}; a]$ is the tree-level action. So the true nth-loop-level effective action is given by $\Gamma^{(n)} + \Delta\Gamma^{(n)} \equiv \Gamma^{(n)}_{\text{total}}$. The tree-level action S_2 is the most general gauge-invariant dimension-2 term, so that the $A_{2\text{GI}}[\phi, V]$ term in Eq. (16) can be canceled by suitably chosen counterterms, $\delta a^{(n)}\frac{\partial S}{\partial a}$. The second term $-S * Y$ in Eq. (16) just represents a "canonical transformation" of S generated by $-Y$. Therefore we choose the nth-order field counterterms $\delta\Phi^{(n)}$ and $\delta\mathbf{K}^{(n)}$ to be equal to the canonical transformations of Φ and $\mathbf{K}$ generated by $+Y$; $\delta\Phi^{(n)} = \Phi * Y$, $\delta\mathbf{K}^{(n)} = \mathbf{K} * Y$. Then the first and the second terms in Eq. (18) just give $S * Y$ and precisely cancel the second term in Eq. (16). Thus we have completed the proof of our statements (2) and (3).

In conclusion, we have shown in the covariant gauges that our tree-level dimension-2 action $\int d^4x(\mathcal{L}_A + a\mathcal{L}_V)$, if written in terms of renormalized parameters and fields, already gives the exact action Γ_2 including all the loop effects. This form of the effective action (in particular the $\mathcal{L}_V$ part) implies that the previously derived relation [5,6]

$$\frac{g_V(p^2)}{g_{V\pi\pi}(p^2, p_{\pi_1}^2 = p_{\pi_2}^2 = 0)}\bigg|_{p^2=0} = 2f_\pi^2 \tag{19}$$

is actually an exact low energy theorem valid at any loop order. Of course, this theorem concerns off-shell quantities at $p^2 = 0$, and hence is not physical as it stands. However, as Georgi [7] discussed, the consequence of HLS is physical when the vector meson can be regarded as light (e.g., in the "vector limit"). Suppose that the vector mass $m_V^2 = ag^2f_\pi^2$ is sufficiently small compared with the characteristic energy scale Λ^2 of the system, which is customarily taken as $\Lambda^2 \sim 16\pi^2 f_\pi^2$. Then we expect that the on-shell value of $g_V/g_{V\pi\pi}$ at $p^2 = m_V^2$ can deviate from the left-hand side of Eq. (19) only by a quantity of order $m_V^2/\Lambda^2 \sim ag^2/16\pi^2$, since the contributions of the dimension-4 or higher terms in the effective action Γ (again representing all the loop effects) are suppressed by a factor of p^2/Λ^2 at least. Therefore as far as the vector mass is light, namely, when either a or $g^2/16\pi^2$ is small, our theorem is truly a physical one. In the actual world of

QCD, the ρ meson mass is not so light ($ag^2/16\pi^2 \sim 1/2$) so that the situation becomes a bit obscure. Nevertheless, the fact that the KSRF I relation $g_\rho/g_{\rho\pi\pi} = 2f_\pi^2$ holds on the ρ mass shell with good accuracy strongly suggests that the ρ meson is the hidden gauge field and *the KSRF I relation is a physical manifestation of our low energy theorem.*

Some comments are in order.

(1) Our conclusion in this paper remains unaltered even if the action S contains other dimension-4 or higher terms, as far as they respect the symmetry. This is because we needed just $(S * \Gamma)_2$ and $(S * \Gamma)_4 |_{\text{K-linear}}$ parts in the WT identity to which only S_2 and K-linear part of S_4 can contribute.

(2) When we regard this HLS model as a low energy effective theory of QCD, we must take account the anomaly and the corresponding Wess-Zumino-Witten term Γ_{WZW}. The WT identity now reads $\Gamma * \Gamma = $ (anomaly). However, the RHS is saturated already at the tree level in this effective Lagrangian and so the WT identity at loop levels, which we need, remains the same as before. The WZW term Γ_{WZW} or any other intrinsic-parity-odd terms [16] in S are of dimension 4 or higher and hence do not change our conclusion as explained above.

(3) Since the low energy theorem concerns off-shell quantities, we should comment on the gauge choice. In the covariant gauges which we adopted here, the G_{global} and H_{local} BRS symmetries are separately preserved. Accordingly, the V_μ field is multiplicatively renormalized (recall that $\delta V_\mu^{(n)} = V_\mu * Y = \alpha V_\mu$), and the above (off-shell) low energy theorem Eq. (19) holds. However, if we adopt R_ξ gauges (other than Landau gauge), these properties are violated; for instance, $\phi \partial_\mu \phi$ or the external gauge field $\mathcal{V}_\mu$ gets mixed with our V_μ through the renormalization, and our off-shell low energy theorem Eq. (19) is violated. This implies that the V_μ field in the R_ξ gauge generally does not give a smooth off-shell extrapolation; indeed, in R_ξ gauge with gauge parameter $\alpha \equiv 1/\xi$, the correction to $g_\rho/g_{\rho\pi\pi}$ by the extrapolation from $p^2 = m_\rho^2$ to $p^2 = 0$ is seen to have a part proportional to $\alpha g^2/16\pi^2$, which diverges when α becomes very large. Thus, in particular, the unitary gauge [17], which corresponds to $\alpha \to \infty$, gives an ill-defined off-shell field.

(4) Our argument is free from infrared divergences at least in Landau gauge. This can be seen as follows. In this gauge the propagators of the NG bosons, the hidden gauge bosons, and the FP ghosts (after rescaling the FP antighost $\bar{C}$ into $f_\pi^2 \bar{C}$) are all proportional to $1/f_\pi^2$ in the infrared region. Therefore, a general L-loop diagram, which includes V_4 dimension-4 vertices and K BRS source vertices, yields an amplitude proportional to $(1/f_\pi^2)^{(L-1+V_4+K)}$ [12]. Thus, from dimensional consid-

eration we see that there is no infrared contribution to $\Gamma_0^{(n)}[\phi]$, $\Gamma_2^{(n)}[\phi, V]$, and $\Gamma_4^{(n)}[\Phi, K]|_{\text{K-linear}}$. In other covariant gauges, there appears a dipole ghost in the vector propagator, which is to be defined by a suitable regularization.

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