Evidence for Nonconventional Vortex Dynamics in an Ideal Two-Dimensional Superconductor

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Impedance measurements performed on a weakly frustrated triangular array of Josephson junctions over a wide range of frequencies and at temperatures such that vortex pinning is irrelevant reveal that vortex dynamics in an ideal two-dimensional (2D) superconductor does not obey Drude's classical prediction for a 2D Coulomb gas of free and independent vortex charges. An analysis in terms of a complex vortex dielectric constant implies that the vortex mobility vanishes logarithmically in the limit of small frequencies, thereby pointing to anomalous vortex diffusion.

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The concept of vortex is essential to understand the physics of two-dimensional (2D) superfluids [1]. In neutral superfluids and, under appropriate conditions, in charged superfluids the interaction between two vortices depends logarithmically on their separation, a feature leading to a natural description of the vortex medium in terms of a 2D Coulomb gas analog [2]. Detailed insight into the physics of 2D superfluids emerging from this picture is provided by studies of their response to a time-dependent perturbation. The Andronikashvili torsional oscillator has proven to be quite successful to investigate vortex dynamics near the Kosterlitz-Thouless (KT) transition of liquid-helium films [3]. A corresponding probe for 2D superconductors relies on a two-coil mutual inductance technique [4] which allows us to extract the dynamical properties of the vortices from measurements of the sample's complex sheet impedance [5,6]. However, the experiments performed so far on liquid-helium films could not systematically explore the response as a function of frequency, while the investigations carried out on superconducting films [7] were almost unvariably affected by pinning effects masking the intrinsic 2D Coulomb-gas properties of the vortex medium. In this Letter, we report a study of the complex dielectric constant $\varepsilon(\omega)$ of a dilute system of vortices created by a small perpendicular magnetic field H in an almost pinning-free triangular array of Josephson junctions. Our data, taken over a wide range of driving angular frequencies ω, reveal novel and unexpected aspects of the dynamics of vortex excitations in an ideal, i.e., pinning-free, 2D superconductor.

Compared to superconducting films, Josephson junction arrays (JJA) prepared with modern microfabrication techniques provide nearly ideal systems in which vortex pinning due to ever present disorder can be kept at extremely low levels. Moreover, in *triangular* arrays intrinsic pinning effects resulting from the periodic nature of the system are much weaker than in other lattice structures and become totally irrelevant at temperatures T appreciably lower than the zero-field KT transition temperature T_{KT} [8]. Thus, if T is not too far below T_{KT} and H corresponds to small values (<0.05) of the frustration parameter f, defined as the number of flux quanta per elementary triangular cell, one would expect the vortex medium in a triangular JJA to behave as a 2D Coulomb gas of free (i.e., unpinned) and independent (i.e., noninteracting) charges with $\varepsilon(\omega)$ obeying Drude's classical prediction [9]. Surprisingly, however, we find that $\varepsilon(\omega)$, as inferred from measurements of the linear complex sheet impedance $Z=R+i\omega L$ of the array, is highly nonconventional at low frequencies. More precisely, the low-frequency superfluid component, $\mathrm{Re}[1/\varepsilon(\omega)]$, of $1/\varepsilon(\omega)$ is found to be proportional to ω, in striking contrast with the quadratic frequency dependence predicted by Drude's theory. This unusual dynamic response implies that the vortex mobility $\mu_v(\omega)$ vanishes as $1/|\ln\omega|$ in the limit $\omega\rightarrow 0$, a feature pointing to anomalously slow ("sluggish") vortex diffusion.

Although our results, in particular the logarithmic frequency dependence of $\mu_v(\omega)$, suggest a few attractive, though speculative, theoretical interpretations, the exact nature of the microscopic mechanism responsible for anomalous vortex transport has not been identified yet. We notice, however, that features identical to those observed in our measurements have been seen [10] in numerical simulations, based on a time-dependent Ginzburg-Landau approach, of the dynamic response of thermally nucleated free vortices in unfrustrated ($f=0$) square arrays. This supports the idea that anomalous vortex diffusion is a general *intrinsic* property of ideal 2D superfluids.

The JJA studied in this work is a system of $\sim 10^6$ *SNS* junctions consisting of star-shaped superconducting (S) Pb islands at the sites of a triangular lattice with lattice spacing $a=15$ μm and proximity effect coupled to each other by an underlying Cu normal (N) layer. In zero magnetic field the array undergoes a KT transition at $T_{\mathrm{KT}}\approx 3.70$ K. Real and imaginary parts of Z were extracted [4] from measurements of the mutual inductance change of a drive-receive coil system induced by the supercurrents flowing in the JJA in response to a weak ac

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field creating a maximum rms flux smaller than $10^{-4}\phi_0$ per unit cell in the center of the array (ϕ_0 is the superconducting flux quantum). Combined with a suppression of ambient magnetic fields to ~ 1 mG, this low-level excitation allowed f to be tuned with a precision better than 10^{-3}. Data were taken over three decades in frequency, from 10 Hz to 10 kHz, with a SQUID-operated ac bridge allowing one to detect inductance changes of the order of 0.1 pH over most of the spectral range. In the following, temperatures will be expressed in terms of the reduced temperature $\tau = kT/E_J(T)$, where the Josephson coupling energy $E_J(T) = (\phi_0/2\pi)^2/[L_k(T)\sqrt{3}]$ was inferred from measurements of the "bare" sheet kinetic inductance $L_k(T)$ of the unfrustrated ($f=0$) array at temperatures well below $T_{\rm KT}$ [6].

In Fig. 1 the inverse inductive component L^{-1}, proportional to the effective (i.e., renormalized by various kinds of topological excitations) areal superfluid density in the system, is shown as a function of f at $\tau = 0.09$ ($T = 2.65$ K). The extraordinary richness of the fine structure with sharp peaks corresponding to commensurate states at $f=0$, $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{12}$, and $\frac{1}{2}$ and incipient higher-order maxima at $\frac{3}{7}$ and $\frac{7}{16}$ is a striking demonstration of the excellent uniformity of the array. With increasing temperature and/or decreasing frequency the fine structure in $L^{-1}(f)$ sharpens quite dramatically into a sequence of δ-function-like peaks [11]. We interpret this behavior as clear evidence that, at sufficiently high temperatures, phase coherence in the neighborhood of a commensurate state (where the vortex lattice is pinned) is drastically disrupted by excess (or missing) vortices moving *freely* on a pinned vortex background. Near $f=0$, this is consistent with the observation that at $\tau = 0.5$ ($T = 3.27$ K), the temperature at which the impedance measurements reported below were performed, the escape probability $I_0^{-2}(\Delta/2kT)$ of a single vortex over the potential-energy barrier $\Delta(T) = 0.043E_J(T)$ opposing its motion in a triangular JJA [8] is very close to 1 [$I_0^{-2}(0.043/2\tau) = 0.999$, $I_0(x)$ is the modified Bessel function of order zero]. In the following, we focus on the central ($f=0$) quantum structures in $R(f)$ and $L(f)$ and study in detail how they evolve with frequency in the pinning-free regime at $\tau = 0.5$. Notice that, since τ is appreciably lower than $\tau_{\rm KT} \approx 1.5$, effects due to thermally nucleated free vortices can be ignored.

Real and imaginary parts of $Z(f)$ measured at three different frequencies in a narrow range ($|f| < 0.05$) of frustrations centered about $f=0$ are shown in Fig. 2. Surprisingly, if f is not too small, $R(f)$ increases with ω, reaching saturation only near the upper limit of our spectral range. This is clearly inconsistent with the frequency independent flux-flow prediction $R(f) \approx R_n f$ for free vortices (R_n is the normal-state sheet resistance). Moreover, with decreasing frequency $R(f)$ and, in a more limited interval ($|f| < 0.005$, see inset), $L(f)$ exhibit growing negative curvature resulting in appreciable deviations from linearity. This shows that, even in the absence of pinning, vortex correlations are important, presumably because of the long-range logarithmic interaction be-

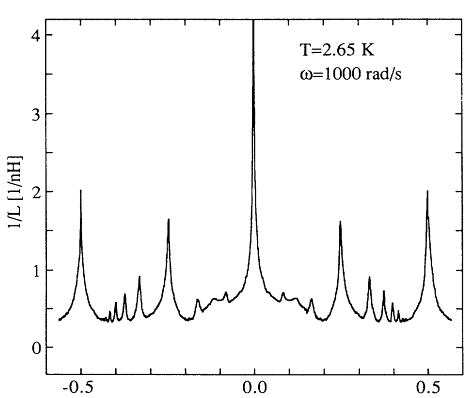

FIG. 1. Inverse sheet inductance of the triangular Josephson junction array as a function of frustration. See text for peak assignment to rational values of f.

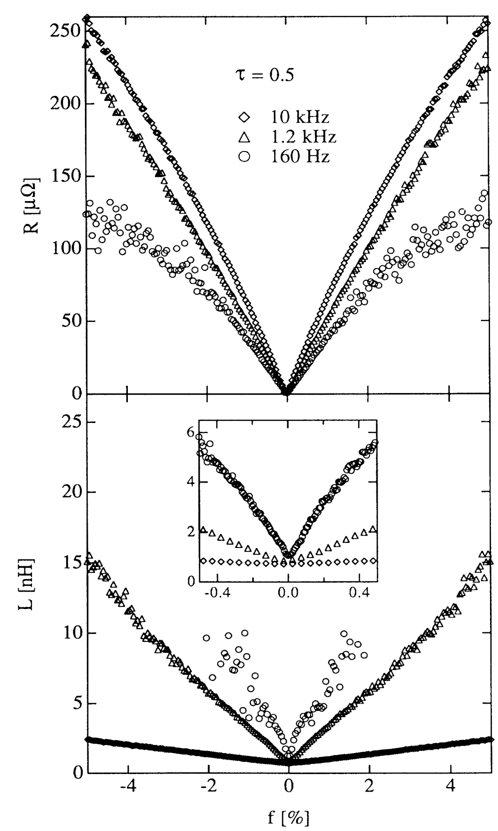

FIG. 2. Sheet resistance and sheet inductance of the triangular JJA at three different frequencies as a function of frustration in the range $|f| < 0.05$. The inset shows the $L(f)$ data in the interval $|f| < 0.005$.

tween the vortices. Notice, however, that $R(f)$ becomes linear and almost independent of ω in the limit $f \to 0$, as one expects for a plasma of noninteracting free vortices.

Deeper insight into this unexpected behavior is provided by an analysis of the impedance data near $f=0$ in terms of a complex dielectric constant $\varepsilon(\omega,f)$ defined by $Z(\omega,f)=i\omega L_k \varepsilon(\omega,f)$ [6,7]. According to Drude's prediction

$$\varepsilon(f,\omega)=1+2\pi(\sigma_0/i\omega) \tag{1}$$

for a 2D Coulomb plasma of free and independent vortices characterized by a *frequency independent* conductivity σ_0 proportional to the vortex density [9], i.e., $\sigma_0 \sim f$, $\varepsilon(\omega,f)$ is expected to be a function of f/ω only. Using $L_k(\tau=0.5)=0.69$ nH, the superfluid component $\mathrm{Re}(1/\varepsilon)$, which is the dynamical analog of the helicity modulus [2], and the dissipative component $\mathrm{Im}(1/\varepsilon)$ of $1/\varepsilon$ were inferred from a collection of impedance data taken at four different frequencies in the range $|f|<0.05$. The results are shown in Fig. 3 as a function of f/ω on a log-log plot. Within experimental accuracy [scattering of the data at large f/ω values arises from the extreme sharpness of the $f=0$ structure in $L(f)$ at the lowest driving frequencies], scaling of the data with f/ω is indeed observed over more than four decades in f/ω.

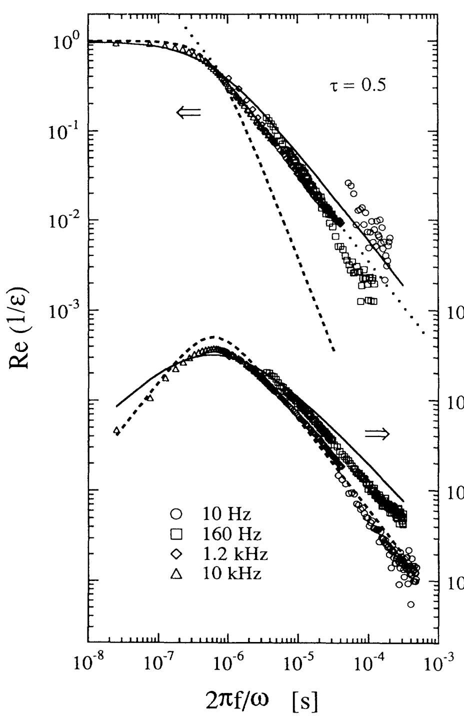

FIG. 3. Real and imaginary parts of the inverse complex vortex dielectric constant vs ratio of frustration to frequency on a log-log plot. Solid curve: Minnhagen's model; dashed curve: Drude's model; dotted straight line: $(f/\omega)^{-1}$ fit to the $\mathrm{Re}(1/\varepsilon)$ data at large f/ω values.

However, the log-log plot immediately reveals that, at large values of f/ω, $\mathrm{Re}(1/\varepsilon)$ scales as $(f/\omega)^{-1}$, in striking contrast to the $(f/\omega)^{-2}$ dependence predicted by Eq. (1). This unusual response of the vortex medium at low frequencies implies that $\mathrm{Re}(1/\varepsilon)$ is nonanalytical at $\omega=0$, thereby pointing to anomalous vortex diffusion. To see this in more detail, we begin by noticing that the functional dependence on f/ω emerging from the $\mathrm{Re}(1/\varepsilon)$ data of Fig. 3 can be described, to a first approximation, by an expression of the form

$$\mathrm{Re}(1/\varepsilon)=[1+(\omega_0/\omega)]^{-1}, \tag{2}$$

where ω_0 is an as yet unspecified characteristic frequency proportional to f. We pretend Eq. (2) is rigorous only at low frequencies ($\omega \ll \omega_0$) and provides a reasonable extrapolation to high frequencies. Recalling that $\mathrm{Im}(1/\varepsilon)$ is related to $\mathrm{Re}(1/\varepsilon)$ by a Kramers-Kronig relation, it follows [2] that

$$\mathrm{Im}(1/\varepsilon)=(2/\pi)(\omega_0/\omega)\ln(\omega_0/\omega)/[(\omega_0/\omega)^2-1]. \tag{3}$$

It is then straightforward to show that $\varepsilon(f/\omega)$, as given by Eqs. (2) and (3), can be cast into the Drude form Eq. (1), however, with σ_0 replaced by a *frequency dependent* complex vortex conductivity $\sigma_v(\omega)$. Expressing $\sigma_v(\omega)$ in terms of the vortex charge $q_v=(\pi E_J)^{1/2}$ [1,2], of the areal vortex density $n_v=4f/(a^2\sqrt{3})$ and of the complex vortex mobility $\mu_v(\omega)$ as $\sigma_v(\omega)=q_v^2\mu_v(\omega)n_v$, we find at low frequencies ($\omega \ll \omega_0$)

$$\mu_v(f/\omega)=(\pi/2)\mu_0[1+i(\pi/2)/\ln(\omega_0/\omega)]/\ln(\omega_0/\omega), \tag{4}$$

where $\mu_0=\omega_0/2\pi q_v^2 n_v$ is the (constant) mobility of an isolated vortex. Thus, the nonconventional scaling properties of $\varepsilon(f/\omega)$ imply that, for a given f, $\mu_v(\omega)$ vanishes as $1/|\ln\omega|$ in the limit $\omega \to 0$, thereby showing that vortex diffusion becomes anomalously slow (or "sluggish") at large time scales. In view of the fact that this remarkable feature emerges from the study of a pinning-free system, we believe it to be a general intrinsic property of ideal 2D superconductors, an interpretation corroborated by recent simulations [10] of the dynamics of thermally nucleated free vortices above T_{KT} in square arrays at zero frustration.

Additional evidence for anomalous vortex diffusion is provided by a quantitative analysis of the data of Fig. 3 based on Eqs. (2) and (3). Using the Bardeen-Stephen relation $\mu_0 \approx R_n a^2/\phi_0^2$ appropriate for arrays [6,8,12], we find $\omega_0=C(2/\sqrt{3})(2\pi/\phi_0)^2 R_n(kT/\tau)f$, where the numerical factor C, the only adjustable parameter of the model, is expected to be somewhat larger than 1. With this expression for ω_0, Eqs. (2) and (3) become identical to those obtained by Minnhagen [2] in his phenomenological treatment of Coulomb-gas dynamics. Using $R_n=2$ mΩ, as inferred from resistive measurements, and $C=5.45$ to fit the position of the maximum in $\mathrm{Im}(1/\varepsilon)$ at $\omega=\omega_0$, we find (see Fig. 3) that Minnhagen's approach provides a good description of $\mathrm{Re}(1/\varepsilon)$ and, with the exception of the high-frequency range ($\omega \gg \omega_0$) where the model predictions are expected to be less accurate, also of $\mathrm{Im}(1/\varepsilon)$. For comparison, in Fig. 3 we also show the result of a calculation based on Drude's model [Eq. (1)]. Notice that, if the impedance is expressed as $Z \approx \phi_0^2 \mu_v n_v$ [12], Eq. (4) qualitatively accounts for the frequency dependence of the impedance data of Fig. 2, in particular for the weak negative curvature observed at low frequencies.

At the present level of understanding, we can only speculate about the nature of the microscopic mechanisms responsible for sluggish vortex motion. It is well known that diffusion in disordered systems does not obey the classical laws thereby leading to anomalous transport properties [13]. However, on account of the extreme uniformity of the array emerging from Fig. 1, we exclude disorder-induced pinning as a possible source of anomalous vortex diffusion. The key feature of randomness needed to explain the *nonanalytic* behavior of $1/\varepsilon$ would be some very peculiar hierarchical distribution of energy barriers [13], a requirement difficult to reconcile with the exceptional richness of the fine structure of $Z(f)$. The only conceivable pinning mechanism operating in our system at $\tau = 0.5$ is edge pinning [12] due to the finite size of the sample, but it appears to be unlikely that size effects could significantly affect vortex dynamics in experiments relying on the two-coil technique [4]. A more plausible mechanism which could intrinsically reduce the vortex mobility at low frequencies results from the coupling of the vortices to the 2D acoustic modes (the spin-wave excitations of the XY model) of the phases associated with the superconducting islands of the array. This mechanism has been studied in detail in connection with the dissipation generated by moving "massive" vortices in the quantum regime [14], but it should also be relevant to determine the friction of massless vortices in the overdamped classical regime studied in this work. In particular, it reproduces the low-frequency behavior $\mu_v(\omega) \sim 1/|\ln\omega|$ in two dimensions [15]. Moreover, if one takes into account vortex correlations through a liquidlike vortex structure factor with a main peak at a wave number related to the mean vortex distance, this model also predicts the scaling of μ_v with f/ω, thereby providing an explanation of the nonlinearity of $Z(f)$ shown in Fig. 2.

In conclusion, ac impedance measurements performed over a wide range of frequencies on an almost pinning-free triangular array of Josephson junctions exposed to a weak magnetic field have disclosed novel and unexpected features of vortex dynamics in an ideal two-dimensional superconductor. Our analysis in terms of a complex vortex dielectric constant implies that, in striking contrast with Drude's classical prediction for a two-dimensional Coulomb gas of free and independent vortex charges, the vortex mobility vanishes logarithmically in the limit of small frequencies, thereby pointing to anomalous vortex diffusion.

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