Scaling of the Hall Resistivity in High-T_c Superconductors

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A scaling behavior of the Hall resistivity in a mixed state of high-T_c superconductors is shown to be a general feature of any vortex state with disorder-dominated dynamics (thermally assisted flux flow, vortex glass, etc.). The universal scaling law $\rho_{xy} \propto \rho_{xx}^2$ is found. The presented theory agrees with recent experimental data.

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One of the most striking features of the vortex motion in the oxide superconductors is the behavior of the Hall resistivity which has attracted intense recent attention. The Hall resistivity has been found to undergo a sign change when cooling the sample down from the normal state [1-7]. Moreover a second change of sign back from a negative to a positive resistivity has been discovered in strongly anisotropic materials [4,8]. Furthermore, a puzzling scaling behavior of the Hall versus longitudinal resistivities has been observed as a function of temperature [7]: $\rho_{xy} \propto \rho_{xx}^{\beta}$ with $\beta \approx 1.7$.

The scaling behavior of the Hall resistivity has been attributed to the flux pining [7], whereas the origin of the sign change generated a variety of possible explanations [5,9-11], but which one is the most preferable to accept is presently unclear.

Very interesting ideas to explain the scaling behavior of the Hall resistivity have been put forward by Dorsey and Fisher [12]. In particular, the Hall effect itself was attributed to a "particle-hole" asymmetry. The scaling behavior of the Hall against the longitudinal resistivity has been explained in terms of the more general picture of the glassy scaling near the vortex-glass transition. They related the exponent β to the specially introduced particle-hole asymmetry exponent λ_v, which has to be chosen $\lambda_v \approx 3$ in order to produce $\beta \approx 1.7$.

In this paper we calculate the effect of flux pinning on the Hall resistivity and show that the pinning straightforwardly gives rise to the scaling $\rho_{xy} \propto \rho_{xx}^{\beta}$, with $\beta = 2$. Such a scaling law arises in a natural way without the need to invoke the hypothesis of the vortex-glass scaling. The scaling law is universal, independent of the specific vortex structure, whether it is a vortex liquid or a vortex glass. The considerations below suggest that the scaling behavior in the Hall resistivity resulting from pinning is of a different origin than its change of sign. This is well illustrated by the recent experimental data of Samoilov [8], who observed the scaling behavior $\rho_{xy} \propto \rho_{xx}^2$ in the regime of the *positive* Hall effect. Combination of this finding with the data of Luo *et al.* [7], who observed a scaling behavior in the regime of the *negative* Hall resistivity, seems to rule out a possible link between scaling and sign change of the Hall resistivity.

We start with the equation of the momentum balance for a stationary moving vortex line in the absence of the quenched disorder [13]:

$$\eta \mathbf{v}_L + \alpha \mathbf{v}_L \times \mathbf{n} = \frac{\Phi_0}{c} \mathbf{j} \times \mathbf{n} . \tag{1}$$

Here, $\mathbf{j} = e n_s \mathbf{v}_s$ is the transport current density, $\mathbf{v}_L$ is the velocity of the vortex with respect to the crystal lattice, $\mathbf{v}_s$ and n_s are the superfluid velocity and the density of superconducting electrons, respectively, $\mathbf{n}$ is the unit vector along the vortex line, and $\Phi_0 = \pi \hbar c/e$ is the flux quantum. The friction coefficient η can be estimated by means of the Bardeen-Stephen formula $\eta = \Phi_0 H_{c2}/c^2 \rho_n$, where ρ_n is the normal state resistivity. The coefficient α determines the sign and the magnitude of the Hall angle Θ_H via the relation $\tan\Theta_H = \alpha/\eta$. The explicit expression for α has to be found from microscopic considerations; it is the particular behavior of α that determines, for example, such an effect as the sign change of the Hall resistivity [10]. In what follows we explore the phenomenological level of the problem and the specific form of α is irrelevant.

Let us discuss briefly the origin of the different terms in Eq. (1). The forces acting on the vortex line arise either from the interaction with the underlying crystal lattice (depending, thus, upon $\mathbf{v}_L$) or from the interaction with the superfluid flowing by. In the latter case the force is determined by the relative velocity $\mathbf{v}_L - \mathbf{v}_s$; the Lorentz force acting on the individual vortex is, therefore, $(e/c) n_s \Phi_0 (\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{n}$ (when writing down the force balance the part proportional to $\mathbf{v}_L \times \mathbf{n}$ has been extracted from the Lorentz force and then combined with the drag force acting on the vortex line).

Note that the force balance equation does not include a term like $\eta' \mathbf{v}_s$, which can be easily perceived from the following *Gedanken experiment.* Let us consider a vortex line in a superconducting cylinder ($z \parallel \mathbf{n}$) exposed to a circular current flowing in the plane perpendicular to z. Moving the vortex adiabatically along a closed loop around the cylinder (i.e., along the flowing current) leaves the energy of the system unchanged, hence no force with the component $\eta' \mathbf{v}_s$ is exerted on the vortex line. An alternative argument is the following. Let us assume the force $\eta' \mathbf{v}_s$ to exist and the vortex to be at rest in the lattice frame of reference (i.e., to be pinned strongly). Then the counterforce $-\eta' \mathbf{v}_s$ exerted by the vortex on the superfluid would decelerate the superflow giving rise to the dissipation of energy in an obvious conflict with the original assumption about the fixed at-rest vortex line. Another point to be made is that although the formal solution to Eq. (1) can be written in the form

$$\left(1+\frac{\alpha^2}{\eta^2}\right)\eta \mathbf{v}_L=\frac{\Phi_0}{c}\mathbf{j}\times\mathbf{n}+\frac{\Phi_0}{c}\frac{\alpha}{\eta}\mathbf{j}\,,$$

the last term in the right-hand side should not be interpreted as the force acting on the vortex.

As a next step we consider the system of interacting vortices in the presence of the quenched disorder and thermal noise. By adding the terms describing interactions, $\mathbf{F}_{\mathrm{int},i}$, pinning, $\mathbf{F}_{\mathrm{pin},i}$, and thermal noise, $\mathbf{F}_{T,i}$ the force balance equation for a given ith vortex acquires the form

$$\eta \mathbf{v}_i+\alpha \mathbf{v}_i\times\mathbf{n}=\frac{\Phi_0}{c}\mathbf{j}\times\mathbf{n}+\mathbf{F}_{\mathrm{int},i}+\mathbf{F}_{\mathrm{pin},i}+\mathbf{F}_{T,i}\,. \qquad (1\mathrm{a})$$

Note that η and α here are the bare quantities introduced above for the pure system. We show below that pinning renormalizes the friction η leaving the Hall coefficient α unchanged.

On averaging Eq. (1) over disorder, thermal fluctuations, and vortex positions one arrives at

$$\eta \mathbf{v}+\alpha \mathbf{v}\times\mathbf{n}=\frac{\Phi_0}{c}\mathbf{j}\times\mathbf{n}+\langle F_{\mathrm{pin},i}\rangle\,, \qquad (2)$$

where $\mathbf{v}$ denotes the average velocity of vortices. The averaged interaction force is zero due to Newtons' third law. The determination of the average pinning force is a highly nontrivial issue. This problem has been first investigated by means of a perturbation theory with respect to disorder [14,15]. The central point in the discussion of the resistivity scaling is the question of the *direction* of the averaged pinning force. The pinning force is the gradient of the random potential. This random potential is determined by the relative positions of the vortices with respect to the pins and is invariant under time reversal. Hence, the averaged pinning force is invariant under reversal of the magnetic field. Then, after averaging the only vectorial quantity that characterizes the vortex motion and is independent of the sign of magnetic field is the vortex velocity $\mathbf{v}$. Therefore, pinning force can be written in the form

$$\langle \mathbf{F}_{\mathrm{pin},i}\rangle=-\gamma(v)\mathbf{v}\,, \qquad (3)$$

where the coefficient $\gamma(v)>0$ can be in principle calculated by the summation of the perturbation series. The above statement can be illustrated by the lowest order calculation in the perturbation expansion over disorder potential:

$$\langle \mathbf{F}^{(1)}_{\mathrm{pin},i}\rangle=-\left\langle\sum_i V(\mathbf{r})[\mathbf{u}_{\mathrm{pin}}(\mathbf{r},t)\cdot\nabla]\nabla p(\mathbf{r}_\perp-\mathbf{r}_{\perp i}-\mathbf{v}t)\right\rangle\,, \qquad (4)$$

where $V(\mathbf{r})$ is the short range disorder potential, $p(\mathbf{r})$ is the vortex core form factor, $\mathbf{u}_{\mathrm{pin}}(\mathbf{r},t)$ is the disorder-induced displacement field of the vortex configuration [16], and the summation is performed over the positions of the vortex lines $\mathbf{r}_{\perp i}$. Straightforward calculation shows that the perpendicular to $\mathbf{v}$ terms cancel in Eq. (4). Generally this cancellation follows from the symmetry considerations presented above. Hence the equation of vortex motion now reads:

$$[\eta+\gamma(v)]\mathbf{v}+\alpha\mathbf{v}\times\mathbf{n}=\mathbf{f}_L\,, \qquad (5)$$

with $\mathbf{f}_L=(\Phi_0/c)\mathbf{j}\times\mathbf{n}$. We see that as we have mentioned before pinning renormalizes the friction coefficient γ only, whereas the Hall "conductivity" α remains unchanged. A solution to Eq. (5) is easily found:

$$\mathbf{v}=\frac{1}{\tilde\gamma(v)}\left(1+\frac{\alpha^2}{\tilde\gamma^2}\right)^{-1}\left(\mathbf{f}_L-\frac{\alpha}{\tilde\gamma(v)}\mathbf{f}_L\times\mathbf{n}\right)\,, \qquad (6)$$

where $\tilde\gamma(v)=\gamma(v)+\eta$. In what follows we will take into account that the Hall angle and, therefore, the ratio $\alpha/\tilde\gamma(v)$ are very small, and, hence, we will neglect the term $\alpha/\tilde\gamma(v)^2$ with respect to 1. The electric field induced by the vortex motion is $\mathbf{E}=(1/c)\mathbf{B}\times\mathbf{v}$ and from Eq. (6) we obtain straightforwardly $E_x=[B\Phi_0/c^2\tilde\gamma(v)]j$, $E_y=[\alpha B\Phi_0/c^2\tilde\gamma^2(v)]j$. For resistivity defined as $\rho=E/j$ one immediately gets

$$\rho_{xy}=\rho_{xx}^2\frac{c^2\alpha}{B\Phi_0}\,. \qquad (7)$$

The relation (7) is equivalent to an inversion of the conductivity tensor: $\rho_{xy}=\sigma_{xy}/(\sigma_{xy}^2+\sigma_{xx}^2)$ with $\sigma_{xy}=c^2\times\alpha/B\Phi_0$. The main result of our analysis is that Hall conductivity σ_{xy} does not depend on disorder. This result is general and applies to the regimes of flux flow, thermally assisted flux flow (TAFF), and vortex glass behavior (creep) as well. The temperature dependence of the Hall resistivity ρ_{xy} is determined by that of the longitudinal component ρ_{xx} and α. In particular the sign of ρ_{xy} follows the sign of α. In the flux flow regime the temperature dependence of α in Eq. (6) is of equal importance as that of ρ_{xx} and no simple relation between the temperature dependences of the resistivity components can be observed. On lowering the temperature pinning becomes relevant, and ρ_{xx} displays thermally activated behavior decreasing exponentially with temperature. In this regime the temperature dependence of ρ_{xy} is dominated by that of ρ_{xx} (i.e., the exponential temperature dependence of ρ_{xx} dominates the comparatively weak temperature dependence of α which is uninfluenced by disorder), and we arrive at the scaling relation

$$\rho_{xy} = A\rho_{xx}^2 \,. \tag{8}$$

The results (7) and (8) should be compared with the available experiments. The data of Iye, Nakamura, and Tamegai [3], Chien *et al.* [6], and Luo *et al.* [7] consistently show a rapid drop in ρ_{xy} as soon as ρ_{xx} becomes small due to pinning in the TAFF regime. Introducing pinning sites artificially by irradiation with heavy ions shifts the irreversibility line as measured by the onset of longitudinal resistivity ρ_{xx} [17]. The drop in the Hall resistivity ρ_{xy} follows this shift and always occurs near the crossover from the flux flow to the TAFF behavior of ρ_{xx}. These data are in agreement with the relation (6), which shows that the fast decrease of ρ_{xy} should follow that of ρ_{xx}.

Regarding the origin of the sign change in the Hall resistivity, ρ_{xy}, one can conclude from the data reported by Artemenko, Gorlova, and Latyshev [2], Hagen *et al.* [4], and Luo *et al.* [7], that the sign change is not related to the pinning and may even take place above T_c provided the magnetic field is low enough. The data of Hagen *et al.* and Samoilov [8] indicate that ρ_{xy} is positive both at high and low temperatures and experiences a local depression, which can cause an excursion into a negative value, near T_c. Upon entering the TAFF regime where pinning is relevant when decreasing the temperature, ρ_{xy} drops very fast below the threshold of sensitivity. Therefore, the existence of a second sign change depends on whether the pinning is strong enough as to suppress ρ_{xy} before the second sign change occurs [2–4,7], or afterwards [4,8]. This explains in a very natural way the occurrence of the double sign change in the strongly layered Tl and Bi compounds, where pinning is reduced due to the large anisotropy, whereas the second sign change in YBCO is suppressed by the stronger pinning.

The strongest experimental support for the above ideas comes from the recent finding of Samoilov [8]. Samoilov observed a scaling $\rho_{xy} = A\rho_{xx}^{\beta}$, with $\beta = 2.0 \pm 0.1$, and a field independent factor A. The result (6) shows that the simultaneous measurements of the resistivities ρ_{xx} and ρ_{xy} allow for the determination of the *bare* coefficient α. Comparing (7) to Samoilov's result we then find $\alpha \propto H$ in agreement with the Bardeen-Stephen result for the Hall resistivity, $\tan\Theta_H \propto H$. Note that in the Bardeen-Stephen approach the Hall effect is due to the normal carrier Hall effect in the vortex cores, which in turn is proportional to the magnetic field within the core. Recently Budhani, Liou, and Zhang [18] studied the mixed-state Hall resistivity in Tl-based films containing columnar defects. They observed that while the sign anomaly diminishes with the increasing defect concentration, the power law with $\beta = 1.85 \pm 0.1$ holds even after irradiation, introducing columnar defects; i.e., scaling law *does not depend on the type of defects.*

In the pinned regime the Hall resistivity behavior ρ_{xy} is dominated by the temperature dependence of ρ_{xx}. The latter is determined by the effective pinning-induced damping $\gamma(v)$ in Eq. (3). As we have mentioned before the explicit calculation of $\gamma(v)$ is difficult and remains an unresolved problem for the moment. It is very tempting to try to infer the functional dependence of $\gamma(v)$ from the current-voltage characteristics obtained for the various pinning regimes. In the TAFF regime $\gamma(v) \propto \exp(U_{\mathrm{TAFF}}/T)$. In the vortex glass phase $\rho_{xx} \propto \exp[-(j_0/j)^{\mu}]$, and we find $\gamma(v) \propto 1/v[\ln(1/v)]^{1/\mu}$. Within first order perturbation theory the above result for $\gamma(v)$ in the TAFF regime has been found in [16] in the course of the analysis of the pinning of vortex liquid. The first order approximation for the vortex solid gives a divergent algebraic dependence $\gamma(v) \propto v^{-1/2}$ indicating the vortex glass behavior [16].

A basic assumption made in the derivation of the scaling behavior (6) was the time reversal symmetry of the pinning potential. This condition holds for conventional superconductors with nonmagnetic impurities, but can be violated in the presence of magnetic impurities or in unconventional superconductors with broken time reversal symmetry, e.g., in anyon superconductors. It would have been tempting to view the scaling exponent $\beta = 1.7$ found by Luo *et al.* [7] as a manifestation of an exotic type of superconductivity in YBCO, but more straightforwardly, though more ordinarily, the deviation of β from the expected value 2 should be attributed to the temperature dependence of α coming into play near T_c (note that Luo *et al.* [7] probed temperatures much closer to T_c than Samoilov [8]).

Finally we would like to point out that the obtained independence of Hall conductivity σ_{xy} upon disorder is a dual analog to the behavior of dirty semiconductors where the Hall *resistivity* ρ_{xy} remains unaffected by disorder [19–21]. Indeed, in the mixed state of superconductors dissipation is caused by the motion of vortices, and, therefore, in this case the roles of conductivity and resistivity are reversed.

In conclusion, we have presented a simple and straightforward explanation of the scaling behavior of the Hall resistivity in a mixed state of type II superconductors as a result of flux pinning. We have found the universal scaling law $\rho_{xy} \propto \rho_{xx}^2$. The theory developed is in complete agreement with the recent experimental data on BSCCO compounds by Samoilov [8] and with data on TBCCO by Budhani *et al.*

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[1] M. Galffy and E. Zirgiebl, Solid State Commun. **68**, 929 (1988).
[2] S. M. Artemenko, I. E. Gorlova, and Y. I. Latyshev, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 352 (1988) [JETP Lett. **49**, 403 (1988)].
[3] Y. Iye, S. Nakamura, and T. Tamegai, Physica (Amsterdam) **159C**, 616 (1988)].
[4] S. J. Hagen, C. J. Lobb, R. L. Greene, M. G. Forrester, and J. H. Kang, Phys. Rev. B **41**, 11630 (1990).
[5] Z. D. Wang and C. S. Ting, Phys. Rev. Lett. **67**, 3618 (1991).
[6] T. R. Chien, T. W. Jing, N. P. Ong, and Z. Z. Wang, Phys. Rev. Lett. **66**, 3075 (1991).
[7] J. Luo, T. P. Orlando, J. M. Graybeal, X. D. Wu, and R. Muenchausen, Phys. Rev. Lett. **68**, 690 (1992).
[8] A. V. Samoilov, Phys. Rev. Lett. **71**, 617 (1993).
[9] A. Freimuth, C. Hohn, and M. Galffy, Phys. Rev. B **44**, 10396 (1991).
[10] R. A. Ferrel, Phys. Rev. Lett. **68**, 2524 (1992).
[11] E. Z. Meilikhov and R. M. Farzetdinova (to be published).
[12] A. T. Dorsey and M. P. A. Fisher, Phys. Rev. Lett. **68**, 694 (1992).
[13] W. F. Vinen and A. C. Warren, Proc. Phys. Soc. **91**, 409 (1967).
[14] A. Schmid and W. Hauger, J. Low Temp. Phys. **11**, 667 (1973).
[15] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **65**, 1704 (1973) [Sov. Phys. JETP **38**, 854 (1974)].
[16] V. M. Vinokur, M. V. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Phys. Rev. Lett. **65**, 259 (1991); Zh. Eksp. Teor. Fiz. **100**, 1104 (1991) [Sov. Phys. JETP **73**, 610 (1991)]; Eq. (5) of the latter paper contains an error, the correct form is given by Eq. (4) of the present paper.
[17] F. Rullier-Albenque (private communication).
[18] R. C. Budhani, S. Liou, and Z. X. Zhang (to be published).
[19] P. Hopkins *et al.*, Phys. Rev. B **39**, 12708 (1989).
[20] V. Goldman *et al.*, Phys. Rev. Lett. **61**, 881 (1988).
[21] S.-C. Zhang, S. Kivelson, and D.-H. Lee, Phys. Rev. Lett. **69**, 1252 (1992).