How Transparent are Hadrons to Pions?

B. Blättel,[1] G. Baym,[1] L. L. Frankfurt,[2],[a] and M. Strikman[3]
[1] *Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801*
[2] *Department of Physics, University of Washington, Seattle, Washington 98185*
[3] *Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802*
(Received 20 October 1992)

The scattering of hadrons at high energies can be described by a distribution function of cross sections, which incorporates the phenomena of color transparency and color opacity. We infer here a cross-section distribution for pionic projectiles on nucleons consistent with the data on the moments of the distribution. Using this distribution we calculate the probability for a pion to be in a pointlike configuration and compare with predictions based on a combination of perturbative QCD and QCD sum rules. Comparison with the corresponding distribution for nucleon-nucleon scattering shows that color transparency effects should be more pronounced for a meson beam.

PACS numbers: 13.85.Lg, 12.40.Pp, 24.85.+p, 25.80.Hp

At high energies, the substructure of hadrons leads to cross-section fluctuations in hadronic collisions. In scattering, the instantaneous hadronic configurations can be considered frozen, and the scattering process should be calculated first for the particular configuration, and then averaged over all configurations. The cross section σ has to be replaced by a distribution of cross sections $P(\sigma)$, whose mean is the experimentally determined cross section. In Refs. [1–3] we showed how moments of $P(\sigma)$ can be extracted from experimental data on diffractive dissociation of hadrons and inelastic shadowing corrections to total cross sections. In particular, we found that the distribution function for pion-nucleon collisions has a much larger width than that for nucleon-nucleon scattering. Furthermore, quark-counting rules suggest a different asymptotic behavior of $P(\sigma)$ for π-N than for N-N at small cross sections. As suggested in [2] this behavior implies a much higher probability for the pion than a nucleon to be in a "pointlike configuration," in which the constituents are much closer together than on average. In the following, we present quantitative estimates for this probability and discuss the form of the cross-section distribution function for pion-nucleon collisions.

The criterion that a pion configuration remain "frozen" between collisions is that the expansion length scale, $l \sim 2E_\pi/\Delta M^2$, be large compared to the target dimensions. Here ΔM^2 is a measure of how far off shell on average the quark-gluon configuration is in the pion. The typical scale for ΔM^2 is the inverse slope of the corresponding Regge trajectory, $1/\alpha'_R \sim 1.1$ GeV^2, or, equivalently, the difference of mass squared of the pion and the nearest excited pion state. Since the slopes of the meson and baryon trajectories are close, the expansion scales for baryons and mesons should be roughly equal.

For nucleon-nucleon collisions we find [2] that the variance

$$\omega_\sigma \equiv \frac{\langle\sigma^2\rangle - \langle\sigma\rangle^2}{\langle\sigma^2\rangle} \tag{1}$$

grows logarithmically with energy, at least up to $p_{\rm lab} \approx 1$ TeV. A typical value for ω_σ^N at around $p_{\rm lab} \approx 200$ GeV is 0.25. From proton-deuteron diffraction experiments we find that the cubic moment

$$\kappa_\sigma \equiv \frac{\langle\sigma^3\rangle - \langle\sigma\rangle\langle\sigma^2\rangle}{\langle\sigma\rangle^3} \tag{2}$$

is $\approx 2\omega_\sigma^N$. Further information about $P(\sigma)$ for $\sigma \to 0$ can be obtained from the fact that the cross section for a small-size configuration with transverse radius $r_\perp$ is

$$\sigma \sim r_\perp^2 \ \text{ for } \ r_\perp \ll (\bar\sigma/\pi)^{1/2}, \tag{3}$$

and that quark-counting rules imply that the distribution of sizes is given by $|\Psi(r_\perp^2)|^2 \sim r_\perp^{2(N_q-2)}$ for small $r_\perp$; then

$$P(\sigma) \sim \sigma^{N_q-2} \ \text{ for } \ \sigma \ll \bar\sigma, \tag{4}$$

where N_q is the number of valence quarks. Thus the nucleon-nucleon distribution $P(\sigma)$ is $\sim \sigma$ for small σ. Based on this information we approximated the distribution function in the form

$$P(\sigma) = N\frac{\sigma/\sigma_0}{\sigma/\sigma_0 + a}e^{-[(\sigma-\sigma_0)/(\Omega\sigma_0)]^n}. \tag{5}$$

The distribution is shown in Fig. 1, for the parameter set n=6, a=0.1, σ_0= 34.8 mb, Ω=1.10, N=0.0017 mb^{-1}, for which ω_σ=0.25 and $\kappa_\sigma = 2.00\omega_\sigma$; an alternative parameter set, n=10, a=1.0, σ_0= 6.07 mb^{-1}, Ω=11.0, N=0.0184 mb^{-1}, yields a nearly identical distribution, not shown in the figure, for which ω_σ=0.25 and $\kappa_\sigma = 1.99\omega_\sigma$. As discussed in [2], the rather small value of κ_σ^N requires the distribution to fall off sharply at large values of σ. This falloff becomes plausible if one takes into account that (3) is valid only for small $r_\perp$, whereas the dependence should be much weaker for larger configurations, e.g., $\sigma \sim r_\perp^\alpha$ with $\alpha \ll 2$, a natural behavior in constituent quark models of hadrons. The resulting effect for $P(\sigma)$ can be seen if we assume a particular shape for the distribution of sizes, e.g., a Gaussian falloff as in [1],

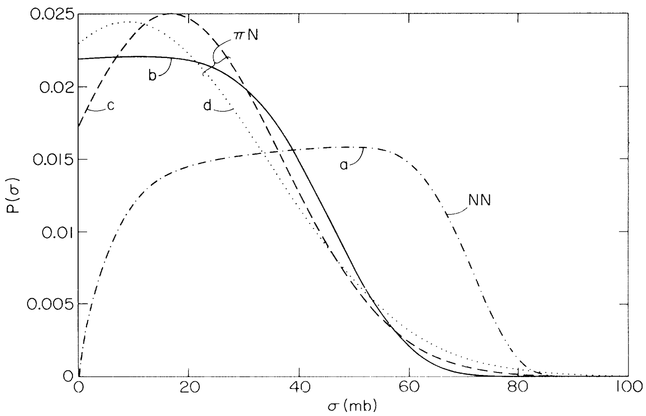

FIG. 1. Cross-section distribution functions for NN scattering, Eq. (5), and πN scattering, Eq. (8), as functions of σ. For NN scattering (from Ref. [2] with $\bar{\sigma}$ = 39 mb), curve a (dash-dotted curve) corresponds to parameters n=6, a=0.1, σ_0= 34.8 mb^{-1}, Ω=1.10, N=0.0017 mb^{-1}, for which ω_σ=0.25 and $\kappa_\sigma = 2.00\omega_\sigma$. For πN, curve b (solid) corresponds to n=4, σ_0 = 5.25 mb^{-1}, Ω=8.4, N=0.0221 mb^{-1}, for which ω_σ = 0.4 and $\kappa_\sigma = 2.24\omega_\sigma$; for c (dashed), n=2, σ_0 = 17.4 mb^{-1}, Ω=1.6, N=0.025 mb^{-1}, for which ω_σ = 0.4 and $\kappa_\sigma = 2.40\omega_\sigma$; for d (dotted), n=2, σ_0 = 9.6 mb^{-1}, Ω=3.7, N=0.024 mb^{-1}, for which ω_σ = 0.5 and $\kappa_\sigma = 2.60\omega_\sigma$.

$$|\Psi(r_\perp^2)|^2 \sim r_\perp^{2(N_q-2)} \exp(-r_\perp^2/r_c^2), \text{ so that for } N_q = 2,$$

$$P(\sigma) \sim |\Psi(r_\perp^2)|^2 \sim e^{-r_\perp^2/r_c^2} \sim e^{-(\sigma/\sigma_0)^{(2/\alpha)}} \quad \text{for } \sigma \geq \bar{\sigma}, \tag{6}$$

which falls off sharply with $n = 2/\alpha \gg 1$.

Although the data indicate a rather broad distribution for the nucleon-nucleon cross section, current data on inelastic shadowing in pion-deuteron scattering indicate an even larger variance $\omega_\sigma^\pi \approx 0.4$ for π-N reactions. This result can be understood as arising from the fewer number of degrees of freedom in the pion, or a meson in general. As discussed in [2] it also follows from the factorization principle and the triple-Pomeron limit, which leads to

$$\frac{\omega_\sigma^\pi}{\omega_\sigma^N} = \frac{\sigma^{NN}}{\sigma^{\pi N}} \approx \frac{40 \text{ mb}}{25 \text{ mb}} = 1.6. \tag{7}$$

Unfortunately, there does not yet exist experimental information about κ_σ^π, which would be accessible from measurements of diffractive hadron production in π-d reactions. However, we argue that the relation $\kappa_\sigma = \omega_\sigma\delta$ with $\delta \approx 2$, derived in [2] for the nucleon, should also be applicable to the pion. The "rescattering parameter" δ, extracted from experimental p-d diffractive scattering in [2], is related to the scattering amplitude of the nucleon to inelastic states X, and can be approximated by $\delta = 1 + \sigma_{XN}/\sigma_{NN}$. This formula is valid in the inelastic eikonal approximation to the total cross section of Kaidalov and Ter-Martirosyan [4], when transitions between different inelastic states are neglected. The result, $\delta \approx 2$, implies that a hadronic projectile h colliding with a nucleon produces inelastic states X with total cross section σ_{XN} roughly the same as the elastic cross section σ_{hN}. This approximate result is reasonable at high energies in constituent quark models of hadrons. However, one should keep in mind that this estimate of δ is only rough; in the following we vary κ_σ within reasonable limits to see the effect of the uncertainties on the distribution function.

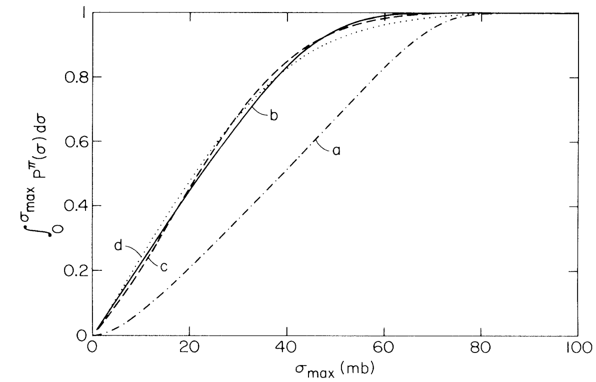

FIG. 2. Probabilities of pointlike configurations obtained by integrating $P(\bar{\sigma})$ up to $\bar{\sigma}_{max}$ as a function of $\bar{\sigma}_{max}$. Curves are the same as in Fig. 1.

An important difference of the π-N distribution compared to the N-N distribution is the limiting behavior following from the discussion above, which suggests that for small σ, $P(\sigma) \sim$ const. Based on the information available, we suggest the following shape of the distribution function [5]:

$$P_\pi(\sigma) = N e^{-(\sigma-\sigma_0)^n/(\Omega\sigma_0)^n}. \tag{8}$$

In Fig. 1 we show several distributions that lead to $\omega_\sigma^\pi \sim$ 0.4–0.5 with different values for κ_σ^π in the range (2.24–2.60)ω_σ. Note that one expects $P(\sigma)$ to approach the limiting value $P(0)$ from above: In addition to the lowest $q\bar{q}$ component, other components present lead to a decrease in $P(\sigma)$ as $\sigma \to 0$ at least as fast as $\sim \sigma$. Assuming that for $\sigma \sim \bar{\sigma}$ the higher components are comparable to the lowest component, we expect $P(\bar{\sigma})$ to be significantly larger than $P(0)$. Thus the model corresponding to the dashed curve c in Fig. 1, with ω_σ = 0.4 and $\delta = 2.40$, appears to be most realistic.

However, even within these uncertainties, Fig. 1 shows the very different behavior of $P_\pi(\sigma)$ compared to $P_N(\sigma)$ for small cross section. To illustrate this point we estimate the probability for a pion to be in a pointlike configuration by calculating the integral $\int_0^{\sigma_{max}} P_\pi(\sigma)d\sigma$ as a function of the upper limit σ_{max}. The result is shown in Fig. 2, where we also compare with the corresponding curve from integrating $P^N(\sigma)$. We find that the pion has a much larger probability to be in pointlike configuration than the nucleon. If, to be quantitative, we regard a configuration as pointlike if $\sigma < 5$ mb, the probability is about 10% for the pion, but only $\approx 2\%$ for the nucleon. Figure 3, the ratio of the probabilities for the pion and the nucleon, shows the higher probability for the pion, especially for very small configurations.

The value of $P_\pi(\sigma = 0)$ that follows from our fit is in reasonable agreement with theoretical estimates. The physics at small σ is dominated by small-size valence $q\bar{q}$ Fock state configurations of the pion, measured by the wave function $\Psi(x, r_\perp)$, where x is the light-cone momentum fraction, and $r_\perp$ is the transverse size of the configuration. At small $r_\perp$, color is highly localized in the pion. Assuming σ to depend only on $r_\perp$, as $r_\perp \to 0$, we may write

$$P_\pi(\sigma = 0) = \int_0^1 dx |\Psi(x, r_\perp^2 = 0)|^2 \left(\frac{\pi d r_\perp^2}{d\sigma}\right)_{r_\perp = 0} . \quad (9)$$

Following Bertsch *et al.* [6], we calculate $\Psi(x, r_\perp^2 = 0)$ using the asymptotic QCD wave function of the pion of Ref. [7] and find

$$\Psi(x, r_\perp = 0) = \sqrt{48}\pi x(1-x) f_\pi , \quad (10)$$

where $f_\pi = 93$ MeV is the pion-decay constant; this component of the wave function is normalized to reproduce the width of pion decay into muon and neutrino [8]. Using this wave function, we have

$$P_\pi(\sigma = 0) = \frac{8\pi^2}{5} f_\pi^2 \left(\frac{d r_\perp^2}{d\sigma}\right)_{r_\perp = 0} . \quad (11)$$

With the interpolation formulas, $\sigma = \langle\sigma\rangle r_\perp^2 / \langle r_\perp^2\rangle$, and neglect of the x dependence of σ as $r_\perp \to 0$, we estimate $P_\pi(\sigma = 0) = (8\pi^2 f_\pi^2/5)\langle r_\perp^2\rangle/\langle\sigma\rangle \approx 0.031$ mb^{-1}. With the more realistic Chernyak-Zhitnitski wave function [9] based on QCD sum rules, we obtain a value 0.037 mb^{-1}, a factor of 25/21 higher. These estimates are of similar magnitude to the range of values that follows from our

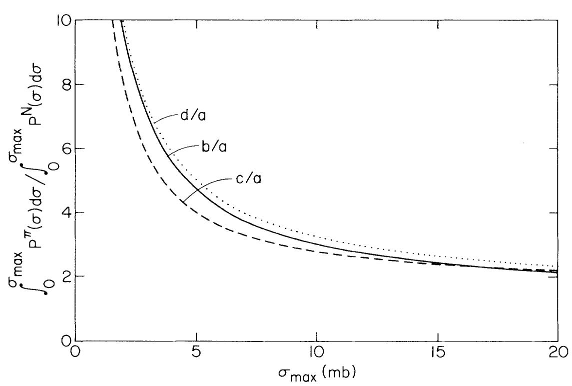

FIG. 3. Ratios of the probabilities of pointlike configurations, shown in Fig. 2, for the pion relative to that of the nucleon. Curves are the same as for the pion distributions in Fig. 1.

cross-section distribution functions, 0.017–0.023 mb^{-1}, with the lower number preferred, as discussed above.

Since color effects dominate for small transverse separation, $r_\perp$, of the valence $q\bar{q}$ pair in the pion, we can in fact apply perturbative QCD directly to calculate $\sigma(r_\perp^2)$ at $r_\perp \ll \langle r_\perp^2\rangle$. For small-size configurations, the dominant process in πN scattering is exchange of two gluons, a process analogous to the box diagram for γ-hadron scattering, where instead of the $\gamma \to q\bar{q}$ coupling, the $\pi \to q\bar{q}$ wave function enters. Taking the leading gluon exchange diagrams into account in the limit in which the center of mass energy $\sqrt{s}$ is large compared with $r_\perp^{-2}$, we find the πN cross section [10],

$$\sigma(r_\perp^2) = \frac{4\pi^2}{3}\left[r_\perp^2 \alpha_s(Q^2)\bar{x} G_N(\bar{x}, Q^2)\right]_{\bar{x} = 1/s r_\perp^2, Q^2 = 1/r_\perp^2} , \quad (12)$$

where $\alpha_s(Q^2)$ is the QCD running coupling constant, and $G_N(x, Q^2)$ is the gluon distribution in the nucleon. For simplicity we give this result, which numerically depends only weakly on $x_q/x_{\bar{q}}$, for the case $x_q \sim x_{\bar{q}} \sim 1/2$ in the pion. Since $G_N(x, Q^2)$ can be measured independently, Eq. (12) is an exact, parameter-free result in the limit $1/s \ll r_\perp^2 \ll \langle r_\perp^2\rangle$.

Neglecting, for a first estimate, the dependences of α_s and G_N on $r_\perp^2$, we derive $P_\pi(\sigma = 0) \approx 6 f_\pi^2/5\alpha_s(Q^2) x G_N(x, Q^2)$. For $\alpha_s \approx 0.3$–0.4, and $x G_N(x, Q^2) \approx 2.0$–2.5 we estimate from Eq. (11) that $P_\pi(\sigma = 0) \approx 0.027$–0.044 mb^{-1} with the wave function (10), and ≈ 0.032–0.053 mb^{-1} with the Chernyak-Zhitnitski wave function, values again comparable to those extracted from inelastic shadowing corrections to $\sigma^{\pi d}$.

The arguments presented here should also apply to a large extent to other mesons, not only because of the similar asymptotic behavior for small σ, but also because of the larger ω_σ compared to nucleons. For example, the factorization principle also suggests a large variance for the kaon: $\omega_\sigma^K/\omega_\sigma^N = \sigma^{NN}/\sigma^{KN} \approx 1.8$.

What do our results imply for experiment? First, they suggest that the physics of color transparency should be much more pronounced for pion (or, in general, meson) induced reactions. Quasielastic wide angle processes off nuclei, as suggested by Mueller and Brodsky [11, 12] and performed by Carroll *et al.* [13] with protons as projectiles, should occur more frequently with pionic projectiles, since pions could have a factor ~ 5 larger probability to penetrate the nucleus without any initial state interactions. Another way to see the effect is in the transparency of the target to a produced pion. Even at relatively small momentum transfers $-t \sim 5$–10 GeV^2, the pion expansion length $l \sim 2E_\pi/\Delta M^2$ can be made sufficiently large, at large beam energy and forward angles, that the produced pion has no time to expand and thus would exhibit significant transparency. This effect could be visible in high-energy exclusive nuclear pion photoproduction.

In addition, measurements of diffractive high p_t dijet production in πA scattering would provide an interesting handle on the dominance of color effects at small distances. Since the role of color fluctuations increases with energy, color coherence effects should be clearly revealed at Fermilab fixed target energies, leading to cross-section dependences $\propto A$. Equation (12) is valid also for nuclear targets with $G_N(x,Q^2)$ replaced by $G_A(x,Q^2)$. However, since the gluon distribution in nuclei is expected to be shadowed at small x, i.e., $G_A(x,Q^2)/AG_N(x,Q^2) < 1$ for $x < 10^{-2}$, at sufficiently large energies that $\bar{x}$ in Eq. (12) is $\lesssim 10^{-2}$, nuclear shadowing would decrease the A dependence of diffractive high p_t dijet production, and effects of color transparency would be diminished.

Finally, one should note that besides pionic color transparency, cross section fluctuations should also be very pronounced in color opacity effects, e.g., in fluctuations in multiplicity or E_t measured in π-nucleus scattering. Those implications, discussed in Ref. [1], should be considered in future work on meson-induced reactions. Measurements of the inelastic shadowing correction to the total cross section, the number of wounded nucleons, etc., in meson-nucleus scattering, would provide a check on our distribution.

To summarize, we have outlined a determination of the cross-section distribution function for π-N reactions, based on experimental information about its moments. Although the available data do not allow a very accurate determination of the distribution function, we can infer some of its features, e.g., a very large probability for a pion to be in a pointlike configuration, 10% for $\sigma < 5$ mb. In particular, this probability is roughly a factor of 5 larger than for a nucleon, which should be seen experimentally in pion-induced color-transparency experiments. Estimates from experiment are in good agreement with estimates based on perturbative QCD and QCD sum rules, illustrating well how phenomena of color transparency can be calculated directly using modern methods of QCD.

This work was supported by NSF Grant No. PHY89-21025 and in part by DOE Grant No. DE-FG02-93-ER40771. B.B. is grateful for support from the Alexander von Humboldt Stiftung.

(a) Current address: Tel Aviv University, Tel Aviv, Israel. Also at St. Petersburg Institute of Nuclear Physics, St. Petersburg, Russia.

[1] H. Heiselberg, G. Baym, B. Blättel, L.L. Frankfurt, and M. Strikman, Phys. Rev. Lett. **67**, 2946 (1991).

[2] B. Blättel, G. Baym, L.L. Frankfurt, H. Heiselberg, and M. Strikman, Phys. Rev. D (to be published).

[3] B. Blättel, G. Baym, L.L. Frankfurt, H. Heiselberg, and M. Strikman, Nucl. Phys. **A544**, 479c (1992).

[4] A.B. Kaidalov and K.A. Ter-Martirosyan, Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984) [Sov. J. Nucl. Phys. **39**, 979 (1984); **40**, 135 (1984)].

[5] The general problem is to determine the most likely $P(\sigma)$ from only a knowledge of its first moments, here at most 4. A possible guide is the principle of maximum entropy from which the most likely distribution is inferred by maximizing the "entropy," $S = -\int d\sigma\, P(\sigma) \ln P(\sigma)$, constrained by the known moments. Minimizing the functional $F = -\int d\sigma\, P(\sigma) \ln P(\sigma) - \sum_{i=0}^{3} \lambda_i \int d\sigma\, \sigma^i\, P(\sigma)$, leads to a minimal information distribution of the form $P(\sigma) \sim \exp(-\lambda_1\sigma - \lambda_2\sigma^2 - \lambda_3\sigma^3)$, where the Lagrange multipliers λ_i are determined from the known moments. We are grateful to James Gubernatis for discussion of this approach.

[6] G. Bertsch, S.J. Brodsky, A.S. Goldhaber, and J.G. Gunion, Phys. Rev. Lett. **47**, 297 (1981).

[7] G.P. Lepage and S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[8] Note that the present result for $\Psi(x, r_\perp = 0)$ differs from that given in Ref. [6] for $\Psi(x, b = 2r_\perp = 0)$ by a factor of 2.

[9] V.L. Chernyak and A.R. Zhitnitski, Phys. Rep. **112**, 173 (1984).

[10] L. Frankfurt and M. Strikman (to be published).

[11] A.H. Mueller, in *Proceedings of the Seventeenth Rencontre de Moriond*, edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, 1982), Vol. I, p. 13.

[12] S.J. Brodsky, in *Proceedings of the Thirteenth International Symposium on Multiparticle Dynamics, Volendam, 1982*, edited by W. Kittel *et al.* (World Scientific, Singapore, 1983), p. 963.

[13] A.S. Carroll *et al.*, Phys. Rev. Lett. **61**, 1698 (1988).