Driving Operators Relevant: A Feature of the Chern-Simons Interaction

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By computing anomalous dimensions of gauge invariant composite operators $(\bar{\psi}\psi)^n$ and $(\phi^*\phi)^n$ in matter–Chern-Simons models, we address the fact that the Chern-Simons interactions make these operators more relevant, or less irrelevant, in the low energy region. We suggest a critical Chern-Simons fermion coupling, $1/\kappa_c^2 = 6/19$, for a phase transition at which the leading irrelevant operator $(\bar{\psi}\psi)^2$ becomes marginal, and a critical Chern-Simons boson coupling, $1/\kappa_c^2 = 6/34$, for a similar phase transition for the leading irrelevant operator $(\phi^*\phi)^4$. We see the phenomenon also in the $1/N$ expansion.

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One of the remarkable features of matter–Chern-Simons gauge theories [1] is that the Chern-Simons interactions attach statistical flux tubes to particles, by which a fermion can be transmuted into a boson or an anyon (and vice versa), depending on the strength of the Chern-Simons coupling, $1/\kappa$ (κ the statistical parameter) [2]. In the relativistic quantum field theory scheme, an Abelian Chern-Simons term receives *no* correction from massive matter beyond one loop [3, 4], and receives only *finite* corrections from massless matter starting from two loops [4, 5]. This results in an identically vanishing beta function for Chern-Simons couplings. The insensitivity of Chern-Simons couplings to energy scales is attributed to the topological nature of Chern-Simons actions. However, it does not imply a triviality of the whole theory in the sense of renormalization. In fact, as shown in [6], matter fields in Chern-Simons models *do* need infinite renormalization and receive anomalous dimensions. (A non-Abelian Chern-Simons field receives an anomalous dimension through interaction with matter as well [6], while the Chern-Simons coefficient keeps finite and quantized, as required by the "large" gauge symmetry.) This observation leads naturally to a conjecture that the asymptotic behavior of gauge invariant operators in a matter–Chern-Simons theory is nontrivial. Recently, matter–Chern-Simons models have been used in describing the phase transition between quantum Hall states and insulators in [7] and [8], where among other observations is the fact that the composite mass operators of matter fields receive as well an anomalous dimension, and therefore the critical exponents η and ν are both dependent on Chern-Simons couplings. In the present Letter, we address another feature of matter–Chern-Simons systems: namely, that the effect of the Chern-Simons coupling on the anomalous dimensions of gauge invariant composite matter operators is to drive these operators in the direction of increased relevance.

By dimensional analysis, not many operators in a matter–Chern-Simons model are relevant. For instance, the Euclidean action of a Chern-Simons fermion model, if it contains the relevant and marginal operators only, reads

$$S = \int d^3x \bigg(\bar{\psi}\, \gamma \cdot \partial\psi + im\bar{\psi}\psi - i\bar{\psi}\gamma \cdot A\psi + i\frac{\kappa}{4\pi}\epsilon^{ijk}A_i\partial_j A_k \bigg). \tag{1}$$

The Chern-Simons field can be normalized so that κ is dimensionless. In particular, the fermion self-interactions $(\bar{\psi}\psi)^n$, for $n \geq 2$, in three spacetime dimensions are irrelevant [9]. On the other hand, these operators, as expected, may receive an anomalous dimension in quantization. If this is the case, the signs of the anomalous dimensions become crucial to the asymptotic behavior of the operators. We shall see below that the anomalous dimensions of these composite operators are dependent on the Chern-Simons coupling and are *negative.* Namely, due to the Chern-Simons interaction, they become less irrelevant. Moreover, our calculation suggests a phase transition with a critical effective coupling

$$1/\kappa_c^2 = 6/19, \tag{2}$$

at which the leading irrelevant operator $(\bar{\psi}\psi)^2$ becomes marginal. The critical value in (2) shows that if such a phase transition would occur, it occurs only in a region with a rather strong Chern-Simons interaction. We make several remarks at this point. First, it is not difficult to check that, at the order $1/\kappa^3$, all associated Feynman diagrams are *finite* (in the regularization by dimensional reduction), and the next to leading contribution to the anomalous dimensions are of the order $1/\kappa^4$ (four and higher loops). According to (2), therefore, there exists a multiplicative factor $1/\kappa_c^4 \simeq 1/10$ in the next order. If the numerical coefficient of the next order would be comparable to (or smaller than) that of the leading order, the loop expansion near the transition point is, to some extent, acceptable. Second, once a phase transition like this happens, a four-fermion operator, now marginal or relevant, is switched on to the system. In turn, this four-fermion self-interaction develops a *positive* anomalous dimension that has the potential to make the operator irrelevant. As a combining effect of the Chern-Simons–fermion and four-fermion interactions, a balance would be reached somewhere near the Gaussian fixed point for the four-fermion interaction [10]. Finally, a bare four-fermion interaction in three dimensions has a coupling constant that carries a *negative* dimension of mass and this makes the operator nonrenormalizable. However, once the Chern-Simons interaction turns the operator marginally relevant, the effective coupling of the four-fermion interaction has zero or a positive dimension and therefore the four-fermion operator becomes renormalizable or super-renormalizable.

We also conduct a parallel discussion for the Chern-Simons boson model. We shall see that the composite operators $(\phi^*\phi)^n$ gain anomalous dimensions which are also dependent on the Chern-Simons coupling and are *negative*. To drive the leading irrelevant operator $(\phi^*\phi)^4$ marginal, the Chern-Simons coupling must not be weaker than

$$1/\kappa_c^2 = 3/17. \tag{3}$$

In the bosonic case, the operators $g_2(\phi^*\phi)^2$ and $g_3(\phi^*\phi)^3$ are relevant and marginal, respectively. If one perturbs the theory near the Gaussian fixed point $g_2 = g_3 = 0$, the self-interactions can be turned off. On the other hand, if a perturbation is performed near the infrared fixed points of the self-interactions $g_2(\phi^*\phi)^2$ and/or $g_3(\phi^*\phi)^3$, these are the driving forces and cannot be ignored. The self-interaction $(\phi^*\phi)^2$, for example, at the infrared fixed point may be so strong that it makes itself irrelevant. A sufficiently strong Chern-Simons interaction may possibly draw it back and the corresponding infrared fixed point may be significantly shifted.

Other interesting cases are systems that involve N species of matters with some symmetry, $O(N)$, for instance. A plausible expansion in this case is over $1/N$. We shall see the same phenomenon in this expansion at order $1/N$, before concluding the Letter.

Now we turn to renormalization. Though, in a procedure of renormalization, one normally deals with the ultraviolet divergences, the resulting (ultraviolet) finite effective theory takes a form that is equally good in exhibiting the asymptotic behavior of the theory in both high and low energy limits. Let us take a simple example. After renormalization, the two-point function of the fermion in the momentum space has an asymptotic form

$$S(p) = \langle \psi(p)\bar{\psi}(-p)\rangle = \frac{1}{i\gamma\cdot p}\left(\frac{p^2}{\mu^2}\right)^{\gamma_\psi},$$

where μ is a reference mass parameter and γ_ψ the anomalous dimension of the fermion field. In the Chern-Simons fermion model, $\gamma_\psi = -\frac{1}{3\kappa^2} \le 0$, at the lowest nontrivial order and in the Landau gauge [6]. This implies that the fermion field in the Chern-Simons quantum field theory has a dimension less than its engineering one. Moreover, the kinetic term $\bar{\psi}\gamma\cdot\partial\psi$ takes an asymptotic form $(\frac{p^2}{\mu^2})^{\gamma_\psi}$. Now we see the kinetic energy of the fermion in the Chern-Simons theory is relevant, instead of marginal, in the lower energy region.

To renormalize the action (1), only *one* nontrivial renormalization constant suffices. This is because $Z_A = 1$ can always be chosen, as the Abelian gauge field A needs no infinite renormalization, and $Z_\psi = Z_{(\bar{\psi}A\psi)}$, due to the gauge symmetry. The fermion wave-function renormalization constant, to the lowest nontrivial order, in the Landau gauge, and under the minimal subtraction, is [6]

$$Z_\psi = 1 + \frac{1}{3\kappa^2}\frac{1}{\epsilon}, \tag{4}$$

with $\epsilon = 3 - \omega \to 0$. Now we consider the composite operators of interest. For simplicity, we shall set matter masses zero ($m = 0$). This will not change the ultraviolet divergence structure of the model. We use the Landau gauge for convenience, the results must be independent of a gauge choice as the operators we are dealing with are gauge invariant. The regularization by dimensional reduction, as used in [6, 8], will be used here. The same results have been reproduced by a regularization with a naive cutoff, which provides a check of consistency. To calculate the renormalization of a composite operator $O_n = \frac{1}{(n!)^2}(\bar{\psi}\psi)^n$ for a given n, we construct a one-particle irreducible (1PI) composite vertex $\Gamma^{2n}_{O_n}$ which contains the operator O_n as a vertex and has n truncated external fermion and n antifermion (and null Chern-Simons) lines. The nontrivial Feynman diagrams at the lowest nontrivial order (two loops) are depicted in Fig. 1.

By power counting, the composite vertex $\Gamma^{2n}_{O_n}$ is dimensionless. The calculation is somewhat tedious but

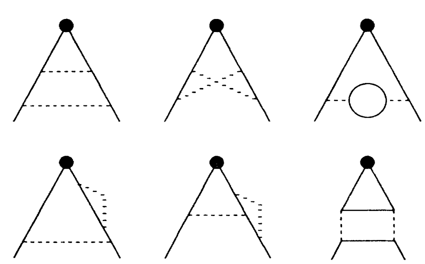


FIG. 1. Nontrivial Feynman diagrams for $\Gamma^{2n}_{O_n}$ at order $1/\kappa^2$ in the fermion model. Solid lines stand for the fermion propagator; dashed lines the Chern-Simons propagator; and dark spots the operator $O_n = \frac{1}{(n!)^2}(\bar{\psi}\psi)^n$ with $2(n-1)$ external fermion lines omitted. There is a symmetric factor of 2 for each of the last three diagrams.

straightforward. The result turns out to be

$$\Gamma^{2n}_{O_n}\left(\mathbf{0},p;\frac{1}{\epsilon}\right) = 1 + \frac{5}{2\kappa^2}\left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{\mu^2}{p^2} + \text{finite}\right), \tag{5}$$

where, without loss of generality, we have set all external momenta but one (carried by a fermion-antifermion pair) to zero. Using the renormalization relation

$$(\Gamma^{2n}_{O_i})_R(p) = Z^n_\psi\left(\frac{1}{\epsilon}\right)(Z_O)_{ij}\left(\frac{1}{\epsilon}\right)\Gamma^{2n}_{O_j}\left(p;\frac{1}{\epsilon}\right) \tag{6}$$

and the fermion wave-function renormalization constant (4), we obtain the renormalization constant and scaling dimension for the operator $(\bar\psi\psi)^n$,

$$Z_{(\bar\psi\psi)^n} = 1 - \frac{2n+15}{6\kappa^2}\left(\frac{1}{\epsilon}\right), \tag{7}$$

$$d_{(\bar\psi\psi)^n} = 2n + \gamma_{(\bar\psi\psi)^n} = 2n - \frac{2n+15}{6\kappa^2}. \tag{8}$$

It is worth noting a simplification appears here. As shown in the renormalization relation (6), in its general form, there may exist operator mixing in renormalization of higher dimensional operators. The operator mixing, if it happens, will make the situation much more complicated. However, operator mixing does *not* appear in the renormalization of the class of operators $(\bar\psi\psi)^n$ of the Chern-Simons fermion model [nor in that of $(\phi^*\phi)^n$ of the Chern-Simons boson model, as we shall see below]. This is because the only primary divergence brought in by the operator $(\bar\psi\psi)^n$ for a given n is that in (5), and therefore any cutoff dependence associated with the vertex O_n can be resolved by one of the counterterms in a form given in (7). We have calculated the composite vertex $\Gamma^{2(n-1)}_{O_n}$, and confirmed that there is indeed no need for any new counterterm—the one given in (7) suffices to resolve it—though the composite vertex $\Gamma^{2(n-1)}_{O_n}$ is of dimension two and presumably had various divergences.

In the Chern-Simons boson model, the composite operators under consideration are $O_n = \frac{1}{(n!)^2}(\phi^*\phi)^n$, which have an engineering dimension n. To perform the renormalization and therefore to calculate their anomalous dimensions, we consider similarly the composite vertex $\Gamma^{2n}_{O_n}$ with the operator O_n and n external charged boson and n anticharged boson lines. Life here seems easier as, at the lowest nontrivial order, there is only one nontrivial diagram, as shown in Fig. 2, for $\Gamma^{2n}_{O_n}$.

Calculating Fig. 2, we obtain

$$\Gamma^{2n}_{O_n}\left(\mathbf{0},p;\frac{1}{\epsilon}\right) = 1 + \frac{1}{\kappa^2}\left(\frac{1}{\epsilon} + \frac{1}{2}\ln\frac{\mu^2}{p^2} + \text{finite}\right). \tag{9}$$

Finally, we have the renormalization constant and scaling dimension of $(\phi^*\phi)^n$

$$Z_{(\phi^*\phi)^n} = 1 - \frac{7n+6}{6\kappa^2}\left(\frac{1}{\epsilon}\right), \tag{10}$$

$$d_{(\phi^*\phi)^n} = n + \gamma_{(\phi^*\phi)^n} = n - \frac{7n+6}{6\kappa^2}. \tag{11}$$

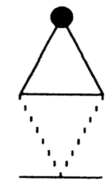


FIG. 2. The only nontrivial Feynman diagram for $\Gamma^{2n}_{O_n}$ at order $1/\kappa^2$ in the boson model. Solid lines stand for the boson propagator; dashed lines the Chern-Simons propagator; and the dark spot the operator $O_n = \frac{1}{(n!)^2}(\phi^*\phi)^n$ with $2(n-1)$ external boson lines omitted. There is a symmetric factor of 2 for the diagram.

To get these, we have used the boson model version of the renormalization relation (6) and the boson wave-function renormalization constant $Z_\phi = 1 + \frac{7}{6\kappa^2}(\frac{1}{\epsilon})$ [6].

To conclude this Letter, we discuss the expansion in a controlling parameter N, assuming there are N species of matter fields which obey a global symmetry O(N). We take the fermion model as an example, and to generalize to the boson model is straightforward. As is known, the one-loop fermion bubble chain is at the same $1/N^0$ order with the bare Chern-Simons propagator, instead of the bare one, one must use a dressed gauge propagator that sums over the one-fermion-loop chains. The dressed gauge propagator in the Landau gauge takes the form (equivalent in both the fermion and boson models)

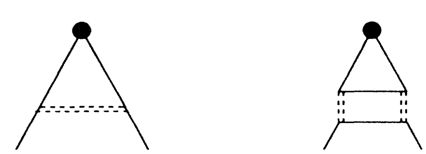
$$\Delta^{\mu\nu}(p) = A\frac{\delta^{\mu\nu}p^2 - p^\mu p^\nu}{p^3} + B\frac{\epsilon^{\mu\nu\lambda}p^\lambda}{p^2}, \tag{12}$$

$$A = \frac{8\pi\theta}{64\theta^2+\pi^2}, \quad B = -\frac{64\theta^2}{64\theta^2+\pi^2}, \tag{13}$$

where $1/\theta$ is the effective Chern-Simons coupling. The fermion wave-function renormalization constant at order $1/N$ and in the Landau gauge is [8]

$$Z_{\psi_i} = 1 + \frac{16}{3\pi\theta(64\theta^2+\pi^2)N}\left(\frac{1}{\epsilon}\right). \tag{14}$$

At order $1/N$, the nontrivial diagrams for the composite vertex $\Gamma^{2n}_{O_n}$ with the composite operator $\frac{1}{(n!)^2}(\bar\psi_i\psi_i)^n$ are given in Fig. 3. Calculating the diagrams in Fig. 3, and



(a) (b)

FIG. 3. Nontrivial Feynman diagram of $\Gamma^{2n}_{O_n}$ at order $1/N$. Real lines stand for the fermion propagator; double dashed lines the dressed gauge propagator; and dark spots the operator $O_n = \frac{1}{(n!)^2}(\bar\psi_i\psi_i)^n$ with $2(n-1)$ external lines omitted. There is a symmetric factor of 2 for the diagram (b).

using (6), (13), and (14), we obtain

$$\gamma_{(\bar{\psi}\psi)^n} = -\frac{16(64n\theta^2 + n\pi^2 + 960\theta^2 - 9\pi^2)}{3(64\theta^2 + \pi^2)^2}\frac{1}{N}. \quad (15)$$

Now we see the anomalous dimensions of $(\bar{\psi}\psi)^n$ are negative and therefore these operators are more relevant (for $n = 1$) or less irrelevant (for $n \geq 2$), only if the Chern-Simons coupling is not unreasonably strong, not stronger than $1/\theta^2 \sim 15$ for $n = 2$, for instance.

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