Attainment of Space-Charge Dominated Beams in a Synchrotron

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For a given rf cavity voltage, there is a maximum attainable peak current in an ion storage ring. This occurs when the electric field from the beam space charge is balanced by the field from the rf cavity. In this limit, the linear charge density distribution is parabolic and incoherent synchrotron motion is suppressed. The beam energy spread cannot be determined from the bunch length which depends only on the beam current and rf voltage. This has been observed using the high energy electron-cooling system at the Indiana University Cyclotron Facility.

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Beams in storage rings are "bunched" using radio frequency (rf) accelerating cavities operating on a harmonic, h, of the beam revolution frequency. The particle beam, however, has a finite momentum spread and consequently a finite frequency spread. All the particles contained within the bucket, or separatrix in longitudinal phase space formed by the rf potential, oscillate in frequency about the synchronous frequency. The frequency of modulation is called the synchrotron frequency. This is the principle of "phase focusing" in a synchrotron [1]. In low energy rings (< 10 GeV), the synchrotron oscillation frequency is on the order of 1 kHz, and is as low as 1 Hz in high energy rings (1 TeV).

Since the cavity produces a conservative force field, it cannot change the beam longitudinal phase space density. Consequently, for a known rf voltage, the beam time spread normally provides a direct measurement of the beam momentum spread and the area occupied by the beam in phase space, or the beam longitudinal emittance which is proportional to the beam temperature. An electron-cooling system [2,3], however, can reduce the ion beam emittance to extremely small values. In the process of cooling, the electrostatic potential energy spread across the beam bunch becomes larger than the moving frame kinetic energy spread. In this new regime, the bunch shape is no longer determined by the longitudinal emittance and rf cavity potential, but by a balance between the space charge and rf cavity forces [4-6].

Since in all cases of interest the bunch lengths are much greater than the radius of the surrounding vacuum chamber, the particle potential energy as a function of longitudinal position $U(s)$ can be found by integrating the forces on the particle due to the magnetic and electric fields produced by the beam, from the vacuum chamber radius, r_v, to the outer edge of the beam, r_b:

$$U(s)=\frac{2\ln(r_v/r_b)}{\gamma^2}mc^2\frac{r_e\rho_l(s)}{e}, \tag{1}$$

where γ is the usual relativistic parameter, m the electron mass, c the speed of light, r_e the classical electron radius, e the electron charge, and $\rho_l(s)$ the beam linear charge density. For the Indiana University Cyclotron Facility (IUCF) Cooler $\ln(r_v/r_b)\approx 3.2$. The space-charge force F_{SQ} causing the beam bunch to lengthen is then $dU(s)/ds$. The force compressing the beam, $F_{\rm rf}$, exerted by the rf cavity, having a voltage wave form given by $V_{\rm rf}\times\sin(hs/R)$ where R is the radius of the synchrotron storage ring ($\approx$ 13.8 m for the IUCF Cooler Ring), is given by

$$F_{\rm rf}=\frac{h}{2\pi}\frac{eV_{\rm rf}}{R}\frac{s}{R}, \tag{2}$$

where we have assumed short bunches, i.e., $\sin(hs/R)\approx hs/R$.

By equating $F_{\rm rf}$ with F_{SQ}, one obtains $d\rho_l(s)/ds$; one can then integrate $d\rho_l(s)/ds$ over s to find the equilibrium beam charge density distribution:

$$\rho_l(s)=\frac{\gamma^2}{8\pi\ln(r_v/r_b)}\frac{heV_{\rm rf}}{mc^2}\frac{e}{r_e}\frac{L_b^2-s^2}{R^2}, \tag{3}$$

where $|s|\le L_b$, $2L_b$ is the bunch length, and the constant of integration is used to set $\rho_l(s=\pm L_b)=0$. Equation (3) shows that the equilibrium linear charge density is parabolic. Had we not made the short bunch approximation in Eq. (2), we would instead have obtained a cosine distribution. Integrating $\rho_l(s)$ in Eq. (3) over s yields a relation between the beam current I the full width at half maximum beam time spread $T_{\rm FWHM}$ and $V_{\rm rf}$:

$$I=\frac{h^2\beta^4\gamma^2}{24\sqrt{2}\pi^2\ln(r_v/r_b)}\frac{eV_{\rm rf}}{mc^2}\frac{ec}{r_e}\frac{c^3T_{\rm FWHM}^3}{R^3}, \tag{4}$$

where $\beta=v/c$.

The time structure of an electron-cooled 45 MeV kinetic energy proton beam bunched on the first harmonic ($h=1$) was measured by recording the amplified signal from a beam position monitor [7] with a digital oscilloscope. This recorded signal is not, however, a true representation of the beam shape. The beam is ac coupled to the electrode, and the ac-coupled amplifiers, power splitters, and isolation transformers also act as high-pass filters. The measured time constant is 212 ns. The signal amplitude is also attenuated as $\approx\exp[-f(66\text{ MHz})]$,

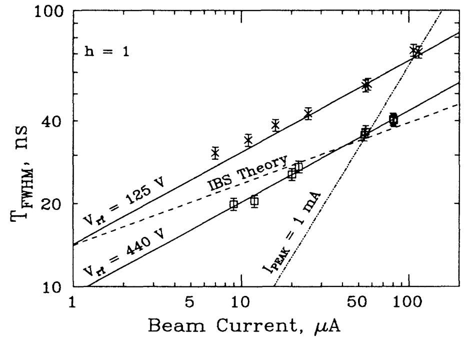

FIG. 1. The measured (×'s and □'s) and theoretical [solid lines, Eq. (4)] values for $T_{\rm FWHM}$ vs I for rf amplitudes of 125 and 437 V, respectively ($h=1$). The dashed curve is what is expected from the intrabeam scattering (IBS) theory [8].

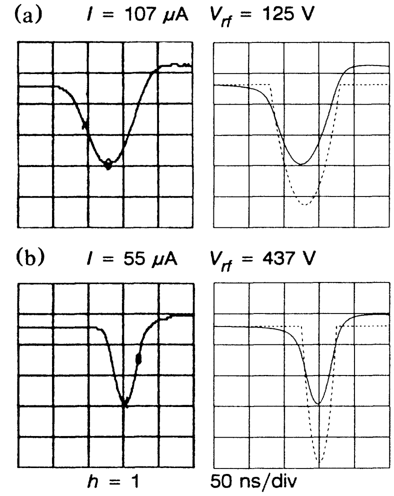

FIG. 2. (a),(b) Measured (left-hand side) and theoretical (right-hand side) beam time structure. The dashed theoretical curve is the parabolic bunch distribution; the solid theoretical curve includes the effect of filtering.

where f is frequency, in the cables between the storage ring and the control room. The theoretical charge distribution, Eq. (3), was modified to simulate the effects of these filters.

Comparisons between the predicted, Eq. (4), and measured $T_{\rm FWHM}$ are summarized in Fig. 1. In Figs. 2(a) and 2(b) the theoretical, Eq. (3), and measured beam time structure is compared for two different cases.

The beam current was measured with a parametric current transformer with 1 μA precision. The rf voltage was measured with a synchronous detector and calibrated by measuring the coherent synchrotron oscillation frequency as a function of rf amplitude and is accurate to about ±5%.

The observed increase in the bunch length within beam intensity has been previously attributed to an increase in the beam momentum spread due to intrabeam scattering [3,8]. These theories, however, predict the beam momentum spread, and consequently time spread, to increase with the cube root of the *peak* beam current. With this scaling, the bunch length should then increase with the $\frac{2}{9}$ power of the beam current rather than as the $\frac{1}{3}$ power [Eq. (4)]. This model, shown in Fig. 1 as the dashed line with the label IBS theory, clearly disagrees with our data. In fact, the data shown in Ref. [8] agree perfectly with the model proposed here. Here we propose that such measurements only give information about the ratio $I/V_{\rm rf}$; i.e., the measurements are consistent with a zero-momentum spread beam. In other words, space charge rather than emittance is the predominant defocusing force.

This cooled beam bunch also has very interesting properties. Although the beam bunch oscillates coherently at the expected synchrotron oscillation frequency, synchrotron motion within the bunch is apparently strongly suppressed, or ceases altogether. The coherent synchrotron oscillation frequency decreases with amplitude, in exact analogy to a gravitational pendulum, due to the nonlinear restoring force. Consequently, the independent particle model predicts large amplitude oscillations of beams with large time spreads to rapidly decohere. In the IUCF Cooler, however, artificially excited coherent oscillations, which the independent particle model predicts to decohere in a few synchrotron oscillation periods, stay coherent for many hundreds of periods; instead the oscillations damp coherently in accordance with the measured [9] electron-cooling damping force. There is an even more dramatic demonstration of this phenomenon: By off-setting the electron-cooling system energy from the energy corresponding to the synchronous energy, the damping of incoherent motion can be maintained without damping the coherent oscillation [10]. In this situation, the independent particle model predicts the beam to uniformly populate the resulting "Mills circle" [11] [illustrated schematically in Fig. 3(b)] since there is no preferred phase and the synchrotron oscillation frequency varies with amplitude. However, in this space-charge dominated regime, the beam remains bunched as illustrated in Fig. 3(c). Experimental observations are shown in Figs. 4(a) and 4(b).

It would be interesting to accelerate such a bunch through the transition (the point where df_0/dp changes sign). In order to maintain phase stability in conventional accelerators, it is necessary to change the rf phase by 180° at this point; the envelope of this space-charge dominated beam, however, should remain stable while passing through transition without the phase jump.

Although the beam time spread, in this situation, is *not* a measurement of the beam energy spread, the ratio of the rest frame energy spread which would normally be associated with this time spread to the beam moving frame electrostatic potential energy spread is $\eta\gamma \approx 1$, where η is

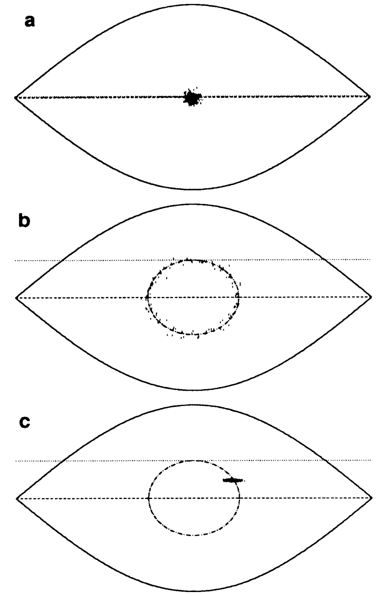

FIG. 3. (a)–(c) Phase space plots (*not data*) characterizing the beam in phase space. The vertical (horizontal) position is proportional to the particle energy (phase with respect to the rf cavity voltage). The solid line is the rf bucket separatrix. The dotted line represents the equilibrium energy to which the beam is cooled in the absence of rf. In (a) the electron beam energy is well aligned with respect to the synchronous energy (dashed line) and beam cools into the center of the rf bucket. In (b) and (c), the electron energy has been misadjusted. In (b) the beam uniformly populates the "Mills circle" (the dash-dotted line) as expected by the independent particle model. (c) shows what happens for a space-charge dominated beam. The actual energy spread is unknown.

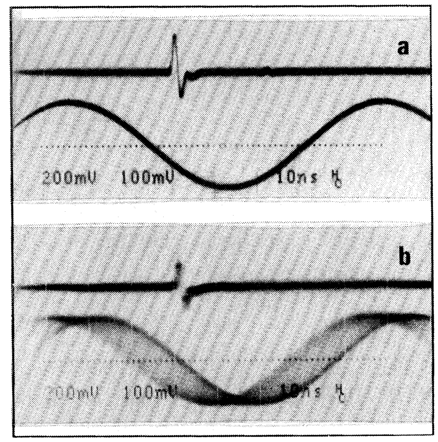

FIG. 4. (a),(b) Oscilloscope recordings of the beam longitudinal density distribution (upper trace) and rf wave form (lower trace). In (a), which corresponds to Fig. 3(a), the electron energy is aligned with the synchronous energy. In (b), which corresponds to Fig. 3(c), the electron energy has been increased 32 eV, corresponding to a 59 keV change in proton beam energy. [$V_{\rm rf}=2.2$ keV, $h=9$, $(\gamma-1)Mc^2=104$ MeV.] The oscilloscope is triggered on the arrival of the beam pulse; coherent synchrotron oscillations cause the rf wave form to be blurred in (b). The rf period is 73.75 ns.

$(df_0/dp)/(f_0/p)$ and is equal to 0.87 for a 45 MeV proton beam in the IUCF Cooler. The ratio of potential to kinetic energy in the transverse plane, assuming that the beam size is a measurement of the beam divergence (in analogy to the beam time spread being a measurement of the beam longitudinal energy spread), yields a similar ratio [5]. We consequently suspect that a similar phenomenon may take place in this dimension, such that incoherent transverse (or betatron) oscillations are also suppressed, and the transverse beam size is determined by space-charge effects rather than the beam emittance.

We are aware of more sophisticated models which include cooling, diffusion, space charge, emittance, and rf. An analytic solution using the Fokker-Planck equation exists and gives solutions which are similar to the much simpler model presented here. In this more sophisticated treatment one sees the beam distribution change from a Gaussian to a parabolic distribution as the ratio of diffusion to cooling forces is changed. Much higher resolution measurements of the beam charge density distribution in the future may be able to resolve small deviations from a pure cosine distribution; one would then be able to estimate the diffusion force, presumably due to intrabeam scattering. It has been theorized that intrabeam scattering may be strongly suppressed due to the mutual correlation of particles in such an ultracold beam [12].

This new form of beam, in which incoherent synchrotron and possibly betatron oscillations are suppressed, is made possible by electron cooling. The standard models describing beam behavior and stability may not be applicable to these beams. The properties of these beams are also of practical interest for planning experiments using internal targets and electron-cooled beams. The beam time spread limits the resolution of neutron time-of-flight experiments, and the energy spread limits the resolution of experiments looking at reactions with definite initial and final states, such pionic atom production. A recent experiment [13] has been approved to study the longitudinal and transverse properties of this space-charge dominated beam in more detail at IUCF.

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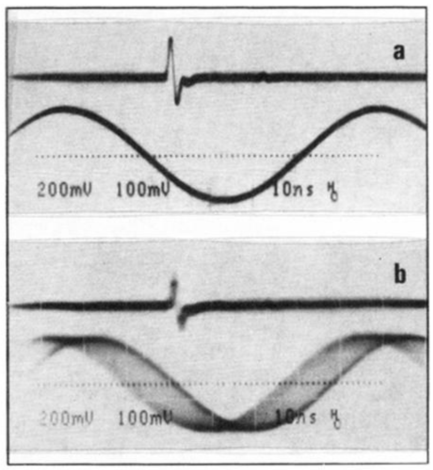

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