Preparation of Fock States by Observation of Quantum Jumps in an Ion Trap

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We propose a technique for the preparation of Fock states of a harmonic oscillator strongly coupled to a single two-level atomic transition based on the observation of quantum jumps. Examples are taken from the fields of cavity QED and ion trapping, where photon number states and number states of the quantized atomic motion may be prepared, respectively.

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The system comprising a harmonic oscillator coupled to a two-level system represents one of the fundamental models of quantum statistical mechanics. In a quantum optics context this model is generally referred to as the Jaynes-Cummings model (JCM) [1] with Hamiltonian

$$H=\hbar\omega_f a^\dagger a+\tfrac{1}{2}\hbar\omega_0\sigma_z+\hbar g(\sigma_+ a+a^\dagger\sigma_-)\,, \qquad (1)$$

where $a^\dagger$ and a are creation and annihilation operators for a mode of the radiation field with frequency ω_f, and $\sigma_{\pm,z}$ are the Pauli spin matrices describing a two-level atom with transition frequency ω_0. The last term in (1) describes the coupling of the field mode to the atom with coupling strength g. Dissipation can be included in this model by coupling the field mode and atom to independent heat baths and using a master equation formulation, in which one introduces damping rates κ and Γ for the field mode and atom, respectively. Of particular interest is the regime in which $g>\kappa,\Gamma$ (i.e., the strong coupling limit), when quantum effects in the coupled oscillator-spin system are most pronounced. Here the spectroscopy of the system is best described in terms of transitions between the "dressed states," or eigenstates, of the Hamiltonian H. The ground state is $|0,g\rangle=|0\rangle|g\rangle$ and, on resonance ($\omega_0=\omega_f$), the excited dressed states are $|n,\pm\rangle=(|n-1\rangle|e\rangle\pm|n\rangle|g\rangle)/\sqrt{2}$ ($n=1,2,\ldots$), where $|g\rangle$ and $|e\rangle$ are the bare atomic ground and excited states, respectively, and $|n\rangle$ are the Fock states of the field mode with excitation number $n=0,1,2,\ldots$. The eigenenergies corresponding to the excited states are $E_{n,\pm}=\hbar[\omega_0(n-\tfrac{1}{2})\pm g\sqrt{n}]$, which show ac Stark splitting proportional to $\sqrt{n}$. The two lowest transitions $|1,\pm\rangle\to|0,g\rangle$ give rise to a doublet structure, the "vacuum" Rabi splitting [2].

Experimental realization of the JCM with dissipation has been demonstrated in the field of cavity QED, in both the microwave and optical domain. Observation of quantum revivals and nonclassical photon statistics with Rydberg atoms and microwave cavities has been reported in [3], and in the optical regime vacuum Rabi splitting has been observed in [4]. In the Rydberg atom experiments radiation damping is negligible and the strong coupling limit is reached by means of weakly damped microwave cavities, whereas in optical experiments radiation damping is significant but strong coupling is achieved via high finesse cavities and very small cavity mode volumes. With this success in cavity QED, a variety of additional possibilities arise; in particular, the quantum nondemolition measurement of small photon numbers with a possibility of preparing photon number, or Fock, states [5].

In this Letter we discuss a different and novel scheme for the preparation of Fock states in a strongly coupled oscillator-atom system. Our proposal is based on the observation of quantum jumps from the manifold of dressed energy levels $|n,\pm\rangle$ to a third weakly coupled atomic level $|r\rangle$. Observation of the quantum jump in the sense of continuous measurement can be identified with the preparation of a Fock state $|n\rangle$ of the harmonic oscillator.

In addition to describing this scheme, we wish also to put forward an alternative experimental configuration for the realization of the JCM. In particular, we consider a *single* trapped ion constrained to move in a harmonic oscillator potential and undergoing laser cooling at the node of a standing light wave. Under conditions in which the vibrational amplitude of the ion is much less than the wavelength of the light (Lamb-Dicke limit), we will show that this problem is mathematically equivalent to the JCM with negligible damping of the oscillator (i.e., $\kappa=0$). In this configuration, preparation of the Fock state corresponds to preparation of a nonclassical state of motion of the trapped ion with fixed energy $n\hbar\omega$, whereas in cavity QED the Fock state corresponds to a nonclassical state of light with no intensity fluctuations and undetermined phase. The significance of our use of the trapped ion configuration is that it demonstrates the potential of the well-established field of ion trapping for investigations of features of the JCM which have thus far been studied only in the context of cavity QED. An analogy between an undamped trapped ion and the JCM has been noted by Blockley, Walls, and Risken [6] in the context of collapses and revivals. However, in contrast to their model, in which only a traveling-wave laser field was considered, our model is fundamentally based upon operation at the node of a standing-wave light field, which, as mentioned above, allows for a very direct connection with the JCM.

The master equation describing a single two-level ion trapped in a harmonic potential and located at the node of a standing-wave light field is given by [7]

$$\frac{d\rho}{dt}=-i\left[\nu a^\dagger a+\frac{\Delta}{2}\sigma_z-\frac{\Omega}{2}(\sigma_++\sigma_-)\sin[\eta(a+a^\dagger)],\rho\right]$$
$$+\frac{\Gamma}{2}(2\sigma_-\tilde{\rho}\sigma_+-\sigma_+\sigma_-\rho-\rho\sigma_+\sigma_-)\,, \qquad (2)$$

where ν is the trap frequency, Δ is the detuning of the two-level transition from the laser frequency, Ω is the laser Rabi frequency, and $\eta=\pi a_0/\lambda$, with a_0 the amplitude of the ground state of the trap and λ the optical wavelength. The term $\tilde{\rho}$ accounts for the momentum transfer associated with the spontaneous emission of a photon and is given by

$$\tilde{\rho}=\frac{1}{2}\int_{-1}^{1}du\, W(u)e^{i\eta(a+a^\dagger)u}\rho e^{-i\eta(a+a^\dagger)u}\,, \qquad (3)$$

where $W(u)$ is the angular distribution of spontaneous emission.

We assume that the trap frequency ν is larger than the decay rate Γ of the excited level. Hence, for laser cooling the sideband limit applies and the ion is cooled to its lowest quantum state [8]. Thus a single ion is localized to a region much smaller than the wavelength of the cooling radiation (Lamb-Dicke limit), provided of course that $\eta\ll 1$ (which can in fact be achieved in current experiments). In this limit, one can make an expansion of (2) and (3) in powers of η, and to first order in η one obtains [noting that $W(u)$ is an even function of u]

$$\frac{d}{dt}\rho=-i\left[\nu a^\dagger a+\frac{\Delta}{2}\sigma_z-\frac{\Omega}{2}\eta(\sigma_++\sigma_-)(a+a^\dagger),\rho\right]$$
$$+\frac{\Gamma}{2}(2\sigma_-\rho\sigma_+-\sigma_+\sigma_-\rho-\rho\sigma_+\sigma_-)\,. \qquad (4)$$

Making the associations $\Delta\leftrightarrow\omega_0$, $\nu\leftrightarrow\omega_f$, and $g\leftrightarrow\eta\Omega/2$, we have a clear analogy with the damped JCM. The additional terms proportional to $\sigma_+a^\dagger$ and σ_-a are usually dropped in the optical regime on the basis of the rotating-wave approximation. In our instance, such an approximation requires that $\nu,\Delta\gg\Omega,\Gamma,|\nu-\Delta|$, which is consistent with the sideband cooling limit assumed above. A significant feature of the JCM produced by this configuration is that dissipation in the system is entirely due to damping of the two-level transition, while the oscillator is undamped (i.e., $\kappa=0$). Further, the effective coupling constant $\eta\Omega/2$ is dependent on the Rabi frequency and could be readily adjusted in experiments to satisfy the strong coupling condition $\Gamma<\eta\Omega/2$ (note that an important consequence of having the ion at the node of the standing wave is that increasing Ω does not lead to heating of the ion; this does not hold for a traveling-wave light field [7]). This control of the coupling strength is in contrast with current cavity QED experiments where the coupling is determined by atomic constants and cavity characteristics which cannot be modified easily.

Under the conditions specified above, we reproduce the JCM, with energy eigenstates as described earlier in the paper, but with Fock states $|n\rangle$ describing the quantized atomic motion. The level scheme of the ion-trap system is shown in Fig. 1. The splitting between the levels $|1,+\rangle$ and $|1,-\rangle$ can be observed by measuring the spectrum of fluorescence emitted by the trapped ion. Figure 2 shows the spectrum computed from a numerical solution of the exact master equation (2) (using a finite basis set truncated at a suitable level). We note first that the contribution to the spectrum at the frequency ω_L is zero. This is due to the fact that the ion is located at the node of a standing wave. Hence, spectral features only appear about the vibrational frequencies $\nu=\pm 10\Gamma$. As can be seen in Fig. 2, the splitting is proportional to Ω. The observation of splitting in the spectrum demonstrates the entanglement of internal and external degrees of freedom, in analogy with cavity QED. The asymmetry observed is caused by non-rotating-wave-approximation terms (e.g., $\sigma_+a^\dagger$), which also lead to enhanced pumping of the levels with increasing Ω, producing larger values of the calculated spectra.

Aside from the doublet structure, additional resonances, corresponding to transitions involving higher ex-

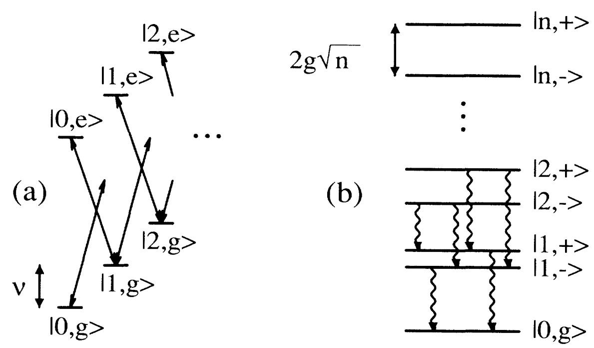

FIG. 1. Level scheme of ion-trap system. (a) Bare states $|n,g\rangle,|n,e\rangle$: The laser-atom detuning is $\omega_L-\omega_0=-\nu$. Arrows pointing to the left indicate the resonant laser coupling with strength $\eta\Omega$; arrows pointing to the right correspond to non-rotating-wave terms. There are no $|n,g\rangle$-$|n,e\rangle$ couplings since the ion is at the node of the standing wave. (b) Dressed states $|n,\pm\rangle$ resulting from the laser coupling.

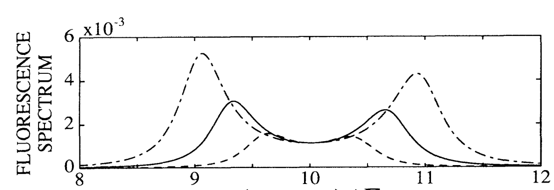

FIG. 2. Spectrum of resonance fluorescence of a trapped ion: emission doublet $|1,\pm\rangle\rightarrow|0\rangle|g\rangle$ with $\eta=0.01$, $\Omega=100\Gamma$, 150Γ, and 200Γ (solid, dashed, and dash-dotted lines, respectively), and $\nu=10\Gamma$.

cited states, may be observed in the fluorescence spectrum. An alternative approach to the observation of the level structure is a measurement of the probe field absorption spectrum. In order to perform such a measurement without perturbing the cooling and strong coupling we propose to employ a third atomic level $|r\rangle$ very weakly coupled to the otherwise strongly coupled $|g\rangle$-$|e\rangle$ transition ("V configuration"), analogous to level schemes used for the observation of quantum jumps [9], where "shelving" of the electron in the third level $|r\rangle$ produces a dark period in the observed fluorescence. The absorption spectrum can be very sensitively measured by observing quantum jumps to and from the level $|r\rangle$ as a function of the detuning of the probe laser from the $|g\rangle \rightarrow |r\rangle$ transition frequency, as indicated by the level scheme in Fig. 3(a).

Figure 3(b) shows an absorption spectrum for such a three-level system. To facilitate excitation of the higher levels $|n,\pm\rangle$ in the JCM ladder of states, we have assumed that additional thermal noise is pumping the ion, which could be easily achieved experimentally by adding weak broadband noise to the laser. The central maximum corresponds to the transition $|0,g\rangle \rightarrow |0,r\rangle$, while the additional maxima correspond to transitions between the dressed states $|n,\pm\rangle$ and the excited levels $|n,r\rangle$. A particularly interesting feature illustrated by this figure is that in the strong coupling limit, $\eta\Omega/2 \gg \Gamma$, each spectral line represents a *particular* transition $|0,g\rangle \rightarrow |0,r\rangle$ or $|n,\pm\rangle \rightarrow |n,r\rangle$. This is a consequence of the unequal spacing of the energy levels in the JCM, which means that at the frequency corresponding to a particular spectral line the probe laser is only resonant with the transition frequency between a *single* pair of levels, as depicted in Fig. 3(a), and thus will only excite the system to a single state $|n,r\rangle$.

This ability to selectively excite a particular transition, together with the state reduction associated with the observation of quantum jumps, offers the intriguing possibility of generating number states of the quantized trap motion. This follows from the fact that the probe laser exciting transitions to the states $|n,r\rangle$ interacts only with the atomic ground state contribution to the particular dressed state $|n,\pm\rangle$ being excited. Given that we are able to distinguish spectroscopically between the different maxima characterizing the absorption spectrum (so that we can identify the dressed state being excited), observation of a quantum jump to the weakly coupled state $|n,r\rangle$ will tell us with certainty that the vibrational state of the ion is $|n\rangle$ (i.e., this is the vibrational state occurring with $|g\rangle$ in the dressed state $|n,\pm\rangle$) and that we have produced a Fock state of the quantized trap motion.

An obvious consequence of having the freedom to choose which transition is excited is the ability to choose the Fock state that is to be produced. As an example, Fig. 4(a) shows the simulation of an experiment with a trapped three-level ion and the calculated random telegraph signal due to the probe excitation. Quantum jumps to the state $|2,r\rangle$ are indicated when emission windows appear in the fluorescence of the strongly coupled atom-trap system. Thus a Fock state of the trap motion with $n=2$ is prepared during these dark periods. Figure 4(a) shows the number of photons as observed in a real experimental situation where the integration time constant is long compared with the decay time of the strongly coupled system. To highlight the internal dynamics, we evaluate the temporal evolution of the mean quantum

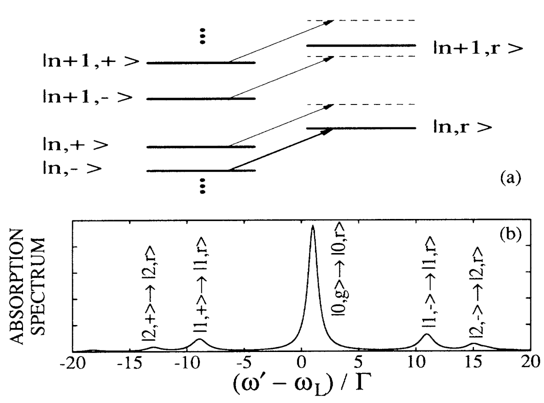

FIG. 3. (a) Level scheme for the two-level system $|g\rangle,|e\rangle$ coupled to a third metastable level $|r\rangle$ by a weak probe laser Ω'. (b) Weak field absorption spectrum to $|r\rangle$ as a function of the probe detuning $(\omega' - \omega_{rs})/\Gamma$ for $\eta=0.05$, $\Omega=400\Gamma$, $\Delta=\nu=50\Gamma$, $\Omega'=0.1\Gamma$, and $\Gamma_{rg}=0.01\Gamma$. Higher levels are excited by adding a thermal light field with mean photon number $N=0.5$. The displacement of the central peak from zero detuning is a result of the Bloch-Siegert shift.

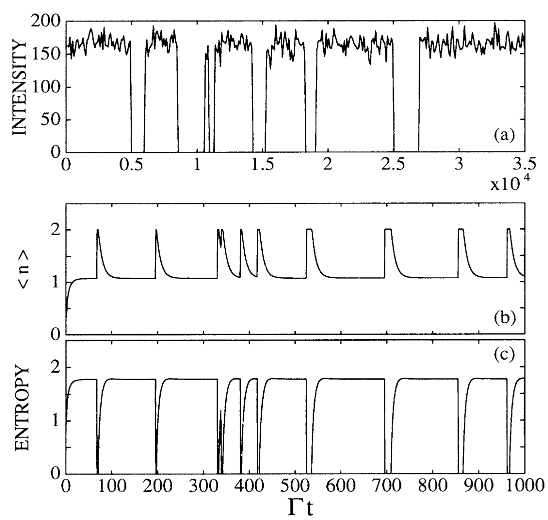

FIG. 4. Simulation of quantum jumps as a function of time. The probe laser is tuned to $|2,-\rangle \rightarrow |2,r\rangle$. (a) Fluorescence intensity on the $|e\rangle$-$|g\rangle$ transition ($\Omega'=2\times10^{-5}\Gamma$, $\Gamma_{rg}=10^{-7}\Gamma$). (b) Time evolution of $\langle n\rangle$, and (c) the entropy, $S=-\mathrm{Tr}[\rho\ln(\rho)]$, of the system ($\Omega'=0.1\Gamma$).

number $\langle n \rangle$ and the corresponding entropy of the system for a different set of parameters and over a time scale in which system relaxation is clearly visible. This is shown in Figs. 4(b) and 4(c) which demonstrate that after a quantum jump to the state $|r\rangle$ the system's entropy is suddenly reduced to zero (indicating the Fock state). After the ion returns to its strongly coupled states $|n, \pm\rangle$ the mean value $\langle n \rangle$ approaches its thermal equilibrium value (here set by the mean photon number N characterizing the broadband thermal noise field) and the entropy increases to its steady state value.

We would like to point out that this generalization of the use of quantum jumps for state reduction to the quantum states of motion in a trap exhibits a number of attractive features and opens the door to a variety of possible applications. First, restating what was said above, we conceive of the possibility of preparing Fock states with a quantum number that can be chosen at will within the limits of the observable number of sidebands simply by tuning the probe laser and observing quantum jumps. If the lasers are switched off after the quantum jump is observed, then, in the absence of strong coupling, the ion will spontaneously decay to its ground state and the final motional state will closely approximate a Fock state provided η is small, since the probability that the spontaneous decay is accompanied by a change in the motional quantum number n is proportional to η. Second, by including small deviations of the ions from the standing-wave node, or by introducing quantum jumps with two (or more) probe laser frequencies, it should be possible to prepare a multitude of nonclassical states, such as superpositions of Fock states. It should be emphasized that all of these predictions are of equal relevance to cavity QED, where analogous nonclassical states of light could be generated.

In summary, we have proposed a novel technique for the preparation of nonclassical states, in particular Fock states, in a strongly coupled oscillator-spin system, with emphasis on the example of quantized motion of a single trapped ion. This example served to highlight the potential of ion-trap experiments for studies of the JCM, thus exposing the well-established field of ion trapping as an alternative testing ground to strongly coupled cavity QED. Since for a trapped ion the coupling parameters can be varied by means of the laser field strength, it is to be expected that a multitude of investigations originating from cavity QED could be performed in an ion-trap configuration. As for an experimental realization, we believe that the parameters required for such an experiment are within the reach of current experiments, e.g., with In^+ or Ba^+ ions. The technical difficulty associated with reaching the strong confinement regime (i.e., $\nu \gg \Gamma$) may be overcome either with new miniaturized trap designs or by employing weak atomic transitions.

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