Universality of the Hall Effect in a Magnetic-Field-Localized Two-Dimensional Electron System

V. J. Goldman, J. K. Wang, and Bo Su
Department of Physics, State University of New York, Stony Brook, New York 11794-3800

M. Shayegan
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544
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We report Hall effect measurements of the localized two-dimensional electron system at low Landau level filling factors ν, including the regime previously identified as a pinned Wigner crystal. We find that the Hall resistivity is universally given by the classical, free-electron gas value of $h/\nu e^2$ to within 5% even when the diverging diagonal resistivity reaches $\sim 100h/\nu e^2$. This implies that both Hall and diagonal *conductivities* approach zero in the localized regime.

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Disorder, always present in experimentally studied two-dimensional electron systems (2DES), leads to magnetic-field-induced localization of 2DES in a high magnetic field B. Phenomenologically, the diagonal resistance (R_{xx}) diverges exponentially as B is increased at a low temperature and is thermally activated at a fixed B. This behavior of R_{xx} was seen when the fractional quantum Hall effect (FQHE) was first observed [1] and in many later experiments [2–6]. At sufficiently small Landau level filling factors ν ($\nu = nh/eB$, where n is the 2DES density) the ground state of a *disorderless* 2DES is expected to be a Wigner crystal (WC) [7]. The other known 2DES ground states in this regime are the FQHE states [8]. Both WC and FQHE states are induced by the interelectron Coulomb interaction $\sim e^2/\epsilon a$, where $a \sim n^{-1/2}$ is the WC lattice constant and ϵ is the host material dielectric constant. At $T=0$, as ν is decreased the ideal (disorderless) 2DES is expected to undergo at least one first-order phase transition between FQHE and WC states. Because WC energy is smooth in ν while the energy of the FQHE states varies nonmonotonically with ν, cusping down at simple rational ν, one or more reentrant WC-FQHE phases are expected; there is experimental evidence for one such reentrant WC phase between the $\frac{2}{9}$ and $\frac{1}{5}$ FQHE states [5,6].

If the temperature is raised at a fixed ν, the WC state is expected to undergo a melting transition, while the FQHE states do not exhibit a phase transition but rather the electron correlations gradually and continuously become less important as kT exceeds $\sim e^2/\epsilon a$. There is no phase transition as a function of the strength of disorder. In Ref. [5] we proposed that two cases should be distinguished: when the correlation length $L^2 \gg a^2$ (localized collective state, pinned WC), and when $e^2/\epsilon a$ is negligible compared with disorder (single-particle localization, Anderson insulator). The fairly wide intermediate range, $L^2 \sim a^2$, is a "Wigner glass." Qualitatively different behavior is exhibited by a localized 2DES depending on whether disorder or Coulomb interaction dominates [7].

The activated R_{xx} behavior can set in just below $\nu=1$ in 2DES samples with relatively high disorder or at $\nu \approx 0.22$ in the currently lowest disorder samples. Much less is known about the behavior of the Hall resistance R_{xy} in the localized regime because of the fundamental (topological) admixture of exponentially large and strongly B- and T-dependent R_{xx} into R_{xy} in experiments. In Ref. [3] we have reported that at 140 mK, R_{xy} of a 2DES remains close to the classical value of $B/en = h/\nu e^2$ in a range of ν around $\frac{1}{7}$. We were able to measure R_{xy} only at a relatively high $T \geq 140$ mK, where R_{xx}, while clearly in the localized regime ($\rho_{xx} > h/\nu e^2$), was not very high so that its admixture into R_{xy} could be subtracted.

In this paper we report our measurements of the Hall resistance R_{xy} in a high-quality 2DES in the regime of the magnetic-field-induced localization at low ν, including the WC phase. The data, for the first time, demonstrate the universality of the Hall resistance. In particular, $R_{xy} = \rho_{xy}$ remains at approximately the classical, free-electron value of $h/\nu e^2$ (excluding the plateau regions), deviating from it by not more than $0.05h/\nu e^2$ even when the diverging ρ_{xx} reaches values as high as $\sim 100h/\nu e^2$.

The measurements were done in an Oxford Instruments top loading into mixture dilution refrigerator and a high-field superconducting magnet. Temperatures were measured by a calibrated ruthenium oxide chip resistor with relatively small and known magnetoresistance and are believed to have an absolute accuracy of 5%. In order to eliminate the admixture of R_{xx} into R_{xy} by reversing direction of B, a high T stability was needed, on the order of 0.1 mK at $T=45$ mK. The wiring cutoff frequency in these measurements is less than 1 kHz (depending on two-terminal sample resistance) and the residual electromagnetic heat leak to the sample is estimated to be $\leq 2\times 10^{-14}$ W.

Several samples cut from a GaAs/AlGaAs wafer show quantitatively identical behavior. A brief illumination by a red light-emitting diode at a low T was used to prepare a 2DES sample. Conventional magnetotransport measurements (see Fig. 1) were used to determine n and to assess 2DES homogeneity and overall quality. It has been found that a nonuniform sample (nonuniform 2DES can be produced in certain illumination–thermal-annealing protocols) will have an R_{xx} admixture into R_{xy} which cannot be removed even by the B reversal. Here we present data obtained from one of the samples lithographically defined as a square "clover leaf" (inset in Fig. 1); four In-Sn alloyed contacts were located on the large square contact pads. This geometry was selected since it minimizes the admixture of R_{xx} into R_{xy} in a uniform 2DES in the sample (in the localized regime, when $\rho_{xx} \gg \rho_{xy}$) and also minimizes the contact resistance.

The swept-B magnetotransport measurements were done with a conventional lock-in ac technique at 3.8 Hz with the current through the sample I_x limited to less than 100 pA. I_x and either the longitudinal V_x or Hall V_y voltage drops were measured simultaneously (and in phase) by two vector lock-in amplifiers. The diagonal and Hall resistances were numerically calculated as $R_{xx}(\uparrow) = V_x(\uparrow)/I_x(\uparrow)$ and $R_{xy} = \frac{1}{2}[V_y(\uparrow)/I_x(\uparrow) - V_y(\downarrow)/I_x(\downarrow)]$, where $\uparrow,\downarrow$ denotes the direction of B. Thus the admixture of R_{xx} into R_{xy} is eliminated [within certain limits and to the first order, since in a uniform sample $R_{xx}(B\uparrow) = R_{xx}(B\downarrow)$, while $R_{xy}(B\uparrow) = -R_{xy}(B\downarrow)$], as well as the error due to the resistance of the contacts, which is usually rather large in these low ν, low T measurements.

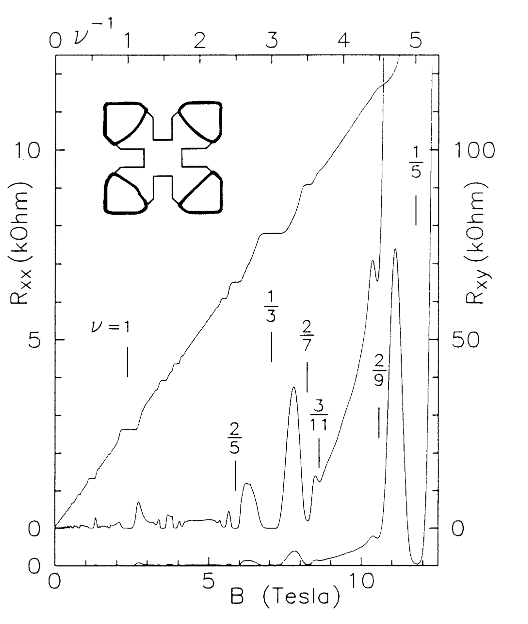

FIG. 1. Diagonal R_{xx} and Hall R_{xy} resistances of the 2DES sample at 60 mK vs magnetic field; the lowest trace shows $R_{xx}/10$. R_{xy} was obtained by averaging the $R_{xy}(B\uparrow)$ and $R_{xy}(B\downarrow)$ data. Electron density $n \simeq 5.71 \times 10^{10}$ cm^{-2}. Inset: The outline of the sample with four In-Sn contacts.

Figure 1 shows magnetoresistance of the sample at 60 mK. At lower B the trace exhibits the integer and FQHE. For $\nu < 0.25$ the magnetic-field-induced localization sets in, although FQHE features are clear at $\nu = \frac{2}{9}$ and $\frac{1}{5}$ (and $\frac{1}{7}$, not shown). Measurements done on samples cut from the same material [5] show that for $\nu > 0.25$ R_{xx} is *not* activated down to $T = 15$ mK. In the localized regime weak temperature dependence at higher T is followed by an activated resistance ($\ln R_{xx} \sim T^{-1}$ with an activation energy ~ 0.2 K) at lower T. Often there is a second linear region of $\ln R_{xx} \sim T^{-1}$ with a lower activation energy after which the T dependence becomes weaker than T^{-1}. This behavior resembles the activation to the conduction band, fixed- and variable-range hopping in bulk (3D) doped semiconductors [9].

Figure 2 illustrates the "raw" $R_{xy}(B\uparrow\downarrow)$ data, with the diagonal resistance admixed in, and the "cleaned" $\rho_{xy}(B) = R_{xy}(B) = \frac{1}{2}[R_{xy}(B\uparrow) - R_{xy}(B\downarrow)]$ data ($\rho_{xy} = R_{xy}$ in two dimensions). The diagonal resistivity $\rho_{xx}(B)$ was obtained as $(\pi/\ln 2)R_{xx}(B)$ from the experi-

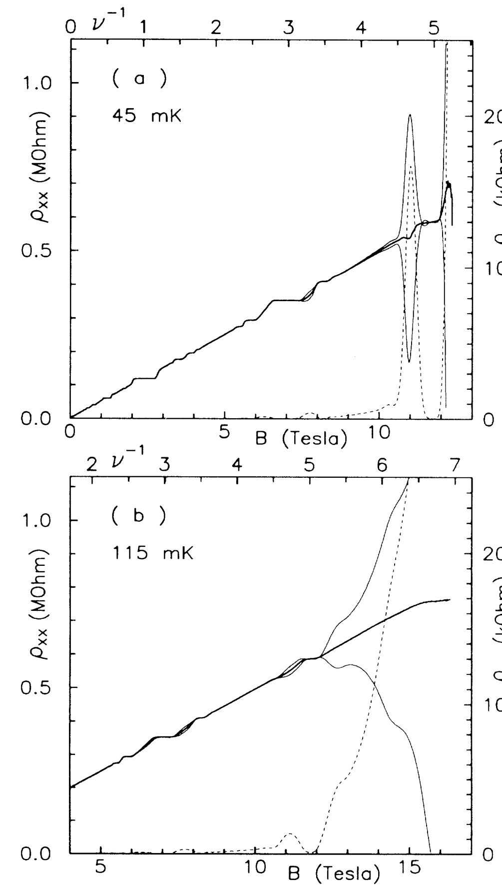

FIG. 2. $R_{xy}(B\uparrow)$ and $-R_{xy}(B\downarrow)$ data for both directions of magnetic field normal to the sample and their average, $\rho_{xy}(B)$. The dashed line gives $\rho_{xx}(B)$; its admixture into $R_{xy}(B)$ is largely eliminated by the averaging.

mental $R_{xx}(B)$ measured in the van der Pauw geometry. It should be noted that transport is nonlocal in a 2DES in the FQHE regime, because of the 1D edge-state conduction [10], and, therefore, the local 2D resistivities ρ_{xx} and ρ_{xy} are largely meaningless. Here, however, we are interested in the localized regime, where $\rho_{xx} > \rho_{xy} \approx h/ve^2$; in this regime there are no conducting edge states, the transport is local just like in the classical, low-B regime, and we can determine the local transport coefficients using the low-B formulas [11]. The absence of conducting edge states, percolating throughout the sample, in the localized regime is evidenced by the large two-terminal resistance $\gg h/ve^2$.

The cleaned $\rho_{xy}(B)$ data obtained at several temperatures are summarized in Fig. 3. These data demonstrate that ρ_{xy} remains at approximately the classical, free-electron value of h/ve^2 (except near the QHE plateau regions) even when the diverging ρ_{xx} reaches values as high as $\sim 100h/ve^2$. This linear behavior of ρ_{xy} of a 2DES is strikingly different from that observed in the magnetic-field-induced localization in the bulk (3D) doped semiconductors, where ρ_{xy} first undershoots the free-electron line at B just below the transition and then significantly overshoots the free-electron line at fields higher than critical [12].

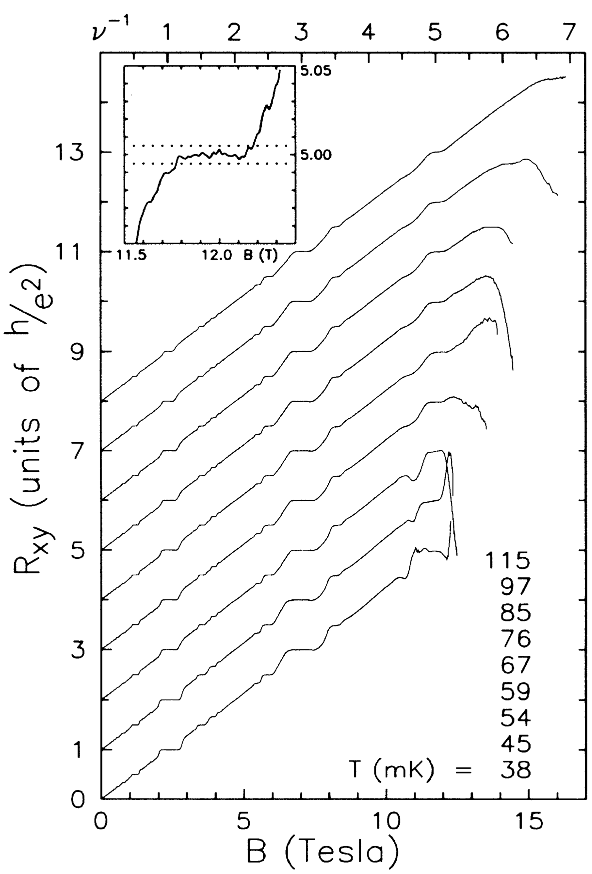

FIG. 3. $R_{xy}(B) = \rho_{xy}(B)$ at several temperatures. The localized regions extend between 10.7 and 11.3 T and for $B > 12$ T. The deviations of $\rho_{xy}(B)$ from the linear behavior at the high-B ends of the traces are instrumental and irreproducible. The traces are offset vertically by $1h/e^2$. Inset: A blowup of $\rho_{xy}(B)$ near $\nu = \frac{1}{5}$; dotted lines are at $\pm 0.1\%$; the plateau is quantized to 0.05% (limited by instrumental noise).

The deviations from the linear behavior at the high-B ends of the ρ_{xy} traces are instrumental and, generally, irreproducible. They occur when the experimental T stability is insufficient and the $R_{xx}(B\uparrow)$ and $R_{xx}(B\downarrow)$ admixtures into R_{xy} do not cancel. The data of Fig. 3 corroborate and extend the more ambiguous, higher T results [3], where ρ_{xx} was limited to less than $2h/ve^2$. Our result, that is, the linearity of $\rho_{xy}(B)$, is consistent with the theoretical work by Zhang, Kivelson, and Lee who have shown, based on the Chern-Simons mean-field theory of the QHE, that ρ_{xy} of a localized 2DES remains *finite* (in the dc limit) at zero T, irrespective of the Coulomb interaction effects [13]. However, it is still not understood why Hall resistance of the localized 2DES has this particularly simple free-electron gas value of h/ve^2.

In Ref. [5] we have argued that an ideal Wigner crystal expected in a disorderless 2DES at low ν can be regarded as a finite correlation length WC in experimental 2DES samples with nonzero disorder. Such WC is pinned, mostly by negatively charged acceptors in the spacer layer, not very far from the 2DES plane, according to the numerical results of Ruzin, Marianer, and Shklovskii [14], and is insulating at zero temperature. Recent calculations show that in our experiments the crystalline order persists over many ($\sim 10^2$) unit cells (correlation length determined from the threshold electric field [5] is between $7a$ and $20a$) [15]. The mechanism of electrical conduction of the pinned WC at finite T is not clear at present; indeed there may be several mechanisms with the relative contributions changing with T. One of the proposed conduction processes is by thermally activated bound charged dislocation pairs [16]. Our present results show that whatever the conduction mechanism of the pinned WC may be, its Hall resistance is given by h/ve^2.

According to the evidence presented in Ref. [5], the reentrant pinned WC phase exists in a narrow region $\frac{2}{9} > \nu > \frac{1}{5}$ as well as for $\nu < \frac{1}{5}$; however, the effects of localization set in at temperatures higher than the WC melting temperature $T_m(\nu)$. Some of the data in Fig. 3 correspond to (T, ν) in the WC phase; for example, the 38 mK data near $B = 11$ T and the 67 mK data between 12 and 13 T. The Hall effect shows no features, such as cusps or kinks, when the FQHE-WC phase transition occurs. It should be also noted that our experiments now provide enough information to allow us to rule out identification of the low ν localized phase as the recently proposed "Hall crystal" [17]. A Hall crystal is predicted to possess the Hall *conductance* equal to $\nu e^2/h$ at $T \rightarrow 0$, while we observe Hall *resistance* equal to $h/\nu e^2$. Since $\rho_{xx} \rightarrow \infty$ as $T \rightarrow 0$ and $\rho_{xy} = h/\nu e^2$, the Hall conductivity in the localized regime $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2) \rightarrow 0$ as $T \rightarrow 0$.

In conclusion, we would like to point out that the classical, free-electron "excluded area" argument [18] predicts that $\rho_{xy} = h/\nu e^2$ in an inhomogeneous 2DES consisting of metallic percolating matrix with insulating inclusions. It appears, however, that such a model is irrelevant to the present case of macroscopically homogeneous magnetic-field localized 2DES, where quantum effects (like tunneling) should be important. Experimentally, the fact that ρ_{xx} diverges at a fixed ν with an activated behavior implies a localized 2DES state and rules out a metallic percolating conduction.

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Note added.—After this work was submitted for publication three of us (V.J.G., J.K.W., and B.S.) received a preprint from T. Sajoto, Y. P. Li, L. W. Engel, D. C. Tsui, and M. Shayegan reporting a study of the Hall resistance of the reentrant insulating phase around the $\frac{1}{5}$ fractional quantum Hall liquid. While they see a deviation of ρ_{xy} from the classical value, $\Delta\rho_{xy}$, they arrive at conclusions similar to ours by an analysis that shows that $\Delta\rho_{xy}$ at $\nu = 0.21$ is linearly proportional to ρ_{xx} at that ν, as the temperature is lowered, for $\rho_{xx} < 10\rho_{xy}$. We did not have to perform such an analysis since we were able to eliminate the admixture of ρ_{xx} into ρ_{xy}, being limited only by the temperature stability (± 0.05 mK at 40 mK) over the time it took to complete the $\rho_{xy}(B\uparrow\downarrow)$ runs at each T.

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