Electron-Electron Correlations and the Aharonov-Bohm Effect in Mesoscopic Rings

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We study the effect of correlations on the Aharonov-Bohm effect in small one-dimensional rings. We use the Luttinger and Hubbard models to describe electrons in the ring with two contacts on opposite sites and a magnetic field passing through the hole of the circuit. We show that correlations change the fundamental periodicity of the transmittance as a function of the magnetic flux. We interpret this new effect as a consequence of the charge and spin separation in the one-dimensional conductors.

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The Aharonov-Bohm (AB) effect [1] in normal metal rings has been the subject of many experimental and theoretical works. The experimental setup consists of a metallic ring with two contacts and a magnetic field through the hole of the conductor as shown in Fig. 1. In these circuits the AB effect consists of a small oscillation of the conductance as a function of the magnetic flux ϕ. In ideal systems the periodicity of the conductance is given by the flux quantum $\phi_0 = hc/e$. Al'tshuler, Aronov, and Spivak [2] predicted that impurities can change the period of the magnetoresistance to $\phi_0/2$. This effect has been observed experimentally in small aluminum and silver rings at low magnetic fields [3].

Most of the theoretical work on this subject was devoted to the study of the effect of impurities on these systems [4]. The correlation effects have not been considered in detail. During the last years there has been great activity concerning the problem of persistent currents in isolated rings [5, 6]. In this context the effect of correlations has been studied in some detail. In this problem of persistent currents in the presence of a magnetic field the whole physics is given by the evolution of the many-body energies with the magnetic flux. In the AB effect not only the energy levels are important, the matrix elements corresponding to the creation and destruction of electrons in the contacts also play an important role.

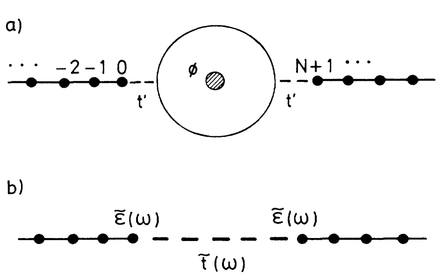

FIG. 1. Schematic circuit used in the calculation: (a) Original circuit including the ring and contacts; (b) equivalent system obtained in second order in t'.

In this Letter we show that in one-dimensional conductors correlations give rise to new effects which are reflected on the effective periodicity of the conductance.

We consider the circuit depicted in Fig. 1(a) consisting of a ring, two contacts, and weak links between the contacts and the ring. Electron-electron correlations are only included in the ring. We first present exact results obtained for the Luttinger Hamiltonian. The case in which the correlated electrons are described by a Hubbard Hamiltonian is also discussed.

The system is described by the following total Hamiltonian:

$$H = H_{\text{ring}} + H_{\text{cont}} + H_{\text{link}}, \tag{1}$$

where H_{ring} is a Hamiltonian describing the correlated electrons in the ring of total length L. The magnetic flux is included by taking boundary conditions given by $2\pi\phi/\phi_0$. The second term in Eq. (1) describes free particles in the contacts and is given by a tight binding Hamiltonian of semi-infinite chains with hopping t and on-site energy ε_0. This energy is chosen to fix the same electron density in the contacts and the ring. Finally, H_{link} is the Hamiltonian of the links between the ring and the contacts, with a hopping matrix element t'.

We calculate the transmittance of the circuit using a Landauer-type formalism. The calculation of the transmittance in mesoscopic systems with interactions has been the subject of many works in the past ten years [7–9]. Although formal exact expressions for the conductance exist in the literature, they involve the knowledge of exact propagators of the system including the contacts. [10]. In what follows we consider the case in which the coupling between the contacts and the ring is weak.

If $t' \ll t$ the part H_{link} of the total Hamiltonian may be considered as a perturbation. In second order in t' the problem is equivalent to the one depicted in Fig. 1(b), where $\tilde{t}(\omega)$ is a frequency dependent hopping that connects sites 0 and $N+1$, and $\tilde{\varepsilon}(\omega)$ is an effective on-site energy. The frequency ω is the energy of the incident electron supposed, to fix ideas, coming from the left. These effective parameters are given by [11]

$$\tilde{t}(\omega) = Lt'^2 G(x = L/2, \omega), \tag{2}$$

$$\tilde{\varepsilon}(\omega) = Lt'^2 G(x = 0, \omega), \tag{3}$$

where $G(x,\omega)$ is the Green function in real space and frequency of the isolated ring. In terms of these quantities the transmittance can be expressed as

$$|T(\omega)|^2 = \frac{4t^2 \sin^2(k)|\tilde{t}(\omega)|^2}{||\tilde{t}(\omega)|^2 - [\omega - \tilde{\varepsilon}(\omega) + te^{ik}]^2|^2}, \tag{4}$$

where k is the momentum of the incident electron taken as the Fermi momentum k_F. For noninteracting systems this expression is exact.

The whole problem then reduces to calculate the Green functions of a finite ring with arbitrary boundary conditions. For the sake of clarity we first discuss the case corresponding to the Luttinger model. The ring Hamiltonian then reads

$$H_{\text{ring}} = v_F \sum_{k,\sigma} k(a^\dagger_{1,k,\sigma} a_{1,k,\sigma} - a^\dagger_{2,k,\sigma} a_{2,k,\sigma}) + 2\pi\phi v_F (N_+ - N_-)/\phi_0 L + H_{\text{int}}; \tag{5}$$

the first term describes the kinetic energy of the bare electrons. Here $a^\dagger_{1,k,\sigma}$ and $a^\dagger_{2,k,\sigma}$ create right and left going electrons, respectively. The momentum k is given by $2\pi n/L$ with n an integer number. In the presence of a magnetic flux all the one-particle energies are shifted by $\pm 2\pi\phi v_F/\phi_0 L$. The effect of the magnetic flux is then given by the second term of Eq. (5) where $N_\pm$ is the total number of electrons traveling in each direction. The last term of the Hamiltonian describes the interaction between electrons.

Following Ref. [12] we obtained for the one-particle Green function the following result for $|\phi| < \phi_0/2$:

$$G(x,t) = \frac{i}{L} e^{ik_F x} e^{i\frac{2\pi\phi}{L\phi_0} v_F t} \Bigg(\frac{\theta(-t)}{[(1 - e^{-\frac{2\pi}{L}[\eta - i(x - v_s t)]})(1 - e^{-\frac{2\pi}{L}[\eta - i(x - v_c t)]})]^{1/2}} \times \frac{1}{[(\frac{L}{2\pi\eta})^2 (1 - e^{-\frac{2\pi}{L}[\eta - i(x - v_c t)]})(1 - e^{-\frac{2\pi}{L}[\eta + i(x + v_c t)]})]^\alpha} - \frac{\theta(t)}{\text{complex conjugate}} \Bigg) + (x \to -x, \phi \to -\phi). \tag{6}$$

Here v_s and v_c are the spin and charge velocities, respectively, η is a convergence factor, and α is a small number that depends on the interactions. This expression is exact for the isolated ring. Note that for $\phi = 0$, finite x and t and for $L \to \infty$ this expression reduces to the one of Refs. [12, 13] and for commensurate charge and spin velocities it is periodic in time.

As we mentioned, Eq. (6) is valid when the magnetic flux ϕ is smaller than half a flux quantum. For the general case one should note that at $\phi = \phi_0/2$ there is a level crossing and for $\phi > \phi_0/2$ the Fermi surface is shifted in $2\pi/L$. The Green function for $\phi_0/2 < \phi < \phi_0$ is given by Eq. (6) if ϕ is replaced by $\phi - \phi_0$ and k_F by $k_F \pm 2\pi/L$ for right and left going electrons, respectively. These changes in k_F give rise to a global factor of the form $\exp(i2\pi x/L)$. We used this expression to calculate the transmittance. The important physical ingredient induced by the correlations is the difference between the charge and spin velocities. These velocities depend on the particle density and the correlations. As we discuss below, the charge-spin deconfinement has important effects in the transmittance.

To calculate the transmittance we have to Fourier transform the Green functions. We resort to an approximation similar to that used in Ref. [14] in which the main idea is to take advantage of the smallness of the parameter α and consider only the dominant poles in $G(x = L/2, t)$ that contribute to the time integral. As an example, we may consider a case in which $v_c = 3v_s$, the dominant poles in $G(x = L/2, t)$ come from the divergencies of the first line in Eq. (6) that are of the form $1/(t - t_n)$ with $t_n = 3L(2n+1)/2v_c$ and n an integer. These are the times at which the spin and charge get together at the right contact of the ring ($x = L/2$). The other divergencies of this term are of the form $(t - t_m)^{-1/2}$ with $t_m = L(6m \pm 1)/2v_c$ and give only small corrections to the Fourier transform.

The more important contributions to $\tilde{t}(\omega)$ are then those in which both the charge and spin excitations propagate from one contact to the other arriving at $x = L/2$ at the same time. However, due to the difference in velocities, charge and spin excitations make a different number of loops in the ring and the time required for this propagation can be much longer than $L/2v_F$. As we discuss below, this effect is the origin of the anomalous field dependence of the conductance.

For the case of charge and spin velocities satisfying $v_s/v_c = p/q$, where p and q are small odd numbers, we obtain

$$\tilde{t}(\omega) = \frac{t'^2 L}{T_1 (v_s v_c)^{1/2}} \Bigg(\sum_{|k| > k_F} \left[\frac{e^{i\frac{kL}{2}}}{\omega - \frac{kL}{T_1} + \frac{2\pi\phi}{\phi_0 T_0} - i\eta} + (\phi \to -\phi) \right] + \sum_{|k| < k_F} [\eta \to -\eta] \Bigg), \tag{7}$$

where T_1 is the period of $G(x = L/2, t)|_{\phi=0}$ and $T_0 = L/v_F$. A similar expression is obtained for $\tilde{\varepsilon}(\omega)$ although for the evaluation of this quantity we take an energy cutoff.

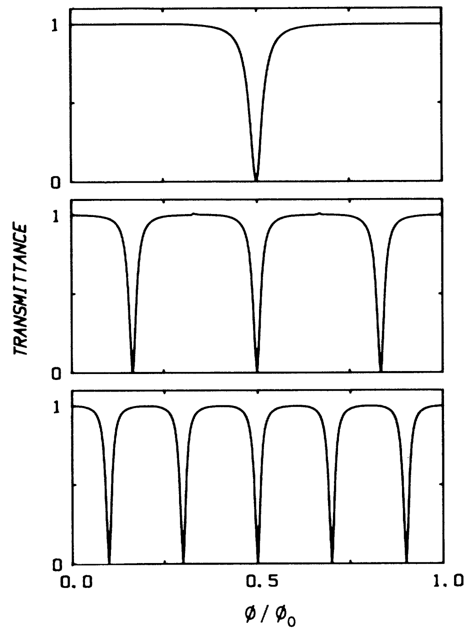

FIG. 2. Transmittance normalized to its value at zero flux for different values of the parameters. The parameters were chosen to get $T_1/T_0 = 1, 3$, and 5 from top to bottom. The case $T_1/T_0 = 1$ corresponds to the noninteracting system.

We evaluate the transmittance $|T(\omega)|^2$ obtained in this way for different values of the parameters. To present the results we have averaged the transmittance over a small ω interval larger than the characteristic energy distance between the levels of the isolated ring.

In Fig. 2 we present results for three different sets of parameters. They were chosen in order to satisfy the relation $T_1 = mT_0$ with m an integer. For the uncorrelated system $v_c = v_s = v_F$ ($T_1 = T_0$) and the transmittance is periodic in ϕ with period ϕ_0 . We get quantitative agreement with the results obtained with conventional methods when the ring is described by a tight-binding Hamiltonian. For the correlated system the transmittance oscillates with period ϕ_0/m. This is a new effect that was missed in the past. In general T_1 will not be a multiple of T_0 and the transmittance oscillates with a fundamental frequency $T_1/T_0\phi_0$ for $-\phi_0/2 \leq \phi \leq \phi_0/2$, at $\phi = \pm\phi_0/2$ there is a change in the phase of the oscillation as shown in Fig. 3. In this case the real period of the transmittance is one flux quantum.

The results presented above correspond to $v_s/v_c = (2n+1)/(2m+1)$. For $(v_s/v_c)^{\pm 1} = 2n$ the transmittance is very small since the charge and spin excitations never get together at $x = L/2$ and the poles of the form $1/(t - t_n)$ never occur in $G(x = L/2, t)$.

For the case of electrons described by a Hubbard model with on site interaction U we expect essentially the same result. On one hand, the Green functions obtained for large x also have the same form as for the Luttinger model [14–16]. On the other hand, the low energy excitations with momentum $\simeq \pm k_F$ that contribute to $G(x = L/2, t)$ are also shifted rigidly by the magnetic flux. Following Ref. [15] we find that this shift is given by $\pm\pi\phi v_c \xi^2/\phi_0 L$, where the dressed charge ξ is a numerical constant that depends on the electron density and on the interaction U. Consequently, in the expression of the effective hopping $\tilde{t}(\omega)$, v_F is replaced by $v_c\xi^2/2$ and the characteristic time T_0 is given by $T_0 = 2L/v_c\xi^2$. In the present case, due to the discretization of the space, we expect the behavior shown in the figures regardless of the rationality of v_c/v_s.

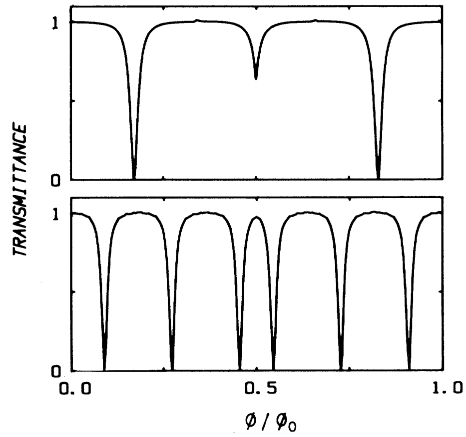

FIG. 3. Same as in Fig. (2) with parameters corresponding to $T_1/T_0 = 2.9$ (top) and 5.5 (bottom).

In summary, we have shown that correlations may change the fundamental AB periodicity of the conductance. We have presented results corresponding to the Luttinger and Hubbard models. In both cases correlations produce charge and spin separation in the one-dimensional conductors and we have shown that leading contributions to the transmittance are given by the propagation of charge and spin excitations that arrive at the same time to the right contact of the ring. When $v_c \neq v_s$ these leading terms correspond to processes in which charge and spin excitations make a different number of loops in the ring. This already suggests that the effective periodicity will be affected. However, a naive picture in which the change in the phase of the wave function is only given by the number of times the charge excitations surround the magnetic flux is not correct. This is clear in a slave boson approach, where the occurrence of the gauge field couples both the charge and spin excitations to effective fields.

In general the ratio v_s/v_c will not be of the form p/q with small p and q. The characteristic frequency of the conductance as a function of ϕ will then be large. That means that experimentally the many-body effects discussed above may only give some structure in the noise spectrum.

Finally, our expression for the transmittance suggests that the charge and spin deconfinement is not an essential ingredient for the change in the periodicity of $|T(\omega)|$. If there were a mechanism able to change the Green function periodicity T_1 and the characteristic time T_0 in a different way without producing charge and spin deconfinement it would give rise to an effect similar to the one discussed above. In any case charge and spin separation is a very efficient mechanism to change T_1 since a relatively small difference in the charge and spin velocities can make $T_1 \gg T_0$ and produce a fundamental period much smaller than ϕ_0.

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