"Dark" Squeezed States of the Motion of a Trapped Ion

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We propose a scheme for preparing coherent squeezed states of motion in an ion trap based on the multichromatic excitation of a trapped ion by standing- and traveling-wave light fields. The squeezed state is produced when the beat frequency between two standing-wave light fields is equal to twice the trap frequency, and is indicated by a "dark resonance" in the fluorescence emitted by the ion.

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In the field of quantum optics, nonclassical states of the electromagnetic field have a well-established theoretical and experimental background. An important example is the squeezed state, which possesses the property that one quadrature phase of the electric field has reduced fluctuations compared to the ordinary vacuum [1]. The ideal squeezed state of a general harmonic oscillator is defined as

$$|\alpha,\epsilon\rangle=\mathcal{D}(\alpha)\mathcal{S}(\epsilon)|0\rangle\,, \tag{1}$$

where $|0\rangle$ is the ground state of the oscillator, $\mathcal{D}(\alpha)=\exp(\alpha a^\dagger-\alpha^* a)$ is the displacement operator, and $\mathcal{S}(\epsilon)=\exp[(\epsilon^* a^2-\epsilon a^{\dagger 2})/2]$ is the "squeeze" operator, with $\epsilon=re^{i\theta}$ (r is the squeezing parameter). Here, $a,a^\dagger$ are destruction and creation operators for the harmonic oscillator, and the real and imaginary parts of α are proportional to the mean position and momentum of the harmonic oscillator, respectively [2]. Defining quadrature phase operators $a_{1,2}$ by $a=(a_1+ia_2)e^{i\theta/2}$ (i.e., position and momentum operators of the harmonic oscillator), measurable variances can be evaluated as $(\Delta a_{1,2})^2=\langle a_{1,2}^2\rangle-\langle a_{1,2}\rangle^2=e^{\mp 2r}/4$, from which it is clear that the noise in one quadrature is reduced below the standard quantum limit, as defined by $(\Delta a_1)^2=(\Delta a_2)^2=\frac{1}{4}$. The computed variances also demonstrate that the ideal squeezed state is a minimum-uncertainty state, i.e., $(\Delta a_1)(\Delta a_2)=\frac{1}{4}$.

Recently, spectacular advances in laser cooling and trapping have enabled the experimental observation of *nonclassical motion* of trapped ions and atoms [3,4], in confirmation of theoretical predictions. In ion trap experiments the trapping potential is provided typically by time-varying electric fields (rf Paul trap), whereas in the optical molasses experiments described in Ref. [4] atoms are confined in optical potentials formed by counterpropagating laser beams with orthogonal linear polarizations. In both cases, the quantized motion of the particles is indicated by the presence of (asymmetric) motional sidebands in probe field absorption spectra or in resonance fluorescence spectra.

This access to quantized states of motion invites further investigation of the possibility of generating novel and interesting nonclassical states, such as squeezed states of a physical harmonic oscillator, in analogy with the quantum theory of light. In this Letter, we propose a scheme for preparing squeezed states of the motion of a single trapped ion which should be realizable with existing or planned technology. This scheme is based on operation in the Lamb-Dicke regime, whereby a trapped two-level ion is localized to a region much smaller than the optical wavelength. Specifically, we consider an ion to be located at a common node of *two* (different frequency) standing-wave laser fields. We show that, in appropriate limits, when the beat frequency between the light fields is equal to twice the trap frequency [5], the *steady state* of the system is a *pure* state described by a product of the ground internal state of the ion and a squeezed state of the quantized motion. Hence, the generation of a squeezed state of the motion is indicated by the cessation of fluorescence emitted by the ion, i.e., by a so-called "dark state." Related dark states, created by coherent population trapping in nonabsorbing atomic states, have been studied in detail previously [6], and velocity-selective trapping states of this kind have been utilized in the context of laser cooling to reduce atomic kinetic energies below the one-photon recoil energy [7].

We note that schemes for the preparation of oscillator squeezed states in an ion trap have been proposed in Ref. [8], involving either a nonadiabatic change in or a parametric driving of the voltage between the electrodes of the ion trap (i.e., the trapping potential). In contrast to these schemes, our proposal utilizes externally applied lasers to produce squeezing, so that the confining potential produced by the voltage between the electrodes need not be modified.

Our analysis is based on the master equation, which, for a single two-level ion trapped in a harmonic potential and interacting with a laser field $E(\hat{R},t)$, can be written in the form [9]

$$d\rho/dt=-i[H,\rho]+\mathcal{L}\rho\,, \tag{2}$$

where $\mathcal{L}\rho$ describes spontaneous emission of the ion and

$$H=\nu a^\dagger a+\frac{\omega_0}{2}\sigma_z-\frac{d}{\hbar}[\sigma_+E^{(+)}(\hat{R},t)+\sigma_-E^{(-)}(\hat{R},t)]\,, \tag{3}$$

where $a,a^\dagger$ are annihilation and creation operators for the trap motion ($\hat{R}\propto a+a^\dagger$ is the position operator of the ion), ν is the trap frequency, $\sigma_{\pm,z}$ are the Pauli spin matrices describing the two-level transition of frequency ω_0, and d is the transition dipole moment. In the above, we have adopted the (optical) rotating-wave approximation. We assume that the ion is located at a common node of two standing-wave laser fields with frequencies located symmetrically about a carrier frequency ω_L, at which we put an additional traveling-wave field (this additional field is not essential, but, as we shall show, it can be used to give the final motional state a nonzero coherent amplitude). Hence, the appropriate form for $E^{(+)}(\hat{R},t)$ is

$$E^{(+)}(\hat{R},t)=E_1\sin(k_1\hat{R})e^{-i(\omega_L-\delta)t-i\phi_1}+E_2\sin(k_2\hat{R})e^{-i(\omega_L+\delta)t-i\phi_2}+E_0e^{ik_0\hat{R}}e^{-i\omega_Lt-i\phi_0}. \tag{4}$$

The operators $k_j\hat{R}$ can be reexpressed in the form $k_j\hat{R}\equiv\eta_j(a+a^\dagger)$, where $\eta_j=\pi a_0/\lambda_j$, with a_0 the amplitude of the ground state of the trap potential and λ_j the optical wavelength. We shall restrict ourselves to the limit $\eta\ll 1$ (which can be achieved in experiments). With the additional assumption that the trap frequency ν is larger than the decay rate Γ of the excited internal state, and hence that sideband cooling may be applied to cool the ion to its lowest quantum state [3], it follows that a single ion can be localized to a region much smaller than the wavelength of the cooling radiation (Lamb-Dicke limit) [10]. Hence, we can make an expansion of the master equation to first order in η_j. After transforming to a rotating frame defined by the unitary transformation $\tilde{\rho}=\mathcal{U}^\dagger\rho\mathcal{U}$ with $\mathcal{U}=\exp[-i(\delta a^\dagger a+\omega_L\sigma_z/2)t]$, we obtain

$$\frac{d\tilde{\rho}}{dt}=-i[H',\tilde{\rho}]+\frac{\Gamma}{2}(2\sigma_-\tilde{\rho}\sigma_+-\sigma_+\sigma_-\tilde{\rho}-\tilde{\rho}\sigma_+\sigma_-)\,, \tag{5}$$

with

$$H'=(\nu-\delta)a^\dagger a+\frac{\Delta}{2}\sigma_z+\left\{g_2[\sigma_+ae^{-2i\delta t}+\sigma_+a^\dagger]+g_1[\sigma_+a+\sigma_+a^\dagger e^{2i\delta t}]+\frac{\Omega_0}{2}\sigma_++\text{H.c.}\right\}. \tag{6}$$

Here $\Delta=\omega_0-\omega_L$, $g_j=\eta_j\Omega_je^{-i\phi_j}/2$, and Ω_j is the (real) Rabi frequency of the jth laser (we have set $\phi_0=0$ without any loss of generality). We have assumed that $\Omega_0\ll\Omega_{1,2}$ so that terms of order $\eta_0\Omega_0$ can be neglected; this simplifies the problem and also prevents significant heating of the ion. We note also that the presence of both red- and blue-detuned laser light leads to a competition between cooling or heating of the ion, so we must assume that $|g_1|>|g_2|$ in order for cooling to predominate.

Further simplification of H' is possible if ν and δ are much larger than any other parameters characterizing the problem (i.e., if $\nu,\delta\gg|g_{1,2}|,|\Omega_0|,|\Delta|,|\nu-\delta|,\Gamma$), for in this case the exponentials appearing in (6) are rapidly oscillating and hence these terms may be neglected, so that H' becomes

$$H'=(\nu-\delta)a^\dagger a+\frac{\Delta}{2}\sigma_z+\left\{\sigma_+(g_1a+g_2a^\dagger)+\frac{\Omega_0}{2}\sigma_++\text{H.c.}\right\}. \tag{7}$$

The coupling terms in H' are associated with cooling and heating of the ion, whereby the vibrational quantum number decreases (e.g., $g_1\sigma_+a$) or increases (e.g., $g_2\sigma_+a^\dagger$), respectively. The coherent competition between these two processes establishes Raman coherences between alternate vibrational states (i.e., between vibrational states $|n-1\rangle$ and $|n+1\rangle$). Squeezing in the harmonic oscillator is associated with coherences of this sort. For the particular choice of detuning $\delta=\nu$, the unitary transformation

$$\tilde{P}=S^\dagger(\epsilon)D^\dagger(\alpha)\tilde{\rho}S(\epsilon)D(\alpha)\,, \tag{8}$$

with ϵ and α given by

$$\epsilon=re^{i\theta},\quad \tanh(r)=|g_2/g_1|<1,\quad \theta=\phi_1-\phi_2\,,$$
$$\alpha=-\Omega_0/2g,\quad g=|g_1|\cosh(r)-|g_2|\sinh(r)\,, \tag{9}$$

reduces Eq. (5) to the form

$$\frac{d\tilde{P}}{dt}=-i\left[\frac{\Delta}{2}\sigma_z+(g\sigma_+a+\text{H.c.}),\tilde{P}\right]+\frac{\Gamma}{2}(2\sigma_-\tilde{P}\sigma_+-\sigma_+\sigma_-\tilde{P}-\tilde{P}\sigma_+\sigma_-)\,. \tag{10}$$

This master equation is equivalent to that of the Jaynes-Cummings model [11] of a two-level atom coupled to a harmonic oscillator with atomic spontaneous emission included. Given an initial density operator $\tilde{P}(t=0)$, spontaneous transitions will cause a downward cascade of population until only the ground atomic ($|g\rangle$) and ground vibrational ($|0\rangle$) states are populated, i.e., $\tilde{P}(t=0)\to\tilde{P}(t=\infty)=|0\rangle|g\rangle\langle g|\langle 0|$. In the case that $|g|>\Gamma$, the time taken for this cascade (or the "cooling" time) will generally be of the order of a few times Γ^{-1}. Inverting our unitary transformation, we therefore find that the steady-state solution of the original master equation (5) is

$$\tilde{\rho}_{ss}=|g\rangle\langle g|\otimes|\alpha,\epsilon\rangle\langle\alpha,\epsilon|\,, \tag{11}$$

where $|\alpha,\epsilon\rangle$ is the squeezed state defined in (1). That is, in the steady state this configuration yields a *pure* state given by the product of a squeezed state of the quantized motion with the ground internal state $|g\rangle$ of the ion. Hence, the choice of detuning $\delta=\nu$, and the consequent generation of a squeezed state, coincide with a dark state for the emitted fluorescence. An analogous dark state, coinciding with the production of a coherent field state,

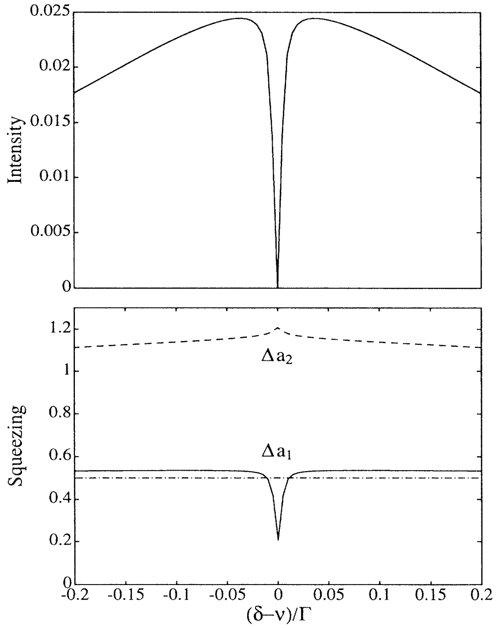

FIG. 1. Fluorescence intensity and quadrature phase variances as a function of detuning δ, with $g_1=0.1\Gamma$, $g_2=0.07\Gamma$, $\theta=0$, and $\Omega_0=\Delta_0=0$. The dot-dashed line indicates the standard quantum limit. At $\delta=\nu$, a squeezed state of the motion is produced with $\langle n\rangle=1$, and a dark resonance occurs in the emitted fluorescence.

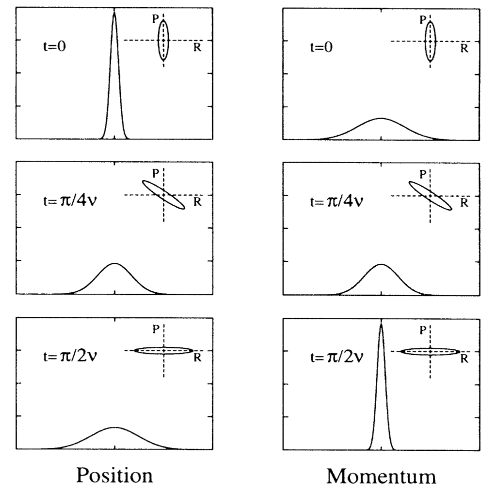

FIG. 2. Time dependence of the position and momentum distributions, and of the position-momentum error ellipse. The figure for $t=0$ corresponds to the situation depicted in Fig. 1 at $\delta=\nu$.

occurs in cavity QED when an externally driven two-level atom is coupled to a lossless cavity mode [12].

The mean vibrational occupation number is $\langle n\rangle \equiv \mathrm{Tr}(a^\dagger a\tilde{\rho}_{ss})=|\alpha|^2+\sinh^2(r)$. For the Lamb-Dicke approximation to hold, we evidently require that $\langle n\rangle$ not be large. The important effect of squeezing is a reduction in the variance of one quadrature phase, which in our case corresponds to a reduction in the uncertainty of either the position or momentum of the harmonically bound ion. In particular, defining quadrature phase (i.e., position and momentum) operators as in the introduction, and choosing $\theta=0$, one finds (for $\delta=\nu$) $(\Delta a_{1,2})=e^{\mp r}/2$. An example is shown in Fig. 1, where we plot the fluorescence intensity (which is proportional to the excited-state population $\langle\sigma_+\sigma_-\rangle$) and the quadrature phase variances as a function of the detuning $\delta-\nu$. The results were obtained from a numerical solution of the master equation with H' given by (7). At $\delta-\nu=0$ a dark resonance occurs, accompanied by a reduction in Δa_1 below the standard quantum limit, indicating that a squeezed state has been generated.

It is important to note that the squeezing exhibited in Fig. 1 is defined in a transformed (rotating) frame and for a particular choice of the phase θ. In the laboratory frame the position-momentum error ellipse will in fact rotate at the frequency ν, as shown in Fig. 2, so that the axis of squeezing oscillates between position and momentum. In Fig. 2, the coherent amplitude α is zero, so the situation depicted in this figure is analogous to a "squeezed vacuum" in quantum optics. The initial distributions at $t=0$ are set by the choice of phase $\theta=0$.

Increasing the amplitude of g_1 has an interesting effect on the intensity and squeezing profiles, as illustrated in Fig. 3. Squeezing now occurs over a broader range of detunings, and the energy-level structure associated with H' becomes more pronounced, with the various maxima corresponding to single-quantum and multiquantum resonant excitation of transitions between various pairs of energy levels. Details and explanations of these resonances will be discussed in another paper.

Further, we have assumed that the two standing-wave fields are collinear. We could also assume that the fields are perpendicular to each other, in which case our scheme would produce "two-mode" (i.e., two-direction) squeezed states of the motion [13].

The observation of a dark resonance in the fluorescence intensity constitutes an indirect detection of the squeezed state. A more direct indication of squeezing could be provided by the observation of "collapses" and "revivals" of the atomic inversion as follows. Once the steady state has been established, one may switch off (suddenly) the blue-detuned standing-wave field ($\Omega_2=0$) and the traveling-wave field ($\Omega_0=0$), so that, in our model, the system is described by the Hamiltonian $H'=(g_1\sigma_+a+\mathrm{H.c.})$ (for $\nu=\delta$ and $\Delta=0$), with the oscillator initially in a coherent squeezed state and the ion in its ground state. The time-dependent (transient) dynamics of such a system has been studied previously [14], and the population inversion of the two-level transition exhibits collapses and revivals (for sufficiently large values of g_1/Γ), which result from interferences between the individual responses

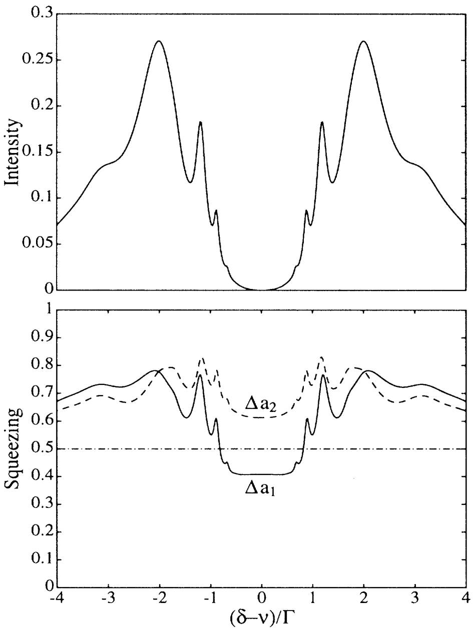

FIG. 3. Fluorescence intensity and quadrature phase variances as a function of detuning δ, with $g_1 = 2\Gamma$, $g_2 = 0.4\Gamma$ ($\langle n \rangle = 0.04$ at $\delta = \nu$), $\theta = 0$, and $\Omega_0 = \Delta_0 = 0$. The dot-dashed line indicates the standard quantum limit.

of the different (discrete) number states associated with the quantized harmonic oscillator. With the oscillator initially in a coherent squeezed state, the collapse and revival times are explicitly dependent on the squeezing parameter r and the phase θ. Collapses and revivals in a quantized ion trap (for a different configuration) have already been predicted with the oscillator assumed to be in an initial coherent state [15]. A very significant feature of the revivals is that they are a uniquely quantum phenomenon arising from the discreteness of the harmonic oscillator states. We have investigated possible experimental configurations for the observation of collapses and revivals in an ion trap, and our scheme should be possible with available ions using a metastable level for the two-level transition and probing the inversion via excitation to a third short-lived level. Details of our investigations, and of the collapse and revival phenomena, will be presented elsewhere.

In conclusion, we have proposed a scheme for generation of squeezed states of the position and momentum of a trapped ion, the signature of these squeezed states being the occurrence of a dark resonance in the fluorescence emitted by the ion for a particular choice of laser frequencies. We believe that such a scheme should be experimentally feasible with, for instance, In^+ ions for which the relevant transition linewidth is $\Gamma/2\pi = 0.3$ MHz. Trap frequencies $\nu/2\pi \sim 10$–20 MHz, which appear to be within reach of experimentalists with new miniaturized trap designs [16], should then suffice to see the effects that we have predicted.

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