No Time Asymmetry from Quantum Mechanics

Don N. Page[(a)]
CIAR Cosmology Program, Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1
(Received 16 February 1993)

With CPT-invariant initial conditions that commute with CPT-invariant final conditions, the respective probabilities (when defined) of a set of histories and its CPT reverse are equal, giving a CPT-symmetric universe. This leads me to question whether the asymmetry of the Gell-Mann–Hartle decoherence functional for ordinary quantum mechanics should be interpreted as an asymmetry of *time* .

PACS numbers: 03.65.–w, 05.70.Ln, 11.30.Er, 98.80.Hw

There are many time asymmetries observed in our Universe (not all unrelated), such as the thermodynamic arrow of time, the arrow of retarded radiation, the psychological arrow, the expansion of the Universe, and the T noninvariance of the K^0 system. The collapse of the wave function through the process of measurement [1] has sometimes appeared to be an independent quantum arrow of time [2], though it has also been ascribed to the thermodynamic time asymmetry of the external measuring apparatus or environment [3].

Aharonov, Bergmann, and Lebowitz [4] have proposed a time-symmetric generalization of ordinary quantum mechanics by using ensembles of histories with both initial and final states. Griffiths [5], and later Unruh [6] and then Gell-Mann and Hartle [7–9], have developed a similar formulation in terms of an initial and a final density matrix. In this formulation, ordinary quantum mechanics corresponds to the case in which the final density matrix is proportional to the identity, which denotes a final condition of indifference and which Gell-Mann and Hartle argue gives ordinary quantum mechanics an arrow of time.

Here I shall prove a theorem implying the CPT invariance of probabilities in ordinary quantum mechanics when the initial density matrix is CPT invariant, which is thus sufficient to give a CPT-invariant universe, assuming, as I shall do throughout, that the Hamiltonian is CPT invariant. I shall follow this with some speculative interpretations of the asymmetry of the Gell-Mann–Hartle formulation of ordinary quantum mechanics.

Gell-Mann and Hartle [10, 11] formulate the laws of generalized quantum mechanics for a closed system in terms of a decoherence functional

$$D(\alpha,\alpha') = \mathrm{Tr}(\rho_f C_\alpha \rho_i C^\dagger_{\alpha'})/\mathrm{Tr}(\rho_f\rho_i), \tag{1}$$

where ρ_i is an initial density matrix, ρ_f is a final density matrix, and

$$C_\alpha = P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1) \tag{2}$$

is a string of projection operators representing the history $\alpha = (\alpha_1,\ldots,\alpha_n)$ in the Heisenberg picture, with $t_1 < t_2 < \cdots < t_n$. Alternatively, C_α could be a sum of such strings. When $\{\alpha\}$ is an exhaustive set of histories, meaning

$$\sum_\alpha C_\alpha = I, \tag{3}$$

and when this set decoheres, meaning

$$\mathrm{Re}\, D(\alpha,\alpha') = 0 \text{ for } \alpha \neq \alpha', \tag{4}$$

then the diagonal elements of the decoherence functional give the probabilities for all histories of that set:

$$p(\alpha) = D(\alpha,\alpha) = \mathrm{Tr}(\rho_f C_\alpha \rho_i C^\dagger_\alpha)/\mathrm{Tr}(\rho_f\rho_i). \tag{5}$$

Ordinary quantum mechanics corresponds to the special case of this in which ρ_i is proportional to the density matrix of the system and ρ_f is proportional to the identity matrix I, giving a final condition of indifference. In this case the difference between ρ_i and ρ_f leads to an asymmetric decoherence functional $D(\alpha,\alpha')$ and set of diagonal elements $p(\alpha)$, which Gell-Mann and Hartle interpret as the (ordinary) quantum-mechanical arrow of time.

To be specific, suppose the initial and final density matrices are separately CPT invariant but not the CPT reverses of each other (so each separate state is time symmetric, by which I shall henceforth mean CPT invariant rather than T invariant in order for the dynamical laws to be time symmetric):

$$\rho_i = \Theta\rho_i\Theta^{-1},\ \ \rho_f = \Theta\rho_f\Theta^{-1}, \tag{6}$$

$$\rho_f \neq \Theta\rho_i\Theta^{-1}, \tag{7}$$

where Θ is the antiunitary CPT operator. Follow Gell-Mann and Hartle [8] in defining the CPT-reversed history $\tilde\alpha$ represented by the string

$$\tilde C_\alpha = \tilde P^1_{\alpha_1}(-t_1)\cdots \tilde P^n_{\alpha_n}(-t_n), \tag{8}$$

with

$$\tilde P^k_{\alpha_k}(-t_k) = \Theta^{-1}P^k_{\alpha_k}(t_k)\Theta, \tag{9}$$

and with the order of the projection operators reversed to put the earlier times on the right and the later ones on the left, $-t_n < \cdots < -t_2 < -t_1$. This gives

$$\tilde C_\alpha = \Theta^{-1}C^\dagger_\alpha\Theta, \tag{10}$$

which is generally true even when C_α is a sum of strings (2). Then Eq. (6) implies that

$$\begin{aligned} D(\tilde\alpha,\tilde\alpha') &= \mathrm{Tr}(\rho_f \tilde C_\alpha \rho_i \tilde C^\dagger_{\alpha'})/\mathrm{Tr}(\rho_f\rho_i) \\ &= \mathrm{Tr}(\rho_i C_{\alpha'} \rho_f C^\dagger_\alpha)/\mathrm{Tr}(\rho_i\rho_f), \end{aligned} \tag{11}$$

0031-9007/93/70(26)/4034(4)$06.00

which would be the complex conjugate of $D(\alpha, \alpha')$ if $\rho_f = \rho_i$ [or if $\rho_f = \Theta\rho_i\Theta^{-1}$ if Eq. (6) is not assumed] but generally is not if the two density matrices are not so related. This is the Gell-Mann–Hartle asymmetry of quantum mechanics with differing initial and final conditions.

However, if the initial and final density matrices ρ_i and ρ_f commute, and if the CPT-reversed set of histories $\{\tilde{\alpha}\}$ obeys the decoherence condition

$$\mathrm{Re}\, D(\tilde{\alpha}, \tilde{\alpha}') = 0 \;\text{ for }\; \tilde{\alpha} \neq \tilde{\alpha}' \tag{12}$$

analogous to (4), so that the diagonal elements

$$p(\tilde{\alpha}) = D(\tilde{\alpha}, \tilde{\alpha}) = \mathrm{Tr}(\rho_f \tilde{C}_\alpha \rho_i \tilde{C}_\alpha^\dagger)/\mathrm{Tr}(\rho_f \rho_i) \tag{13}$$

obey the sum rules necessary for them to be interpreted as probabilities, then the separate CPT invariance of each density matrix implies that the respective probabilities of the corresponding sets of CPT-related histories agree, as the following theorem shows.

Theorem 1.—If the initial density matrix ρ_i and the final density matrix ρ_f commute, if they obey Eq. (6) and hence are each separately CPT invariant, and if the set of histories $\{\alpha\}$ and the corresponding CPT-reversed set $\{\tilde{\alpha}\}$ obey Eqs. (4) and (12) and hence decohere, then the corresponding probabilities of the respective individual histories, $p(\alpha)$ and $p(\tilde{\alpha})$ as given by Eqs. (5) and (13), are equal.

Proof.—Summing the decoherence condition (4) over all α' different from α and using the completeness relation (3) allows one to rewrite Eq. (5) as

$$\begin{aligned} p(\alpha) &= \mathrm{Re}\,\mathrm{Tr}(\rho_f C_\alpha \rho_i I)/\mathrm{Tr}(\rho_f \rho_i) \\ &= \mathrm{Re}\,\mathrm{Tr}(\rho_i \rho_f C_\alpha)/\mathrm{Tr}(\rho_f \rho_i), \end{aligned} \tag{14}$$

where the cyclic property of the trace is used here and below to get the C_α at the right end. Similarly, summing Eq. (12) over all $\tilde{\alpha}$ different from $\tilde{\alpha}'$, using (11) and the analog of (3), and then dropping the prime, converts Eq. (13) into

$$\begin{aligned} p(\tilde{\alpha}) &= \mathrm{Re}\,\mathrm{Tr}(\rho_f I \rho_i \tilde{C}_\alpha^\dagger)/\mathrm{Tr}(\rho_f \rho_i) \\ &= \mathrm{Re}\,\mathrm{Tr}(\rho_f \rho_i \tilde{C}_\alpha^\dagger)/\mathrm{Tr}(\rho_f \rho_i). \end{aligned} \tag{15}$$

Now Eq. (10), the cyclic property, Eq. (6), and the assumption that ρ_i and ρ_f commute give

$$\begin{aligned} \mathrm{Tr}(\rho_f \rho_i \tilde{C}_\alpha^\dagger) &= \mathrm{Tr}(\rho_f \rho_i \Theta^{-1} C_\alpha \Theta) \\ &= \mathrm{Tr}(\Theta \rho_f \Theta^{-1} \Theta \rho_i \Theta^{-1} C_\alpha) \\ &= \mathrm{Tr}(\rho_f \rho_i C_\alpha) = \mathrm{Tr}(\rho_i \rho_f C_\alpha). \end{aligned} \tag{16}$$

Therefore,

$$p(\alpha) = p(\tilde{\alpha}), \tag{17}$$

so the probabilities of CPT-related histories are equal under the assumptions above, even without assuming that the initial and final density matrices are the CPT reverses of each other ($\rho_i = \Theta^{-1}\rho_f\Theta$), Q.E.D.

As an example of a consequence of this theorem, consider the case in which C_α is a single string (2) with $P^1_{\alpha_1}(t_1)$ corresponding to low coarse-grained entropy and $P^n_{\alpha_n}(t_n)$ corresponding to high entropy, so that the history α has entropy increasing from the earliest time t_1 to the latest time t_n. Assuming that the definition of coarse-grained entropy is CPT invariant, so that $\tilde{P}^k_{\alpha_k}(-t_k)$ corresponds to the same entropy as $P^k_{\alpha_k}(t_k)$, then the CPT-reversed history $\tilde{\alpha}$ has entropy decreasing from the new earliest time $-t_n$ (that of the projection operator now adjacent to ρ_i) to the new latest time $-t_1$ (that of the projection operator now adjacent to ρ_f). Then under the conditions above (commuting CPT-invariant ρ_i and ρ_f), the probability of a history α with one thermodynamic time asymmetry is equal to that of the history $\tilde{\alpha}$ with the opposite thermodynamic time asymmetry, so long as both probabilities exist. In other words, the asymmetry of the decoherence functional does not give any preferred direction (in the sense of differing probabilities) for the thermodynamic arrow of time, even if one sticks with the convention [7–11] that the earliest times correspond to the operators nearest to ρ_i in the decoherence functional.

If we do have a final condition of indifference, $\rho_f \propto I$, which corresponds to ordinary quantum mechanics, it obviously commutes with any ρ_i and is CPT invariant. Therefore, in ordinary quantum mechanics the CPT invariance of the initial density matrix is sufficient to imply that the probabilities of a set of histories equal the corresponding probabilities of the CPT-reversed set (if both sets decohere, as is necessary to get probabilities obeying the sum rules). Such a universe would be CPT invariant, according to the definition of Gell-Mann and Hartle [8], even without their alternative sufficient condition

$$\rho_f = \Theta\rho_i\Theta^{-1}. \tag{18}$$

Thus we see that the Gell-Mann–Hartle formulation of quantum mechanics, even with greatly different commuting initial and final conditions (such as ordinary quantum mechanics with its final condition of complete indifference), does not by itself give any time asymmetry for the probabilities. It leads to CPT-symmetric universes if the initial and final conditions are separately CPT invariant. In this formalism, any such asymmetry in the probabilities must lie separately within the initial and/or final density matrix of the closed system. This result is not in conflict with the results of Gell-Mann and Hartle [8], who merely proposed Eq. (18) as a *sufficient* condition for a CPT-invariant universe. However, if one regards the probabilities of decohering sets of histories as basic and does not attach a meaning to the entire decoherence functional (which does have an asymmetry), one can avoid interpreting ordinary quantum mechanics as necessarily having any time asymmetry.

Of course, the time symmetry of the probabilities of CPT-reversed sets of decohering histories does not imply that each history with a significant probability within one of those sets is itself time symmetric, as was illustrated by the example above with changing entropy. It merely implies that the time-reversed history in the other set has the same probability. Thus observers in one of the histories may see that history as being time asymmetric, even if the overall initial and final quantum states are each separately time symmetric and so lead to equal corresponding probabilities for the two CPT-reversed sets of decohering histories. This would also be true under the alternative time-symmetric condition (18) of Gell-Mann and Hartle [8], as they indeed carefully point out.

Thus our observations of an apparently time-asymmetric history for our Universe [12, 13] do not yet appear to rule out either time-symmetric possibility (6) or (18), as is consistent with what Gell-Mann and Hartle [8] noted. Possibility (6) is exemplified by the Hartle-Hawking no-boundary proposal for the quantum state of the Universe [14–18]. Emulating Wheeler [19], one may say that our history of the Universe has "time asymmetry without time asymmetry" of the probabilities. One can summarize the situation by saying that not only do asymmetric boundary conditions in the Gell-Mann–Hartle sense [inequality (7)] not necessarily imply asymmetric probabilities, but also that symmetric conditions [with either the Gell-Mann–Hartle equation (18) or my equation (6)] do not necessarily imply symmetric histories.

The question now arises how to interpret the arrow of ordinary quantum mechanics in the formalism of Gell-Mann and Hartle. In contrast to the analysis above, which does not contradict any of their results, here I shall make some speculative interpretative comments which are my own views and are generally not held by Gell-Mann and Hartle. Before doing this, I should note that, as a consequence of the previous paragraph, both the arrow of their formalism and the time symmetry of the probabilities I have demonstrated (for commuting CPT-invariant initial and final conditions) are not testable within any one individual history of the Universe (e.g., ours) and therefore are both rather metaphysical. Nevertheless, one can say very little if one attempts to be a complete positivist, and therefore I shall continue to consider how a nonphysical metaobserver might view the entire Universe.

It seems to me that the asymmetry of the Gell-Mann–Hartle decoherence functional has more to do with the order and noncommutation of the density and/or projection operators than with any *time* asymmetry. It would exist even when all of these operators are completely stationary as well as CPT invariant, in which case it seems very unnatural to ascribe it to anything involving time.

The asymmetry seems to get associated with time because of the traditional rule of ordering the projection operators in Eq. (2) in time order, which Gell-Mann and Hartle have adopted in their formalism. They do note [7–11] that one would get an equivalent result by a CPT tranformation of the density and projection operators which gives them an antitime ordering, but they do not allow zigzags, in which the times in the successive operators are not monotonically decreasing or increasing.

One might have thought that the probabilities for sequences of alternatives would depend on the order in which the operators are written down to form the string C_α. This may indeed be true for nonadjacent operators (or strings of them). However, it turns out that the order of two *adjacent* substrings within a string does not affect the probabilities (so long as they exist for both orderings), as is shown by the following theorem (a generalization of the penultimate sentence of Sec. III of Hartle [20]):

Theorem 2.—Consider a set of histories $\{\alpha\} = \{(\alpha_1, \alpha_2, \alpha_3, \alpha_4)\}$ represented by

$$C_\alpha = c^4_{\alpha_4} c^3_{\alpha_3} c^2_{\alpha_2} c^1_{\alpha_1} \tag{19}$$

(where each substring α_i is independently allowed to take on all possible values) and a corresponding zigzag set $\{\hat{\alpha}\} = \{(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4)\}$ represented by

$$\hat{C}_{\hat{\alpha}} = \hat{c}^4_{\hat{\alpha}_4} \hat{c}^3_{\hat{\alpha}_3} \hat{c}^2_{\hat{\alpha}_2} \hat{c}^1_{\hat{\alpha}_1} = c^4_{\alpha_4} c^2_{\alpha_2} c^3_{\alpha_3} c^1_{\alpha_1} \tag{20}$$

with $c^2_{\alpha_2}$ and $c^3_{\alpha_3}$ interchanged. Then if both sets decohere, the corresponding probabilities are equal,

$$p(\tilde{\alpha}) = D(\tilde{\alpha}, \tilde{\alpha}) = p(\alpha) = D(\alpha, \alpha). \tag{21}$$

Proof.—To abbreviate the notation, let

$$c_i = c^i_{\alpha_i}, \quad c'_i = \sum_{\alpha'_i \neq \alpha_i} c^i_{\alpha'_i} = I - c_i. \tag{22}$$

Then the weak decoherence condition (4) implies

$$\begin{aligned} 0 &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_2 c_1 \rho_i c_1^\dagger c_2^\dagger c_3'^\dagger c_4^\dagger) \\ &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_2 c_1 \rho_i c_1^\dagger c_2^\dagger c_4^\dagger) - p(\alpha), \end{aligned} \tag{23}$$

$$\begin{aligned} 0 &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_2' c_1 \rho_i c_1^\dagger c_2^\dagger c_3^\dagger c_4^\dagger) \\ &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_1 \rho_i c_1^\dagger c_2^\dagger c_3^\dagger c_4^\dagger) - p(\alpha), \end{aligned} \tag{24}$$

$$\begin{aligned} 0 &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_2' c_1 \rho_i c_1^\dagger c_2^\dagger c_3'^\dagger c_4^\dagger) \\ &= \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_1 \rho_i c_1^\dagger c_2^\dagger c_4^\dagger) \\ &\quad - \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_2 c_1 \rho_i c_1^\dagger c_2^\dagger c_4^\dagger) \\ &\quad - \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_1 \rho_i c_1^\dagger c_2^\dagger c_3^\dagger c_4^\dagger) + p(\alpha). \end{aligned} \tag{25}$$

Combining Eqs. (23)–(25) gives

$$p(\alpha) = \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_3 c_1 \rho_i c_1^\dagger c_2^\dagger c_4^\dagger). \tag{26}$$

Similarly, the corresponding weak decoherence condition $\mathrm{Re}\, D(\hat{\alpha}, \hat{\alpha}') = 0$ for $\hat{\alpha} \neq \hat{\alpha}'$ gives

$$p(\hat{\alpha}) = \mathrm{Re}\,\mathrm{Tr}(\rho_f c_4 c_2 c_1 \rho_i c_1^\dagger c_3^\dagger c_4^\dagger). \tag{27}$$

Now the cyclic property of the trace allows us to move ρ_f to the right end of the matrix of Eq. (27), and then this matrix is the Hermitian conjugate of the matrix of Eq. (26). Thus the real parts of the traces are equal, Eq. (21), Q.E.D.

Note that the c_i's can be projection operators, or strings of them, or even sums of strings, but we do need a coarse graining of $\{\alpha\}$ to include the four histories represented by $C_\alpha = c_4 c_3 c_2 c_1$, $c_4 c_3' c_2 c_1$, $c_4 c_3 c_2' c_1$, and $c_4 c_3' c_2' c_1$ (not just the two histories C_α and $I - C_\alpha$), and similarly for $\{\hat{\alpha}\}$.

To interchange two nonadjacent substrings or sums of strings and get the same probabilities, we would need three permutations to get them through the intermediate substring and through each other. Without assuming that the two intermediate permutations also give decohering sets of histories, the decoherence of merely the initial and final sets is in many cases sufficient for proving the equality of their corresponding probabilities, but not always [21]. Thus a difference in the probabilities appears to be possible.

Therefore, except possibly for the caveat of the last paragraph, the motivation to exclude zigzags and keep the projection operators in time (or antitime) order is lost on me. Thus I am not convinced that the asymmetry that arises from the order of the projection operators relative to that of the density matrices should be associated with the order of time. In other words, I do not see that ordinary quantum mechanics with CPT-invariant initial conditions gives any *time* asymmetry, at least for the probabilities of an CPT-reversed pair of decohering sets of histories, although in a different sense one could say it is indeed quantum mechanics that allows nonunique histories, each of which can be time asymmetric even when the whole set of CPT-reversed pairs is not.

This paper was motivated by discussions with Murray Gell-Mann and James Hartle and was phrased more precisely as a result of many further discussions with them, for which I am deeply grateful, but it by no means represents their interpretation or meaning of time asymmetry, despite the fact that there is no direct contradiction between our basic results. A comment by Stephen Hawking on how his superscattering matrix for black hole formation and evaporation can lead to loss of coherence without being time asymmetric was remembered after the first part of this work was conceived and may have had a subconscious influence.

The hospitality of the University of California at Santa Barbara, of Northwest Airlines over the western United States, and of Nelson and Zena Page in Liberty, Missouri, where this work was first formulated and written up, is gratefully acknowledged. Financial support was provided in part by the Natural Sciences and Engineering Research Council of Canada.

[a] Electronic address: don@page.phys.ualberta.ca

[1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, translated by R. T. Beyer (Princeton Univ. Press, Princeton, 1955).

[2] R. Penrose, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1979), p. 581.

[3] D. Bohm, *Quantum Theory* (Prentice Hall, Englewood Cliffs, NJ, 1951), p. 608.

[4] Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).

[5] R. B. Griffiths, J. Stat. Phys. **36**, 219 (1984).

[6] W. G. Unruh, in *New Techniques and Ideas in Quantum Measurement Theory*, edited by Daniel M. Greenberger (New York Academy of Science, New York, 1986), p. 242.

[7] J. B. Hartle, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989–90 Jerusalem Winter School for Theoretical Physics*, edited by S. Coleman, J. B. Hartle, T. Piran, and S. Weinberg (World Scientific, Singapore, 1991), p. 65.

[8] M. Gell-Mann and J. B. Hartle, in *Physical Origins of Time Asymmetry*, Proceedings of the NATO Workshop, Mazagon, Spain, 30 September–4 October 1991, edited by J. J. Halliwell, J. Perez-Mercador, and W. H. Żurek (Cambridge Univ. Press, Cambridge, to be published) (University of California at Santa Barbara Report No. UCSBTH-91-31, gr-qc/9304023, April 1993); an earlier version appeared in *Proceedings of the First International A. D. Sakharov Conference on Physics, Moscow, U.S.S.R., 27–31 May 1991*, edited by L. V. Keldysh and V. Ya. Fainberg (Nova Science Publishers, Commack, NY, 1992).

[9] J. B. Hartle, in *Gravitation and Quantizations*, Proceedings of the 1992 Les Houches École d'été, Gravitation et Quantifications, 9–17 July 1992, edited by B. Julia and J. Zinn-Justin (North-Holland, Amsterdam, to be published) (University of California at Santa Barbara Report No. UCSBTH-92-21, gr-qc/9304006, April 1993).

[10] M. Gell-Mann and J. B. Hartle, in *Complexity, Entropy and the Physics of Information*, Santa Fe Institute Studies in the Sciences of Complexity Vol. VIII, edited by W. H. Żurek (Addison-Wesley, Reading, MA, 1990), p. 425, or in *Proceedings of the Third International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology*, edited by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1990).

[11] M. Gell-Mann and J. B. Hartle, in *Proceedings of the Twenty-Fifth International Conference on High Energy Physics, Singapore, 2–8 August 1990*, edited by K. K. Phua and Y. Yamaguchi (South East Asia Theoretical Physics Association and Physical Society of Japan, distributed by World Scientific, Singapore, 1990).

[12] P. C. W. Davies and J. Twamley, Classical Quantum Gravity (to be published).

[13] R. Laflamme, Classical Quantum Gravity (to be published).

[14] S. W. Hawking, in *Astrophysical Cosmology: Proceedings of the Study Week on Cosmology and Fundamental Physics, 28 September–2 October 1981*, edited by H. A. Brück, G. V. Coyne, and M. S. Longair (Pontificia Academia Scientiarum, Vatican City, 1982), p. 563.

[15] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).

[16] S. W. Hawking, Phys. Rev. D **32**, 2489 (1985).

[17] D. N. Page, Phys. Rev. D **32**, 2496 (1985).

[18] S. W. Hawking, R. Laflamme, and G. W. Lyons, Phys. Rev. D **47**, 5342 (1993).

[19] J. A. Wheeler, *Frontiers of Time* (North-Holland, Amsterdam, 1979), Chaps. 1 and 6 on "Law without law" and "Causal order without causal order."

[20] J. B. Hartle, in *Directions in General Relativity, Vol. 2: Papers in Honor of Dieter Brill* , edited by B. L. Hu and T. A. Jacobson (Cambridge Univ. Press, Cambridge, 1993), p. 129.

[21] J. B. Hartle (private communication).