Renormalized Perturbation Expansions and Fermi Liquid Theory

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We give a procedure for a renormalized perturbation expansion. It is demonstrated for the Anderson impurity model, but has wide potential application. To zero order it describes the Landau noninteracting quasiparticles. To first order in the renormalized interaction $\tilde{U}$ it gives the exact thermodynamic results for low temperatures, and to second order gives the exact T^2 coefficient for the resistivity. The approach is not restricted to the Fermi liquid regime and provides a framework for systematic corrections to Fermi liquid theory.

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Perturbation expansions in powers of the interaction for systems with strong interactions, such as occur in magnetic impurity systems, heavy fermions, and narrow band transition metal compounds, have proved to be largely intractable. Summation of subsets of diagrams in these expansions have seldom provided the basis for a correct quantitative theory. Finite order perturbation theory works well in some special cases at not too large interaction strengths [1]. Nonperturbative techniques based on the renormalization group, slave boson mean field theory, and for magnetic impurity problems exact solutions have been the main way in which progress has been made in understanding these systems, though much remains to be understood. Scaling and renormalization have been key concepts in this understanding, and form the basis for Wilson's numerical solution of the Kondo model [2] for magnetic impurities. The renormalization group as used by Wilson is based on the idea of progressively eliminating higher energy excitations to arrive at an effective Hamiltonian for the low energy excitations, which for the Kondo problem corresponds to Fermi liquid theory [3]. The idea of renormalization as developed in quantum field theory leading to a reorganized perturbation theory in terms of the physically observed fully dressed particles with the observed effective interactions [4] has not been exploited explicitly for strong correlation systems. Here we show that it is a very effective technique for dealing with strongly renormalized systems. We illustrate the approach by considering the Anderson model for correlated electrons in magnetic impurities. The Anderson model [5] for an impurity with a d level ϵ_d, and an on-site interaction U, hybridized with a matrix element $V_{\mathbf{k}}$ to a band of conduction electrons is given by

$$H=\sum_\sigma \epsilon_{d,\sigma}c^\dagger_{d,\sigma}c_{d,\sigma}+Un_{d,\uparrow}n_{d,\downarrow}+\sum_{\mathbf{k},\sigma}(V_{\mathbf{k}}c^\dagger_{d,\sigma}c_{\mathbf{k},\sigma}+V^*_{\mathbf{k}}c^\dagger_{\mathbf{k},\sigma}c_{d,\sigma})+\sum_{\mathbf{k},\sigma}\epsilon_{\mathbf{k},\sigma}c^\dagger_{\mathbf{k},\sigma}c_{\mathbf{k},\sigma}\,. \tag{1}$$

For this model the Fourier transform of the retarded double-time Green's function for the d electron can be expressed in the form

$$G_{d\sigma}(\omega)=\frac{1}{\omega-\epsilon_d-\Lambda(\omega)+i\Delta(\omega)-\Sigma_\sigma(\omega)}\,, \tag{2}$$

where $\Delta(\omega)=\pi\sum_{\mathbf{k}}|V_{\mathbf{k}}|^2\delta(\omega-\epsilon_{\mathbf{k}})$ is the parameter which controls the width of the virtual bound state resonance at ϵ_d in the noninteracting model ($U=0$), and $\Lambda(\omega)=P\sum_{\mathbf{k}}|V_{\mathbf{k}}|^2/(\omega-\epsilon_{\mathbf{k}})$. In the wide band limit with a flat weighted density of states $\Delta(\omega)$ is independent of ω and $\Lambda(\omega)\to 0$. The function $\Sigma_\sigma(\omega)$ is the proper self-energy within a perturbation expansion in powers of the local interaction U. We will need the corresponding irreducible four-point vertex function $\Gamma_{\sigma,\sigma'}(\omega,\omega')$. Our aim is to reorganize this perturbation expansion into a more convenient form to consider the strong correlation regime (large U, low T, and weak magnetic fields).

Our first step is to write the self-energy in the form

$$\Sigma_\sigma(\omega)=\Sigma_\sigma(0)+\omega\Sigma'_\sigma(0)+\Sigma^{\rm rem}_\sigma(\omega)\,, \tag{3}$$

which is simply a definition for the remainder self-energy $\Sigma^{\rm rem}_\sigma(\omega)$. Using this expression the Green's function given in Eq. (2) in the wide band limit can be written in the form

$$G_{d\sigma}(\omega)=\frac{z}{\omega-\tilde\epsilon_d+i\tilde\Delta-\tilde\Sigma_\sigma(\omega)}\,, \tag{4}$$

where z, the wave function renormalization factor, is given by $z=1/[1-\Sigma'_\sigma(0)]$, the prime denotes a derivative with respect to ω, and $\omega=0$ corresponds to the Fermi level. We assume the general theorem of Luttinger [6] that $\mathrm{Im}\Sigma(0)$ vanishes so that z is real. The "renormalized" quantities, which are denoted by a tilde, are defined by

$$\tilde\epsilon_d=z[\epsilon_d+\Sigma_\sigma(0,0)],\quad \tilde\Delta=z\Delta,\quad \tilde\Sigma_\sigma(\omega)=z\Sigma^{\rm rem}_\sigma(\omega)\,. \tag{5}$$

Two zeros are given in the argument of Σ of (5) to emphasize that this is to be evaluated at $T=0$, and in zero magnetic field, as well as at $\omega=0$. A renormalized four-point vertex function is defined by $\tilde\Gamma_{\sigma,\sigma'}(\omega,\omega')=z^2\times\Gamma_{\sigma,\sigma'}(\omega,\omega')$, and a renormalized interaction $\tilde U$ by the value of $\tilde\Gamma_{\sigma,\sigma'}(\omega,\omega')$ at $\omega=\omega'=0$, $\tilde U=\tilde\Gamma_{\sigma,\sigma'}(0,0)$.

The next step is to introduce rescaled creation and annihilation operators for the d electron via $c^\dagger_{d,\sigma}=\sqrt{z}\,\tilde c^\dagger_{d,\sigma}$, $c_{d,\sigma}=\sqrt{z}\,\tilde c_{d,\sigma}$, and to rewrite the Hamiltonian (1) in the form $H=\tilde H_{\rm qp}-\tilde H_c$, where $\tilde H_{\rm qp}$ will be referred to as the quasiparticle Hamiltonian. It can be written as $\tilde H^{(0)}_{\rm qp}+\tilde H^{(I)}_{\rm qp}$, where $\tilde H^{(0)}_{\rm qp}$ describes noninteracting particles and is given by

$$\tilde{H}^{(0)}_{\rm qp}=\sum_\sigma \tilde{\epsilon}_{d,\sigma}\tilde{c}^\dagger_{d,\sigma}\tilde{c}_{d,\sigma}+\sum_{\mathbf{k},\sigma}(\tilde{V}_{\mathbf{k}}\tilde{c}^\dagger_{d,\sigma}c_{\mathbf{k},\sigma}+\tilde{V}^*_{\mathbf{k}}c^\dagger_{\mathbf{k},\sigma}\tilde{c}_{d,\sigma})+\sum_{\mathbf{k},\sigma}\epsilon_{\mathbf{k},\sigma}c^\dagger_{\mathbf{k},\sigma}c_{\mathbf{k},\sigma}\,, \tag{6}$$

and $\tilde{H}^{(I)}_{\rm qp}$ is the interaction term, $\tilde{H}^{(I)}_{\rm qp}=\tilde{U}\tilde{n}_{d,\uparrow}\tilde{n}_{d,\downarrow}$. The Hamiltonian $\tilde{H}_c$ describes the counterterms and takes the form

$$\tilde{H}_c=\lambda_1\sum_\sigma \tilde{c}^\dagger_{d,\sigma}\tilde{c}_{d,\sigma}+\lambda_2\tilde{n}_{d,\uparrow}\tilde{n}_{d,\downarrow}\,, \tag{7}$$

where λ_1 and λ_2 are given by

$$\lambda_1=z\Sigma(0,0),\quad \lambda_2=z^2[\Gamma_{\uparrow,\downarrow}(0,0)-U]\,. \tag{8}$$

These equations are simply a rewriting of the original Hamiltonian (1). We note that by construction the renormalized self-energy $\tilde{\Sigma}_\sigma(\omega)$ is such that

$$\tilde{\Sigma}_\sigma(0,0)=0,\quad \Sigma'_\sigma(0,0)=0\,, \tag{9}$$

so that $\tilde{\Sigma}_\sigma(\omega)=O(\omega^2)$ for small ω, on the assumption that it is analytic at $\omega=0$. As $\tilde{\Gamma}_{\sigma,\sigma}(0,0)=0$ we also have

$$\tilde{\Gamma}_{\sigma,\sigma'}(0,0)=\tilde{U}(1-\delta_{\sigma,\sigma'})\,. \tag{10}$$

We now identify the Hamiltonian $\tilde{H}^{(0)}_{\rm qp}$ with the noninteracting quasiparticles of the Landau Fermi liquid theory, and $\tilde{H}^{(I)}_{\rm qp}$ as the quasiparticle interaction term. The form of $\tilde{H}_{\rm qp}$ follows from our general prescription which corresponds to that used in many field theoretic treatments of renormalization [4]. We note that $\tilde{H}_{\rm qp}$ in this case is identical in form to the effective Hamiltonian near the strong coupling fixed point for the Kondo model as obtained by Wilson [2] in his numerical renormalization group calculations. The Kondo model corresponds to the regime $\epsilon_d<0$ and $U\gg\pi\Delta$. We propose to show that $\tilde{H}_{\rm qp}$ is an effective Hamiltonian about the Fermi liquid fixed point for all parameter regimes of the Anderson model.

To develop a theory appropriate for the low temperature regime we follow the renormalization procedure as used in quantum field theory so that we can make a perturbation expansion in terms of our fully dressed quasiparticles (see, for instance, Ref. [4]). We take our renormalized parameters $\tilde{\epsilon}_d$, $\tilde{\Delta}$, and $\tilde{U}$ as known and reorganize the perturbation expansion in powers of the renormalized coupling $\tilde{U}$. The full interaction Hamiltonian is $\tilde{H}^{(I)}_{\rm qp}-\tilde{H}_c$. The terms λ_1, λ_2, and z are formally expressed as a series in powers of $\tilde{U}$,

$$\lambda_1=\sum_{n=0}^\infty \lambda_1^{(n)}\tilde{U}^n,\quad \lambda_2=\sum_{n=0}^\infty \lambda_2^{(n)}\tilde{U}^n,\quad z=\sum_{n=0}^\infty z^{(n)}\tilde{U}^n\,. \tag{11}$$

The coefficients $\lambda_1^{(n)}$, $\lambda_2^{(n)}$, and $z^{(n)}$ are determined by the requirement that conditions (9) and (10) are satisfied to each order in the expansion. The perturbation expansion is about the free quasiparticle Hamiltonian given in Eq. (6) so that the noninteracting propagator has renormalized parameters $\tilde{\epsilon}_d,\tilde{\Delta}$.

For most field theoretic models the counterterms are necessary to cancel the divergences which result where there is no high energy cutoff on the integrals. The condition that this procedure gives finite predictions in terms of renormalized parameters when the cutoff is removed from the regularized integrals is the condition that the field theory is renormalizable. In condensed matter systems there is some form of high energy cutoff so this type of reorganization of the perturbation series is not necessary. However, for strong correlation problems, such as magnetic impurity and heavy fermion systems where the quasiparticles are strongly renormalized, it is a very suitable procedure to adopt for low temperature calculations. We propose to show that it makes direct contact with both the Landau phenomenological and the microscopic formulations of Fermi liquid theory.

The Friedel sum rule [7] gives the occupation of the d level, $n_{d,\sigma}$, at $T=0$ and has the form

$$n_{d,\sigma}=\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\left(\frac{\epsilon_{d,\sigma}+\Sigma_\sigma(0,H)}{\Delta}\right), \tag{12}$$

in terms of the self-energy $\Sigma(\omega)$ (in a finite magnetic field H) for the expansion in powers of U for the "bare" Hamiltonian (1). It takes the same form for the renormalized expansion with all the parameters in (12) carrying a tilde (this follows from the definitions and the fact that the common factor of z in the argument of the $\tan^{-1}$ cancels). The quasiparticle interaction plays no role as $T\to 0$ and $H\to 0$ as $\tilde{\Sigma}(0,0)=0$ and so $n_{d,\sigma}$ corresponds to the noninteracting quasiparticle number.

There are two Ward identities [1,8] in the unrenormalized theory,

$$\left.\frac{\partial\Sigma_\sigma(\omega)}{\partial\omega}\right|_{\omega=0}+\left.\frac{\partial\Sigma_\sigma(\omega)}{\partial\mu}\right|_{\omega=0}=-\rho_{d,\sigma}(0)\Gamma_{\uparrow,\downarrow}(0,0) \tag{13}$$

and

$$\left.\frac{\partial\Sigma_\sigma(\omega)}{\partial h}\right|_{\omega=0}-\left.\frac{\partial\Sigma_\sigma(\omega)}{\partial\omega}\right|_{\omega=0}=-\rho_{d,\sigma}(0)\Gamma_{\uparrow,\downarrow}(0,0)\,, \tag{14}$$

where μ is the chemical potential and $h=g\mu_BH$, and $\rho_d(0)$ is the d density of states at the Fermi level. It can be shown that the two equations, (13) and (14), are valid in the renormalized theory (all quantities with a tilde). From Eq. (9) the ω derivative of $\tilde{\Sigma}(\omega)$ vanishes at $\omega=0$ so that the renormalized equations reduce to

$$\left.\frac{\partial\tilde{\Sigma}_\sigma(\omega)}{\partial h}\right|_{\omega=0}=\left.\frac{\partial\tilde{\Sigma}_\sigma(\omega)}{\partial\mu}\right|_{\omega=0}=-\tilde{\rho}_{d,\sigma}(0)\tilde{\Gamma}_{\uparrow,\downarrow}(0,0)\,, \tag{15}$$

where $\tilde{\rho}_{d,\sigma}(0)=\rho_{d,\sigma}(0)/z$ is the quasiparticle density of states at the Fermi level.

As the effects of the quasiparticle interactions go to zero as $T\to 0$ due to the cancellation with the counterterm giving $\tilde{\Sigma}(0,0)=0$ and $\tilde{\Sigma}'(0,0)=0$, the specific heat coefficient of the impurity $\gamma_{\rm imp}$ is due to the noninteracting quasiparticles and so $\gamma_{\rm imp}=2\pi^2k_B^2\tilde{\rho}_d(0)/3$, as in the usual Landau theory. In finite magnetic field the quasiparticle interaction gives an energy shift which is not canceled by the counterterm and the susceptibility is enhanced over the noninteracting quasiparticle value. The spin susceptibility is given by

$$\chi_{\rm imp}=\frac{(g\mu_B)^2}{2}\tilde{\rho}_d(0)\left(1-\frac{\partial\tilde{\Sigma}}{\partial h}\right)$$
$$=\frac{(g\mu_B)^2}{2}\tilde{\rho}_d(0)[1+\tilde{U}\tilde{\rho}_d(0)] \qquad (16)$$

and the charge susceptibility by

$$\chi_{c,\rm imp}=2\tilde{\rho}_d(0)(1+\partial\tilde{\Sigma}/\partial\mu)=2\tilde{\rho}_d(0)[1-\tilde{U}\tilde{\rho}_d(0)]\,. \qquad (17)$$

The well known Fermi liquid relation [1] relating $\gamma_{\rm imp}$, $\chi_{\rm imp}$, and $\chi_{c,\rm imp}$ follows on elimination of $\tilde{U}$ and $\tilde{\rho}_d(0)$.

In the Kondo regime ($U\gg\pi\Delta$, $\epsilon_d<0$, $\chi_{c,\rm imp}\to 0$), $\gamma_{\rm imp}$ can be written in terms of the Kondo temperature T_K, $\gamma_{\rm imp}=\pi^2 k_B/6T_K$, which defines our Kondo temperature. In this limit $n_d\to 1$ and hence from the renormalized Friedel sum rule $\tilde{\epsilon}_d=0$ and $\tilde{\rho}_d(0)=1/\pi\tilde{\Delta}$. This gives us all the renormalized parameters in the strong correlation regime in terms of T_K, $\tilde{U}=\pi\tilde{\Delta}=4k_BT_K$. It is possible to calculate the renormalized parameters perturbatively in powers of the ratio $U/\pi\Delta$ for the symmetric model using the perturbation theory of Yosida and Yamada [1]. In the renormalized theory we see from (17) that the physical requirement $\chi_{c,\rm imp}\geq 0$ implies that the renormalized perturbation expansion is always in the weak coupling regime corresponding to $\tilde{U}\leq\pi\tilde{\Delta}$.

The thermodynamic results can be obtained from the lowest order term in the renormalized perturbation for $\tilde{\Sigma}$, the tadpole diagram shown in Fig. 1(a), which gives

$$\tilde{\Sigma}^{(1)}(\omega,H)=\tilde{U}[n_{d,\sigma}^{(0)}(0,H)-n_{d,\sigma}^{(0)}(0,0)]\,. \qquad (18)$$

There is no wave function renormalization to this order so $z^{(1)}=0$ and to this order $\tilde{\Gamma}^{(1)}(\omega,\omega')=\tilde{U}$, $\lambda_2^{(1)}=0$, and $\lambda_1^{(1)}=\tilde{U}n_{d,\sigma}^{(0)}(0,0)$. The complete cancellation by the counterterm only occurs for $H=0$ and $T=0$. Calculation of the spin susceptibility from (18) gives (16), and similarly the charge susceptibility gives (17). The renormalized Ward identities in Eq. (15) show that higher order terms in $\tilde{U}$ cancel so that the first order calculations of $\chi_{\rm imp}$ and $\chi_{c,\rm imp}$ are exact.

Exact results for the impurity Green's function (2) to order ω^2 follow from the calculation of the second order diagram for $\tilde{\Sigma}$ shown in Fig. 1(b). There is no second order counterterm contribution to this diagram from λ_2 as $\lambda_2^{(1)}=0$. There is a contribution to z which is required to eliminate the contribution from the linear term in ω to this order and is given by

$$z^{(2)}=(\pi^2-12)/4\pi^2\tilde{\Delta}^2\,. \qquad (19)$$

Calculation of this diagram to order ω^2 gives

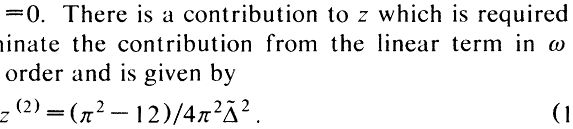

FIG. 1. The (a) first order and (b) second order diagrams within a perturbation expansion in powers of the renormalized interaction $\tilde{U}$ for the impurity Anderson model.

$${\rm Im}\tilde{\Sigma}_\sigma(\omega,0)=\frac{\tilde{U}^2\omega^2}{2\tilde{\Delta}(\pi\tilde{\Delta})^2}+O(\omega^4)\,. \qquad (20)$$

The spectral density of the noninteracting renormalized d Green's function describes the Kondo resonance and the ω^2 terms in $\tilde{\Sigma}_\sigma(\omega,0)$ give the low frequency corrections to this picture.

There is a temperature-dependent contribution to $\tilde{\Sigma}_\sigma(0,T)$ to first order of the form (18) with T replaced by H. For the particle-hole symmetric model, and more generally in the Kondo regime $n_d\to 1$, this vanishes as $n_{d,\sigma}(0,T)=n_{d,\sigma}(0,0)=1$ and the leading order temperature dependence (T^2) arises from the second order diagram 1(b). This result together with (20) can be used to calculate the T^2 contribution to the impurity conductivity $\sigma_{\rm imp}(T)$ and gives

$$\sigma_{\rm imp}(T)=\sigma_0\left\{1+\frac{\pi^2}{3}\left(\frac{k_BT}{\tilde{\Delta}}\right)^2[1+2(R-1)^2]+O(T^4)\right\}, \qquad (21)$$

where $R=1+\tilde{U}/\pi\tilde{\Delta}$ is the Wilson ratio or "χ/γ" ratio. This result is identical to the exact result derived by Nozières [3] in the Kondo regime, $\pi\tilde{\Delta}\to 4k_BT_K$, $R\to 2$, and also to the more general results of Yosida and Yamada [1]. Hence we see that all the Fermi liquid relations can be obtained within the renormalized expansion up to second order in $\tilde{U}$.

We see that the effects of the counterterms play no really significant role in the Fermi liquid regime. The same results can be obtained in this regime by neglecting $\tilde{H}_c$ and working solely with $\tilde{H}_{\rm qp}$ [9] which has to be normal ordered and expressed in terms of particle and hole operators. The quasiparticle interaction only comes into play at finite temperatures and finite magnetic field when these excitations are excited. The molecular field approximation on $\tilde{H}_{\rm qp}$ then corresponds to the Fermi liquid theory. The calculations performed in this way relate closely to the more intuitive phenomenological Fermi liquid approach to Landau [10], and the fact that asymptotically exact calculations are possible as $T\to 0$, $H\to 0$, can be seen to be due to the low density of excitations in this limit ($\delta\tilde{n}\to 0$). The renormalized perturbation theory with the counterterms, however, goes beyond the Fermi liquid regime. In this perturbation theory nothing has been omitted so that in principle calculations can be performed at high temperatures and high fields, allowing the bare particles to be seen. Relatively low order calculations can provide some estimate of the bare parameters in terms of the renormalized ones by inverting (5) and (8),

$$\epsilon_d=(\tilde{\epsilon}_d-\lambda_1)/z,\quad \Delta=\tilde{\Delta}/z,\quad U=(\tilde{U}-\lambda_2)/z^2\,, \qquad (22)$$

where λ_1, λ_2, and z are implicit functions of $\tilde{\epsilon}_d$, $\tilde{\Delta}$, and $\tilde{U}$. Alternatively it should be possible to estimate the renormalized parameters in terms of the bare ones by variational methods.

As the Anderson model is integrable and exact solutions exist for the thermodynamic behavior it is possible to deduce the renormalized parameters exactly over the full parameter regime. For the symmetric Anderson model the parameters $\tilde{U}$ and $\tilde{\Delta}$ have been calculated using the exact Bethe ansatz results [11] from Eqs. (16) and (17) and these are shown in Fig. 2. The asymptotic results at strong coupling correspond to $\tilde{U}=\pi\tilde{\Delta}=4k_BT_K$ with T_K given by

$$k_BT_K=U\left(\frac{\Delta}{2U}\right)^{1/2}e^{-\pi U/8\Delta+\pi\Delta/2U}. \tag{23}$$

The approach has been extended to other impurity models such as the N-fold degenerate Anderson model and the n-channel Kondo model with $n=2S$. The quasiparticle Hamiltonian for the N-fold degenerate Anderson model is of the same form as (6) and specified by the parameters $\tilde{\epsilon}_f$, $\tilde{\Delta}$, and $\tilde{U}$. In the Kondo regime using $\chi_{c,\mathrm{imp}}=0$ we find

$$\tilde{\Delta}=k_BT_K\frac{N^2\sin^2(\pi/N)}{\pi(N-1)},$$
$$\tilde{\epsilon}_f=k_BT_K\frac{N^2\sin(\pi/N)\cos(\pi/N)}{\pi(N-1)}, \tag{24}$$

and $\tilde{U}=k_BT_K[N/(N-1)]^2$, with the Kondo temperature defined by $\chi_{\mathrm{imp}}=(g\mu_B)^2j(j+1)/3k_BT_K$ with $2J+1=N$. The quasiparticle density of states is a Lorenztian resonance at $\tilde{\epsilon}_f$ of width $\tilde{\Delta}$ $\tilde{U}=0$. The narrowing and shifting of the resonance with increase of N from $N=2$ to the large N limit is apparent from (24). In the n-channel model with $n=2S$ the quasiparticle Hamiltonian contains a renormalized Hund's rule coupling $\tilde{J}_H$ in addition to a renormalized on-site interaction $\tilde{U}$ and a hybridized resonance width $\tilde{\Delta}$ which can again be expressed in terms of the Kondo temperature T_K in the localized regime.

To give insight into the renormalized perturbation theory (RPT) we have demonstrated its application here to magnetic impurity models. Clearly there are some simplifying features for impurity problems. One is that the renormalized four-point vertex function $\tilde{\Gamma}$ is a function of frequency only and so $\tilde{\Gamma}(0,0)$ is entirely on-site and takes the form (10). This has the consequence that the quasiparticle Hamiltonian has the same form as the bare Hamiltonian. This will not be the case for the periodic Anderson model for heavy fermions where $\tilde{\Gamma}(0,0)$ will in general be **k** dependent so there will then be off-site terms so that the quasiparticle Hamiltonian will differ from the bare model. The RPT approach should be an effective technique for tackling heavy fermion problems as the renormalizations for these systems are particularly large. For these systems the quasiparticle Hamiltonian $\tilde{H}^{(0)}_{\mathrm{qp}}$ describes electrons in renormalized bands. In the limit of large dimensionality $d\to\infty$ the self-energy is independent of **k** (see Ref. [12]) and, as in the impurity case, it should be possible to use the local form (10) for the quasiparticle interaction term.

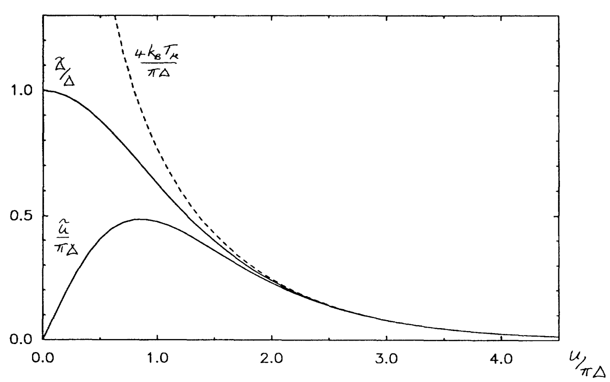

FIG. 2. A plot of the renormalized parameters $\tilde{U}$ and $\tilde{\Delta}$ for the symmetric Anderson model in terms of the bare parameters U and Δ. In the comparison of these parameters with $4k_BT_K$ for $U\gg\pi\Delta$ the value for T_K is given by (23).

One of the major assumptions in the reorganization of the perturbation theory leading to (6) is that the self-energy $\Sigma(\omega)$ is analytic as a function of ω at $T=0$ so that the derivative $\Sigma'(0,0)$ is finite. Clearly when this is not so and Fermi liquid theory breaks down, as in the case of superconductivity or the one-dimensional Luttinger liquid [13], the manipulations leading to (6) are not possible. If a Fermi liquid theory is assumed and calculations carried out, inconsistencies should arise, signaled by singular quasiparticle scattering. In such cases it may be possible to reorganize the series as a renormalized perturbation theory about the appropriate non-Fermi-liquid-theory fixed point.

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