Magnetoresistance in 2D Electrons on Liquid Helium: Many-Electron versus Single-Electron Kinetics

M. I. Dykman,[(1)] M. J. Lea,[(2)] P. Fozooni,[(2)] and J. Frost[(2)]
[(1)]Department of Physics, Stanford University, Stanford, California 94305
[(2)]Department of Physics, Royal Holloway, University of London, Egham, Surrey TW20 0EX, England
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The magnetoresistance ρ_{xx} of two-dimensional electrons, density n, on liquid helium was measured up to the quantum limit, near 1 K where the scattering is due to ^{4}He vapor atoms. Temperature- and density-dependent positive magnetoresistance was observed. Many-electron theory is presented that "restores" the Drude formalism for $\hbar\omega_p \ll kT \ll e^2 n^{1/2}/\varepsilon_0$ [$\omega_p = (e^2 n^{3/2}/2\varepsilon_0 m)^{1/2}$ is the plasma frequency] for classically strong magnetic fields, and shows large magnetoresistance in higher fields, in agreement with the data. The crossover to single-electron kinetics at the highest fields is discussed.

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Electron-electron interactions produce many interesting phenomena in two-dimensional electron systems (2DES) [1], such as the classical electron solid on liquid helium [2], the fractional quantum Hall effect [3], and the proposed magnetically induced Wigner solid in semiconductors [4]. However, there should also be many-electron *dynamic* effects which are not immediately related to long-range order. This Letter discusses the influence of these effects on the magnetoresistivity $\rho_{xx}(B)$ of a nondegenerate 2D electron gas (2DEG), density n, on liquid helium in a perpendicular magnetic field B. For instance, why should the single-electron approximation apply at all, since the electron-electron energy $e^2 n^{1/2}/\varepsilon_0 \gg kT$, the thermal energy, at low temperatures T in this system? Nevertheless the classical Drude model based on elastic (or quasielastic) scattering of independent electrons has been demonstrated to be in good agreement with the experimental data for $B=0$. In a field it predicts a resistivity $\rho_{xx}(B) = \rho_0 = 1/ne\mu_0$ where μ_0 is the zero-field mobility but is commonly recognized to fail for classically strong magnetic fields, $\mu_0 B > 1$, where the density of states is concentrated into a set of *discrete* Landau levels whose spacing exceeds their width. In this case large magnetoresistance would be expected to arise for $B > 1/\mu_0$. The mobility μ_0 is strongly temperature dependent, increasing from 2 m^2/V s at 2 K (where electrons are scattered by ^{4}He atoms) to over 10^3 m^2/V s below 0.5 K where ripplon scattering dominates. Above 1.5 K positive magnetoresistance for $B > 1/\mu_0$ has indeed been observed experimentally and is described [5–8] by a single-particle self-consistent Born approximation (SCBA). However, for lower temperatures (and higher mobilities) there is no large magnetoresistance for $B \sim 1/\mu_0$, and we have previously shown that many-electron effects are important in quantizing magnetic fields in electron-ripplon scattering below 1 K [9–11]. We now present new measurements and theory for $\rho_{xx}(B)$ near 1 K, in fields up to the quantum limit $\hbar\omega_c/kT \gg 1$, where the scattering is dominated by ^{4}He vapor atoms whose density increases exponentially with T and which act as short-range scattering centers, ideal for comparison with theoretical models. The data indicate the many-electron character of the kinetics and the theory resolves the controversy about the applicability of the Drude model to many-electron systems.

We measured the magnetoresistance of a 2DEG on superfluid helium using the Sommer-Tanner technique [12] with coplanar electrodes (see Fig. 1) in a rectangular geometry 120 μm below the electron sheet. An ac voltage V_0 at a frequency f $(=\omega/2\pi)$ between 0.27 and 5 kHz was applied to electrode A and the ac current I to electrode D was measured. For a perfectly conducting electron sheet at $B=0$ the phase of the capacitively coupled current I is $\pi/2$ with respect to V_0. The phase shift $\phi(B)$ away from $\pi/2$ was measured as a function of B for a range of electron densities, for temperatures $0.9 < T < 1.3$ K. The data cover a wide range of $\mu_0 B$ values and

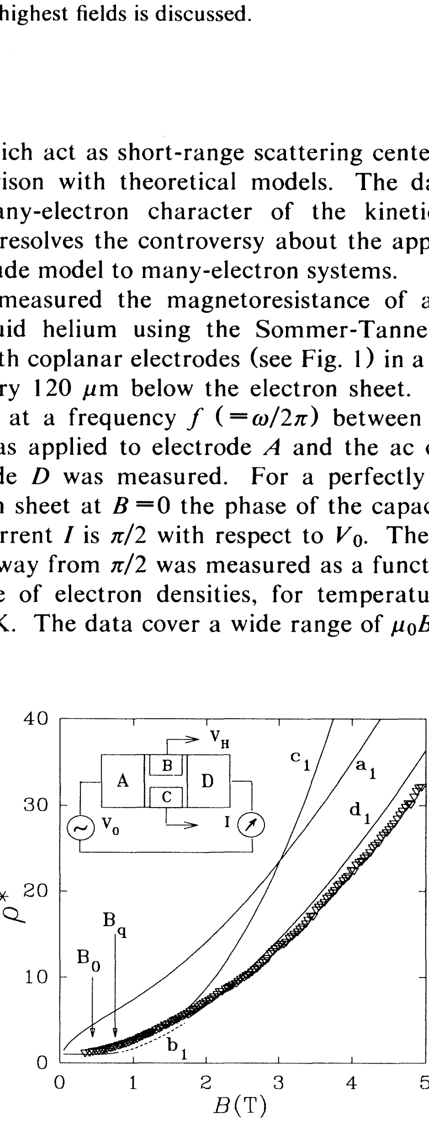

FIG. 1. The normalized resistivity $\rho^*(B)$ vs B for $n = 0.6\times10^{12}$ m^{-2}, $T=1.003$ K. The lines show the theory for independent electrons, ρ_s^* (line a_1), the classical many-electron theory ρ_{mc}^*, Eq. (5) (line b_1), the quantum many-electron theory ρ_{mq}^*, Eq. (6) (line c_1), and the total resistivity, ρ_t^* (line d_1). The onset field B_0 and the quantum limit, $\hbar\omega_c/kT=1$ at B_q, are marked. (Inset: electrode geometry.)

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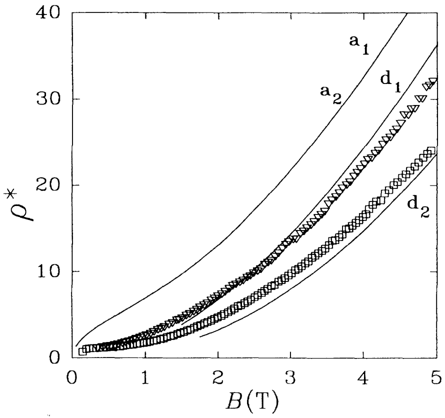

FIG. 2. The density dependence of $\rho^*(B)$ vs B for $n=0.6$ (∇, data set 1) and 2.1 ($\square$, data set 2) $\times 10^{12}\ \mathrm{m}^{-2}$ at $T=1.003$ K. The a and d lines show the theory for independent electrons, ρ_s^*, and the total resistivity, ρ_t^*, from Eq. (7), respectively, for data sets 1 and 2.

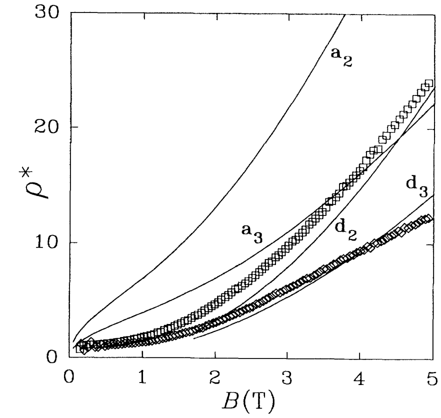

FIG. 3. The temperature dependence of $\rho^*(B)$ vs B for $n=2.1\times 10^{12}\ \mathrm{m}^{-2}$ at 1.003 K ($\square$, data set 2) and at 1.215 K ($\diamond$, data set 3). The a and d lines show the theory for independent electrons, ρ_s^*, and the total resistivity, ρ_t^*, from Eq. (7), respectively, for data sets 2 and 3.

extend into the quantum limit $\hbar\omega_c/kT \gg 1$. The electrons were produced by a glow discharge and held in place by dc voltages on the electrodes, a surrounding guard ring and a top plate 1.93 mm above the helium surface. The electron density was determined from the linear Hall voltage $V_H \propto B/ne$ as measured between electrodes B and C and calibrated using the transition to the 2D electron solid phase at $T_m = 0.216\times 10^{-6} n^{1/2}$ K [1,13,14].

The phase shift $\phi(B)$ in a magnetic field for rectangular electrodes is given by [15,16]

$$\phi(B) = K\omega\rho_{xx}[1+(a\rho_{xy}/\rho_{xx})^2]$$
$$= \phi_0\rho^*[1+(a\mu_0 B/\rho^*)^2]\,, \qquad (1)$$

where K and a ($=0.225$ in this case) depend on the electrode geometry. The second expression, for the normalized resistivity $\rho^* = \rho_{xx}(B)/\rho_0$, is obtained by using the Hall resistance $\rho_{xy} = B/ne$ as confirmed experimentally [5,7] in agreement with theoretical arguments [5,17]. Hence we can obtain $\rho^*(B,T)$ from the measured $\phi(B)$. In very low fields, where $\rho^* \approx 1$, the initial slope of $\phi(B)/\phi_0$ is proportional to $(\mu_0 B)^2$ and directly gives the zero-field mobility. Typical values of μ_0 were 26, 34, 52, 92, and 140 $\mathrm{m}^2/\mathrm{V\,s}$ for $T=1.252$, 1.215, 1.082, 1.003, and 0.924 K, respectively, in agreement with other experiments [18] and single-electron calculations in zero field [19].

Positive magnetoresistance was observed for all densities and temperatures investigated, cf. Figs. 1 to 3. The main features of the data are the same. In low fields the normalized resistivity ρ^* is almost independent of field, though it increases slowly and quadratically with field. At higher fields the resistivity increases rapidly and the value of ρ^* at a given field increases with decreasing density (Fig. 2) and temperature (Fig. 3). The data sets shown were taken at 956 Hz; ρ^* was independent of frequency below 5 kHz. At much higher fields, ≈ 60 T at 1 kHz, an edge magnetoplasmon resonance should be observed [20].

A remarkable feature of the observed magnetoresistance is that it is relatively *small* for classically strong magnetic fields: $\rho^* < 1.1$ even for $B=0.4$ T where $\mu_0 B > 10$ and hence the Landau-level spacing is more than an order of magnitude larger than the level width as given by $\hbar/\tau_0$ (τ_0 is the momentum relaxation time for $B=0$). In the conventional single-electron theory based on the SCBA the width of the levels $\hbar/\tau_B$ is due to electron collisions with ^{4}He vapor atoms and, since the density of states increases due to the "squeezing" of the energy spectrum into Landau levels, $\tau_B^{-1} = (2\mu_0 B/\pi)^{1/2}\tau_0^{-1}$ for $\mu_0 B \gg 1$ and increases sharply with B [6,17]. In the classical limit, $\hbar\omega_c/kT \ll 1$, this leads to $\rho_s^* \approx (\mu_0 B)^{1/2}$ while in the quantum limit, $\hbar\omega_c/kT \gg 1$, $\rho_s^* \approx (\mu_0 B)^{1/2} \times(\hbar\omega_c/kT)$. The full theoretical expression for ρ_s^* using the SCBA for a nondegenerate 2DES has been given by van der Heijden *et al.* [6] and is shown (line a) in the figures. In each case ρ_s^* lies above the data and shows a stronger field dependence. It is this striking observation which indicates the importance of many-electron effects and is the subject of this paper. Moreover, ρ^* displays a density dependence that also indicates the influence of many-electron effects.

The onset of many-electron magnetoresistance and the agreement between the single-electron theory and experiment for $B=0$ can be understood since the result of the strong electron-electron coupling for $e^2 n^{1/2}/\varepsilon_0 \gg kT$ is that an electron is driven by a fluctuational (and fluctuating) electric field $\mathbf{E}$ from the other electrons. If this field is weak so that the change of kinetic energy over a thermal wavelength $\delta\varepsilon = eE\lambda_T \ll kT$ [$\lambda_T = \hbar/(2mkT)^{1/2}$; $E = \langle \mathbf{E}^2\rangle^{1/2}$], the electron motion is semiclassical [21]. A quasielastic collision with a short-range scatterer is then not influenced by many-electron effects (provided $\mathbf{E}$ does not change over the collision time $\hbar/kT$) and the single-particle approximation holds for $B=0$.

For $\mu_0 B \gg 1$ the fluctuational field $\mathbf{E}$ can dramatically change the magnetotransport, compared to the single-electron theory, as the energy spectrum of an electron in crossed $\mathbf{B}$ and $\mathbf{E}$ fields is continuous (not discrete as for $\mathbf{E}=0$), and an electron can be scattered quasielastically by a short-range scatterer (a He atom). Since only one electron is immediately involved in a collision (neglecting momentum transfers of order $\hbar n^{1/2}$), the expression for magnetoconductivity to the lowest order in the coupling is the same as the Drude theory [22], with the only (but important) difference that the electron-electron interaction influences the motion of an electron in the course of a collision and hence

$$\rho^* \equiv \rho_m^* = (\lambdabar_T/\lambda)^2(\tau_0/\tau_B);\quad \tau_B^{-1} = \frac{1}{2}\lambda^2\hbar^{-2}\sum_{\mathbf{q}} q^2 \overline{|V_{\mathbf{q}}|^2}\xi(q),$$
$$\xi(\mathbf{q}) = \int_{-\infty}^{\infty} dt\, \xi(\mathbf{q},t);\quad \xi(\mathbf{q},t) = \langle \exp[i\mathbf{q}\cdot\mathbf{r}_j(t)]\exp[-i\mathbf{q}\cdot\mathbf{r}_j(0)]\rangle . \tag{2}$$

Here, $\overline{|V_{\mathbf{q}}|^2}$ is the mean squared Fourier component of the random quasistationary scattering field, $\mathbf{r}_j$ is the position vector of the jth electron, and λ is a characteristic wavelength of an electron; $\lambda = \lambdabar_T$ for $\hbar\omega_c \ll kT$ and $\lambda = l = (\hbar/eB)^{1/2}$, the magnetic length, for $\hbar\omega_c \gg kT$.

The electron-electron coupling determines the kinetics of an individual electron and thus the value of the correlator $\xi(\mathbf{q},t)$. In other words, Eq. (2) gives the magnetoconductivity due to the *momentum transfer from the "electron community" via the scattering of individual electrons.* The scattering rate for a nondegenerate 2DES has been given previously in the extreme quantum limit $\hbar\omega_c \gg kT$ [23,24]. Here we present a theory of magnetoresistance for arbitrary magnetic fields.

In the single-electron approximation at $B=0$, $\xi(q) = (2\pi m/kTq^2)^{1/2}\exp(-\hbar^2 q^2/8mkT)$, whereas for finite B the integral in Eq. (2) diverges as an electron moves along a closed loop in classical terms. This is no longer true if the electron is also driven by an electric field from the other electrons. The value of $\xi(q)$ and the physics of the scattering depend on the value of the parameter $\eta = \omega_c(2mkT)^{1/2}/eE$ which gives the ratio of the cyclotron radius $R_c = (2kT/m\omega_c^2)^{1/2}$ to the shift of the orbit in the crossed $\mathbf{E}$ and $\mathbf{B}$ fields over the time ω_c^{-1}, or in quantum terms, the ratio of $\hbar\omega_c$ to the uncertainty $eE\lambdabar_T$ in the kinetic energy of an electron. For $\eta \ll 1$ (small fields) the Landau levels are smeared out and the scattering is basically as for $B=0$. The evaluation of $\xi(q)$ in this limit requires the solution of the equations of motion for small times $\sim\hbar/kT$ with the magnetic field and the electron-electron interaction as perturbations. This leads to a small quadratic magnetoresistance

$$\rho_{mc}^* = 1 + (5/96)(\hbar\omega_c/kT)^2 . \tag{3}$$

The E-dependent correction $[\sim \hbar^2 e^2 E^2/48m(kT)^3]$ to τ_B^{-1} is additive, and therefore does not enter ρ^* in this regime.

A different situation arises for $\eta \gg 1$ (still $\hbar\omega_c/kT \ll 1$) when, before the many-electron field drives it away, an electron performs several rotations about a scatterer, increasing the probability of scattering. The correlator $\xi(q)$ was evaluated by the steepest descent method, with saddle points at $2\pi s/\omega_c - i\hbar/2kT$. The many-electron field causes drift of the cyclotron orbit by $(E/B)t$, with $t = 2\pi s/\omega_c$, and therefore

$$\xi(q) \approx \left(\frac{2\pi m}{kTq^2}\right)^{1/2} \sum_s \exp\left(\frac{-\hbar^2 q^2}{8mkT}\right) \times \left\langle \exp\left(\frac{i2\pi s e}{m\omega_c^2}\mathbf{E}\cdot\mathbf{q}\right)\right\rangle . \tag{4}$$

The values of the integer s which contribute to the sum are limited to $|s| \approx \zeta = \hbar\omega_c/eER_c = \eta(\hbar\omega_c/2kT)$. The magnetoresistance increases rapidly for $B > B_0$ ($\zeta = 1$ for $B = B_0$). The onset field B_0 is the field above which the electron drift over the time ω_c^{-1} is less than the thermal wavelength $\lambdabar_T$. If the distribution of the many-electron field is Gaussian and the electrons are scattered by vapor atoms then Eqs. (2)–(4) give

$$\rho_{mc}^* = \sum_{s=-\infty}^{\infty} [1 + 4\pi^2 s^2 (B_0/B)^4]^{-3/2} . \tag{5}$$

The onset field B_0 depends on the fluctuational electric field which can be estimated by assuming short-range order in the electron system (which seems reasonable for $e^2 n^{1/2}/\varepsilon_0 \gg kT$) by equating kT to the energy $e^2E^2/m\omega_p^2$ of electron vibrations in the field of other electrons at a characteristic 2D plasma frequency $\omega_p = (e^2 n^{3/2}/2\varepsilon_0 m)^{1/2}$ [23]; one arrives at $E \approx 0.84(kTn^{3/2}/\varepsilon_0)^{1/2}$ [24], $\eta \sim \omega_c/\omega_p$ and $B_0 = 1.66\times 10^{-5} n^{3/8} T^{1/2}$.

The expression for $\xi(q)$ that follows from quantum theory is of the form $\xi(q) \approx 2Bq^{-1}\langle E^{-1}\rangle \Xi_q \exp(-l^2 \times q^2/2)$ where Ξ_q [and Ξ in Eq. (6) below] allows for the filling of the excited Landau levels and $l = (\hbar/eB)^{1/2}$ is the magnetic length. In the limit of a quantizing field, $\hbar\omega_c/kT \gg 1$, $\Xi_q = \Xi = 1$. The reduced resistivity is then given by

$$\rho_{mq}^* = 0.15(\omega_c/\omega_p)(\hbar\omega_c/kT)^{3/2}\Xi , \tag{6}$$

which exactly coincides with Eq. (5) in the range $B \gg B_0$, $\hbar\omega_c/kT \ll 1$, where both the above classical and quantum theories apply.

We now compare this theory with three data sets [set 1: $n = 0.6\times 10^{12}$ cm^{-2}, $T = 1.003$ K; set 2: $n = 2.1\times 10^{12}$

cm^{-2}, $T=1.003$ K; set 3: $n=2.1\times10^{12}$ cm^{-2}, $T=1.215$ K] in Figs. 1, 2, and 3 where the normalized resistivity is plotted versus B. At low fields (though $\mu_0 B>1$ for all the data shown) the magnetoresistance is in qualitative and quantitative contradiction with the SCBA theory (lines a_1, a_2, and a_3, where the subscripts refer to the corresponding data sets). Figure 1 shows the calculations for the many-electron theories for data set 1. The low field classical theory, Eq. (3) (line b_1), predicts a small magnetoresistance for $\eta\ll 1$ which corresponds to $B\ll 0.2$ T. The onset field for magnetoresistance B_0 from the classical theory, Eq. (5), is 0.44 T for data set 1 ($B_0=0.69$ and 0.76 T for data sets 2 and 3). But quantum effects are already important in this region as $\hbar\omega_c=kT$ at $B_q=0.75$ T at 1 K. For $B>2$ T we can use the quantum magnetoresistance ρ^*_{mq}, Eq. (6), shown by line c_1. However, for $\rho^*>5$, the results of the many-electron theory and of the single-electron SCBA differ by less than a factor of 2 and the combined total resistivity ρ_t is then calculated from the expression

$$\frac{1}{\rho_t}=\frac{1}{\rho_m}+\frac{\rho_t}{\rho_s^2}\,, \tag{7}$$

which is derived from the Einstein diffusion equation in which the scattering rate is proportional to $\hbar\omega_c/\Gamma$, where Γ is the energy uncertainty of each Landau level. Equation (7) incorporates the self-consistent nature of the SCBA. The normalized total resistivity ρ_t^* is plotted as line d_1 and shows good agreement with the measurements, particularly since there are no adjustable parameters in these calculations. Figure 2 demonstrates the density dependence of ρ^* at a fixed temperature where the SCBA result is independent of density while ρ_t^* increases as the density decreases as shown by lines d_1 and d_2. Figure 3 demonstrates the temperature dependence of ρ^* at a fixed density compared to the SCBA results (lines a_2 and a_3) and ρ_t^* (lines d_1 and d_2).

At the highest fields the SCBA, and collision broadening of the Landau levels, will dominate the magnetoresistance. The crossover from many-electron kinetics to the independent electron approach of the SCBA occurs with increasing B and T since Eq. (2) only holds if the duration of a collision $\tau_c\ll\tau_B$, the relaxation time. For classically strong fields τ_c is equal to $\hbar/kT$ for $\eta\ll 1$ and $\lambda_T B/E$ for $\eta\gg 1$ and $\zeta>1$, respectively, while for quantizing fields $\tau_c=lB/E\propto B^{1/2}$. But the scattering rate τ_B^{-1} increases rapidly at high B and T and the electron kinetics and scattering become essentially single electron in nature. At the crossover $\tau_B\approx\tau_c\sim(\tau_B)_{\rm SCBA}$ [note that $(\tau_B)_{\rm SCBA}\sim(\tau_B\tau_c)^{1/2}$] or $\rho\backslash O(*,mq)\approx\rho^*_{\rm SCBA}$. In terms of the zero-field mobility μ_0, the SCBA will apply in the quantum limit for fields $B\gtrsim 5\times10^{-10}\mu_0^{1/2}T^{1/2}n^{3/4}$ T. In the experiments of Adams and Paalanen [8] and van der Heijden *et al.* [6] this condition was satisfied for $B>1$ T.

In conclusion, we have observed and explained the many-electron character of the magnetoresistance of a 2DEG in classically strong and in quantizing magnetic fields. The crossover from many- to single-electron kinetics has been observed with increasing magnetic fields.

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