Curvature Contribution to the Mass of Strangelets

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The curvature energy is shown to give a very significant contribution to the mass of strangelets. Assuming three massless quark flavors, the energy per baryon is increased by roughly (435 MeV)$A^{-2/3}$ within the MIT bag model, dramatically destabilizing the experimentally accessible strangelets with low baryon number A.

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In a recent paper, Mardor and Svetitsky [1] have pointed out that the curvature energy (a term in the free energy of the form $8\pi\gamma R$ added to the surface energy term $4\pi\sigma R^2$, where R is the radius, and σ and γ are the surface and curvature energy densities) plays a decisive role when studying the thermodynamics of the quark-hadron phase transition. The authors concentrate on a high-temperature environment with chemical potential $\mu=0$, but warn that the term may be important in other circumstances as well (see also Ref. [2]). Below it will be shown that this is indeed the case also when effects of finite chemical potentials are taken into account. In particular it is shown that the curvature energy can have a dramatic destabilizing effect on low-baryon-number strangelets.

Quark matter composed of comparable numbers of u, d, and s quarks may be stable at zero pressure and temperature [3,4]. Within the MIT bag model there is a significant range of parameters for which this is the case in bulk systems. For low baryon numbers, the contribution from surface tension due to the finite s-quark mass destabilizes strangelets [4,5], so for fixed parameters (e.g., bag constant B, strong coupling constant α_s, and strange quark mass m_s) there is a limit on the baryon number, $A_{\rm min}$, below which systems are unstable. (At very low A, shell effects may again help in stabilizing strangelets.)

Berger and Jaffe [5] derived a mass formula for strangelets in the "atomlike" bulk limit, i.e., strangelets with $A<10^7$ for which electrons mainly stay outside of the quark phase. In the applicable regime, the authors found that the minimum baryon number for which strangelets were metastable (stable against neutron emission) was given from the surface energy $E_{\rm surf}=4\pi\sigma R^2 \equiv c_{\rm surf}A^{2/3}$ by

$$A_{\rm min}^{\rm meta}=\left(\frac{2c_{\rm surf}}{3(m_n-\epsilon_0)}\right)^3 \approx 296\left(\frac{c_{\rm surf}}{100\ {\rm MeV}}\right)^3\left(\frac{10\ {\rm MeV}}{m_n-\epsilon_0}\right)^3, \qquad (1)$$

where $m_n-\epsilon_0$ is the binding energy (relative to the neutron mass) for strange matter in bulk. The coefficient $c_{\rm surf}\equiv 4\pi\sigma R^2/A^{2/3}\equiv 4\pi\sigma\rho^2$, where the value of ρ, defined by $R\equiv\rho A^{1/3}$, is slowly varying as a function of strong interaction parameters. For the present purpose we ascribe to ρ a constant value of 1 fm (5.068×10^{-3} MeV^{-1} in units where $\hbar=c=k_B=1$). For a strange-quark mass of order 150 MeV, $c_{\rm surf}\approx 100$ MeV [5], whereas $m_n-\epsilon_0 < 60$ MeV in order not to get into the unphysical situation of having a stable phase of non-strange-quark matter [4]. For $m_s=200$ MeV the corresponding numbers are $c_{\rm surf}\approx 60$ MeV, and $m_n-\epsilon_0<40$ MeV.

A number of heavy-ion experiments at BNL and CERN have recently been performed or proposed to search for strangelets that are either stable or metastable with lifetime exceeding 10^{-8} sec [6]. These experiments can realistically hope to form objects with $A<20$–30 [7,8], a range accessible for moderate bulk binding energies according to Eq. (1). Inclusion of the curvature energy destabilizes strangelets in this regime.

The curvature energy arises (like the surface energy) because of the corrections to the quark density of states stemming from the need to match the wave functions to the bag boundary conditions. For massless quarks the density of states dN/dk in the MIT bag is given by [9]

$$\frac{dN}{dk}=g\left(\frac{k^2V}{2\pi^2}-\frac{1}{3\pi}R\right). \qquad (2)$$

The first term gives the bulk energy. There is no surface energy term (proportional to R^2) for massless quarks, but the term proportional to R leads to a curvature energy

$$E_{\rm curv}=gT\int_0^\infty dk\frac{1}{3\pi}R\ln[1+\exp\{(\mu-k)/T\}] = \frac{gR}{3\pi}\int_0^\infty dk\,k[1+\exp\{(k-\mu)/T\}]^{-1}. \qquad (3)$$

For one quark flavor the statistical weight is $g=6$, and in strangelets typical chemical potentials are of order $\mu\approx 300$ MeV. At zero temperature the Fermi distribution becomes a Heaviside function, and the curvature energy is simply

$$E_{\rm curv}(T=0)=g\mu^2R/6\pi \approx (435\ {\rm MeV})\left(\frac{g}{18}\right)\left(\frac{\mu}{300\ {\rm MeV}}\right)^2 A^{1/3}, \qquad (4)$$

where parameters have been scaled to typical magnitudes for three equal chemical potential massless quark flavors in a strangelet.

In this regime, a typical curvature energy per baryon is therefore $E_{\rm curv}/A \approx (435\ {\rm MeV})A^{-2/3}$, or 94, 59, 45, 32, 20, and 4 MeV, respectively, for baryon numbers of 10, 20, 30, 50, 100, and 1000. Strangelet stability at these baryon numbers requires strange quark matter in bulk to be bound by *at least* as much. Surface tension destabilizes the strangelets further; for an s-quark mass of 100–200 MeV, $E_{\rm surf}/A \approx (100\ {\rm MeV})A^{-1/3}$ becomes dominant for baryon numbers exceeding 10^2 (cf. Fig. 1).

Writing $E/A = \epsilon_0 + c_{\rm surf}A^{-1/3} + c_{\rm curv}A^{-2/3}$, with $c_{\rm surf} \approx 100$ MeV and $c_{\rm curv} \approx 435$ MeV (the Coulomb energy is negligible in comparison, because strangelets are almost neutral [4]), the stability condition $E/A < m_n$ may be written as $A > A_{\rm min}^{\rm abs}$, where

$$A_{\rm min}^{\rm abs} = \left[\frac{c_{\rm surf} + [c_{\rm surf}^2 + 4c_{\rm curv}(m_n - \epsilon_0)]^{1/2}}{2(m_n - \epsilon_0)}\right]^3 . \qquad (5)$$

Stability at baryon number 30 requires a bulk binding energy close to 80 MeV (cf. Fig. 1), which is not within reach for $m_s > 100$ MeV if, at the same time, ud-quark matter shall be unstable [4]. The proposed cosmic ray strangelet candidates with baryon number 370 [10] would for stability require a bulk binding energy per baryon exceeding 22 MeV to overcome the combined curvature and surface energies. Absolute stability relative to a gas of ^{56}Fe corresponds furthermore to using 930 MeV instead of m_n, whereas stability relative to a gas of Λ particles (the ultimate limit for formation of short-lived strangelets) would correspond to substitution of $m_\Lambda = 1116$ MeV.

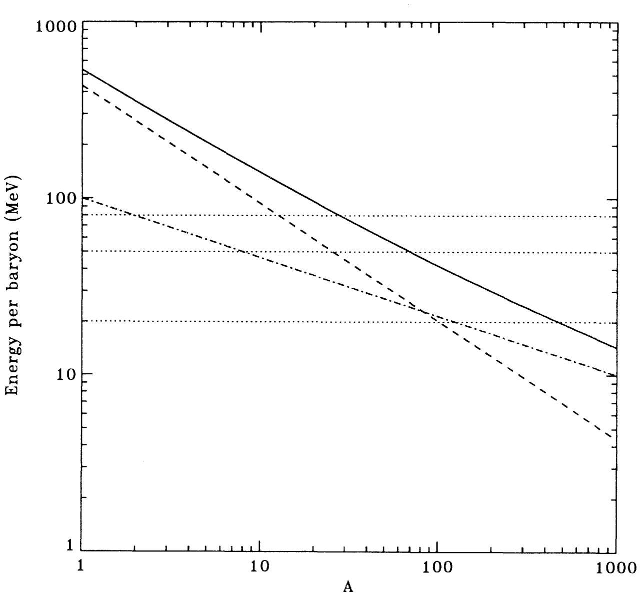

FIG. 1. Surface energy (dot-dashed line), curvature energy (dashed line), and the sum of the two (solid curve), all given in MeV per baryon, as a function of baryon number. Horizontal dotted curves indicate the stability limit for a bulk binding energy $m_n - \epsilon_0$ of 20, 50, and 80 MeV, respectively.

Another way of stating the result is to calculate the minimum baryon number for which long-lived metastability is possible. Identifying this as the limit of neutron emission [as in Eq. (1)] requires $dE_{\rm curv}/dA + dE_{\rm surf}/dA < m_n - \epsilon_0$, or

$$A_{\rm min}^{\rm meta} = \left[\frac{c_{\rm surf} + [c_{\rm surf}^2 + 3c_{\rm curv}(m_n - \epsilon_0)]^{1/2}}{3(m_n - \epsilon_0)}\right]^3 . \qquad (6)$$

[Notice that Eq. (1) is recovered for $c_{\rm curv} = 0$.] To have $A_{\rm min}^{\rm meta} < 30$ requires $m_n - \epsilon_0 > 38$ MeV, which is possible, but only for a narrow range of parameters [4].

It should be mentioned that the $E_{\rm curv}/A \propto A^{-2/3}$ behavior for massless quarks was noted in connection with unstable ud-quark matter by Farhi and Jaffe [4]. However, the authors expected the surface tension term to dominate for strangelets, and the same assumption has been made in all subsequent studies.

Attempts to calculate the mass of small, (meta)stable strangelets by filling up a bag with noninteracting quarks have been undertaken by Farhi and Jaffe [4] and Greiner *et al.* [8]. These quantum-mechanical calculations should implicitly include the surface and curvature energies, since these are ultimately given by the quark boundary conditions in the bag, and they also reveal shell effects, which of course cannot be reproduced by a mass formula derived from expanding the free energy about $R = \infty$, i.e., in powers of $1/R$. The very fact that the curvature term becomes so important even at moderately high baryon numbers is a warning that a $1/R$ expansion is somewhat dangerous, and that quantum-mechanical calculations should be performed for $A \to 1$. However, these calculations are rather complex and have only been performed for a few sets of parameters [4,8], whereas the mass formula, while lacking some of the finer details, gives an easy handle on the general trends. For the parameters $B^{1/4} = 145$ MeV, $m_s = 150$ MeV, a quantum-mechanical strangelet calculation is displayed in Fig. 2(b) of [8]. The surface tension energy per baryon in this regime is roughly $(96\ {\rm MeV})A^{-1/3}$, and that is not nearly enough to fit the quantum-mechanical calculations for $A < 100$. Including also the curvature energy for three massless quarks calculated above gives a very good fit, and an excellent fit (apart from shell effects) results if one picks an effective g value somewhat smaller than 18, corresponding to a reduced contribution from the massive s quarks.

The curvature energy at finite temperature can be found from Eq. (3). For $T \gg \mu$ the integral gives

$$E_{\rm curv}(T \gg \mu) = \frac{gT^2R\pi}{36}\left[1 + \frac{12\ln 2}{\pi^2}\frac{\mu}{T}\right], \qquad (7)$$

and for $T \ll \mu$

$$E_{\rm curv}(T \ll \mu) = \frac{g\mu^2 R}{6\pi}\left[1 + \frac{\pi^2}{3}\left(\frac{T}{\mu}\right)^2\right]. \qquad (8)$$

The curvature energy at finite T and μ can be important in studies of boiling of cosmologically produced strange-quark matter [11], as well as in studies of strange star formation [12] and strangelet production in heavy-ion collisions [7,8]. In high-temperature environments the curvature energy contributions from antiquarks and gluons should also be included.

The results presented above demonstrate that the curvature energy plays a significant role for the stability of strangelets. The assumption of zero strange-quark mass in the curvature term should be relaxed, but presently no calculation of the curvature correction to the density of states for massive quarks exists. Preliminary investigations indicate that the curvature energy contribution for a massive quark flavor is smaller than for a massless flavor, as indeed seems to be demonstrated when fitting to the numerical results in [8]. A more detailed study of the strangelet mass formula in the spirit of Ref. [5], including curvature energy (and the factor of 2 missing in the surface tension term in the original Berger-Jaffe mass formula), is in progress. However, the qualitative effects are likely to remain unchanged: a significant destabilization of the experimentally accessible stranglets of low baryon number.

This should not, however, deter experimentalists from pursuing the proposed searches. After all, the MIT bag model is only an approximation, and in particular shell effects are likely to be important (but very difficult to calculate) in the range of $A < 30$ presently searched for in laboratories. Effects at the 10-MeV level are visible in the numerical results displayed in Ref. [8]. Furthermore, even if a strangelet is unstable according to Eqs. (5) or (6), the time scale for energetically allowed decays has not been calculated. The existence of small-baryon-number strangelets is ultimately an experimental issue, but the conclusion of the present investigation is that curvature energy is a very significant destabilizing factor.

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