Weak Localization of Acoustic Waves in Strongly Scattering Media

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We report the first direct measurement of coherent backscattering of scalar *acoustic* waves by disordered scattering medium. In two and three dimensions an excellent agreement between experimental data and the predictions of simple diffusion theory is found. These measurements have been done for the time dependent and independent cases.

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The propagation of waves in media with randomly distributed scatterers has become a subject of intensive research even since the effect of electron localization in metallic solids has been proposed by Anderson [1] in 1958. The subsequent interpretation of the underlying physics [2–6] in the past decade has led to a significantly new understanding of wave propagation in disordered systems. The localization effects known from weakly disordered electronic systems have universal character and may be used to describe classical as well as quantum mechanical disordered systems. Thus they are a general phenomenon common to any wave propagation. In the recent years there have been numerous experimental and theoretical investigations in several fields of physics, mainly in solid state physics, optics, the microwave region, and also in acoustics. The coherent backscattering, which is regarded as a precursor of strong localization, has been proven in several optical experiments for both the time dependent and independent cases [7–10]. In the regime of strong multiple scattering and for probe sizes L greater than the mean free path (l^*) diffusion theory is a useful starting point to the theoretical treatment of backscattering. The diffusion theory has to be adapted to the experimental arrangements by using proper boundary conditions.

In the field of disordered acoustic systems very little experimental work has been done. Measurements on one- and two-dimensional media confirm that for dimensions $d \leq 2$ acoustic waves are always localized [11]. One- and two-dimensional waveguides (ground mode propagation and rigid walls) with different impurities like Helmholtz resonators or variations of cross section have been reported, as well as stretched strings with irregularly spaced masses [12,13]. Also "third sound" experiments (two dimension) have previously been reported [14]. Localization effects in three-dimensional strongly scattering acoustical systems are expected to be very difficult to achieve experimentally [15], since the basic condition for real scatterers is to fulfill the Ioffe-Regel criterion, which requires the product kl^* to be equal to unity or smaller. Recently acoustic-wave localization in the presence of shear resonances [16] has been reported, which seems to be very promising.

We report here measurements of coherent backscattering in strongly inhomogeneous media by ultrasound, namely, gravel stones (three dimensions) and parallel brass rods (two dimensions), both in water. In this experiment the enhanced backscattering has been investigated for both the time dependent and independent cases. The geometry of the media of anisotropic scatterers was always much larger than the mean free path l^*.

The experimental arrangement is shown in Fig. 1(a). Ultrasonic pulses of 2 MHz center frequency (which corresponds to a wavelength $\lambda = 0.75$ mm in water) and a pulse duration of 8 μs (bandwidth 150 kHz at -3 dB) were injected in a semi-infinite scattering medium by an acoustical beam splitter (BS). The transducer as well as the ultrasonic receiver have been positioned in the far field. The spot illuminated by ultrasound was about 4 cm in diameter. In the three-dimensional case the scattering medium consists of small gravel stones in water. The stones had irregular shapes and an average diameter of 6 mm with a standard deviation of 1 mm; the sample thickness was about 20 cm. The sound velocity in water has been determined to 1480 m/s. For the gravel stone medium the effective sound velocity measured in transmission was 1920 m/s. Its absorption coefficient $\alpha_a = 21\ \mathrm{m}^{-1}$ and scattering coefficient $\alpha_s = (l^*)^{-1} = 121\ \mathrm{m}^{-1}$ have been

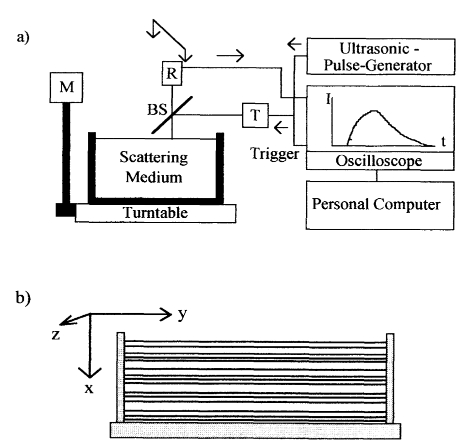

FIG. 1. (a) Experimental arrangements: T, transducer; R, receiver; BS, acoustical beam splitter. (b) Two-dimensional medium.

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determined in a different experiment [17]. Because of the polydispersity of the scatterers this experimental value has to be regarded as an averaged mean free path. In steps of 1° the angle dependent backscattered intensity has been recorded by an ultrasonic receiver. The signal was digitalized by a 100 MSample oscilloscope and transmitted to a personal computer for further signal processing. For ensemble averaging the probe has been rotated out of the receiver axis. The two-dimensional probe is shown in Fig. 1(b). About 2000 rods of brass with diameter of 2 mm ($\alpha_a \approx 0$, $\alpha_s = 71\ \mathrm{m}^{-1}$) are arranged parallel to the y axis and randomly distributed in the x-z plane. The ensemble averaging in that case has been performed by shifting the whole medium parallel to the x-z plane.

Because of the large probe size a sound wave may undergo many scattering events; in this region of multiple scattering the problem can be treated by using diffusion theory. For the unbounded medium the diffusion equation can be written as

$$\left(D\nabla^2 + \frac{\partial}{\partial t}\right) G(\mathbf{R},\mathbf{R}';t,t') = \delta(\mathbf{R}-\mathbf{R}')\delta(t-t'). \quad (1)$$

This formula describes the propagation of energy density released at the point $\mathbf{R}'$ and time t' to the location $\mathbf{R}$ at the time t neglecting any absorption. Considering the boundary conditions of the semi-infinite medium we obtain the following Green's function [9]:

$$G(\mathbf{R},\mathbf{R}';t) = [1/(4\pi Dt)^{3/2}]\exp(-\rho^2/4Dt)\{\exp[-(x-x')^2/4Dt] - \exp[-(x+x'+2x_0)^2/4Dt]\}, \quad (2)$$

where the x axis is oriented perpendicular to the medium surface. $\boldsymbol{\rho} = (y-y')\hat{\mathbf{e}}_y + (z-z')\hat{\mathbf{e}}_z$ describes the lateral distance (parallel to the surface) of the first and the last scatterer and x_0 is a length which enters in the boundary conditions of the Green's function. The diffusion constant can be written as $D = cl^*/d$, where d is the dimension of the medium, neglecting the renormalization of D due to localization effects, c is the effective sound velocity in the disordered medium, and $\mathbf{k}$ and $\mathbf{k}_0$ are the wave vectors of the incoming and outgoing waves.

Convoluting this Green's function with suitable incoming and emerging source distributions the total time and angle dependent albedo defined as the ratio of the backscattered to the incoming intensity per solid angle calculates as

$$\alpha(\mathbf{k}_0,\mathbf{k};t) = (c/4\pi l^{*2}) \int\int dx\, d^2\rho\, dx' \exp[-(x/\mu_0 - x'/\mu)/l^*]\{1+\cos[(\mathbf{k}_0+\mathbf{k})\cdot\boldsymbol{\rho}]\} G(x,x',\rho,t). \quad (3)$$

Referring to Tsang and Ishimaru [18] the exponential term in the description of the coherent part differs for large angles θ ($\mu_0 = \cos\theta_0 = 1$, $\mu = \cos\theta$) between the normal incoming and the emerging wave vectors. In our experimental setup we expect a coherent part of the albedo only for small angles θ. Therefore the difference mentioned above can be neglected. For the incoherent background the integration yields to the well known Lambert law for rough surfaces. In this case we can write the angle dependent exponent approximately as $(\mathbf{k}_0+\mathbf{k})\cdot\boldsymbol{\rho} \cong 2\pi\theta/\lambda$ for normal incidence. After integration one gets for the total albedo in the transient case

$$\alpha(\Theta,t) \cong \frac{(x_0+l^*)(x_0/\mu+l^*)}{8\pi^{3/2}} \times \left\{1+\exp\left[-Dt\left(\frac{2\pi\Theta}{\lambda}\right)^2\right]\right\}. \quad (4)$$

One obtains the albedo in the stationary case by integrating Eq. (3) over time. Thus using $k_\perp$, the magnitude of $\mathbf{k}$ normal to the x axis, one gets

$$\alpha(\Theta) \cong \frac{3\mu}{8\pi}\left\{1+\frac{2x_0}{l^*}+\frac{1}{(1+k_\perp l^*)^2} \times \left[1+\frac{1-\exp(-2k_\perp x_0)}{k_\perp l^*}\right]\right\}. \quad (5)$$

These expressions are in principle valid for isotropic scattering only. An approximate description for the practical case of anisotropic scattering may be obtained [9] by replacing l^* by the transport mean free path l^*_{tr} which can be calculated from

$$\frac{1}{l^*_{\mathrm{tr}}} = n\int_0^\pi \sigma(\Omega)(1-\cos\Omega)\,d\Omega \quad (6)$$

using the angular dependence of the scattering cross sec-

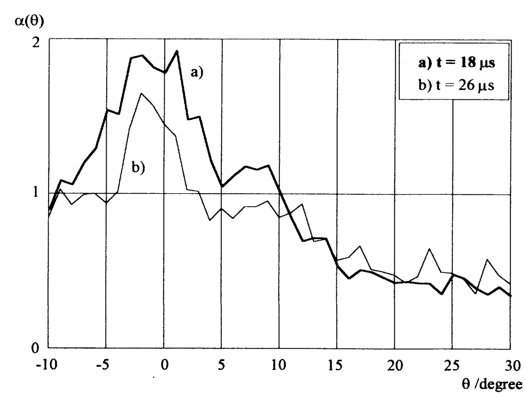

FIG. 2. Experimental data of the albedo for two different times: (a) $t_1 = 18\ \mu$s and (b) $t_2 = 26\ \mu$s; medium gravel stones, 2 MHz, averaged over 300 samples.

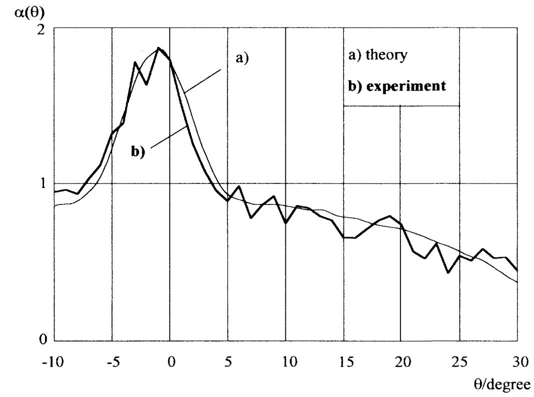

FIG. 3. Time dependent experimental and theoretical results (three dimensions), gravel stones, 2 MHz, $t = 22$ μs, averaged over 300 samples.

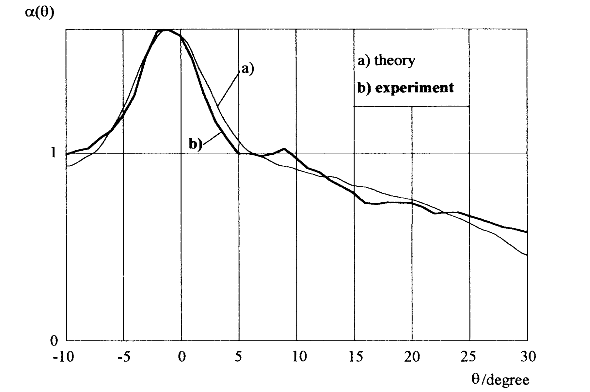

FIG. 5. Stationary experimental and theoretical results in three dimensions, gravel stones, 2 MHz.

tion $\sigma(\Omega)$; n is the volume density of scatterers.

The experimental results obtained with the setup described above are shown in Figs. 2–5. The angular dependence of the backscattered intensity normalized by the incident intensity for two different times $t_1 = 18$ μs and $t_2 = 26$ μs for the 3D medium is shown in Fig. 2. For later times the angular width becomes smaller due to longer scattering paths. Figure 3 compares the experimental curvature and the theoretical predictions. The theoretical results have been convoluted with the receiving directivity patterns and have been corrected by the influence of the experimental arrangement. No adjustable parameter has been used. To compare the experimental and theoretical data they have been matched to the incoherent background. The value for l_{tr}^* and the transport velocity has been determined by a different experiment. Figure 4 shows the experimental and theoretical results ($t = 40$ μs) for the two-dimensional medium.

In the stationary case, i.e., the time integrated backscattered intensity, the theoretical predictions are also confirmed by the experimental data shown in Fig. 5 for three dimensions. It must be pointed out that the integration over time of the experimental data should be started after several scattering events to make sure that the diffusion approximation is valid. In the figure we start the integration after $2l_{\mathrm{tr}}^*$. The same result has been obtained for the two-dimensional disordered medium.

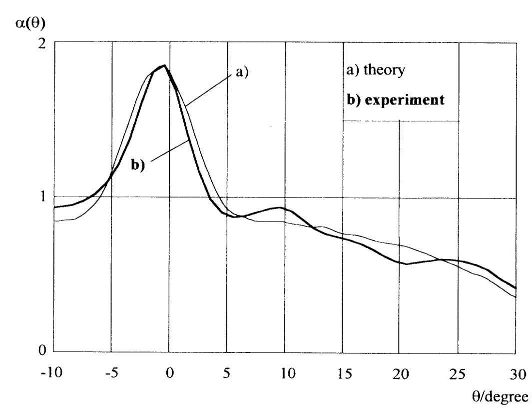

FIG. 4. Time dependent experimental and theoretical results for brass rods (two dimensions), 2 MHz, $t = 40$ μs, averaged over 150 samples (smoothed).

In summary, we have presented observations of the enhanced backscattering due to weak localization effects of acoustical waves. The investigations covered two- and three-dimensional scattering media. It could be shown that the experimental data and the theoretical predictions of diffusion theory are in good agreement.

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