Variable Range Hopping as the Mechanism of the Conductivity Peak Broadening in the Quantum Hall Regime

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We argue that it is the hopping transport that is responsible for broadening of the σ_{xx} peaks. An explicit expression for the width $\Delta\nu$ of the peak as a function of the temperature T is found. A close relation between the characteristic temperature in the dependence $\Delta\nu$ on T and that in the variable-range-hopping exponent at an integer ν is established. Broadening of the peak with increasing current is also explained. The current J is shown to act like the effective temperature $T_{eff} \propto J^{1/2}$ if $T_{eff} \gg T$. The anomalous behavior of two peaks which are close to each other (spin split, e.g.) is considered.

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The integer quantum Hall effect in a disordered two-dimensional electron gas manifests itself more clearly the lower the temperature T. The steps connecting adjacent plateaus in the dependence of the Hall conductance σ_{xy} on the filling factor ν narrow with decreasing T and so do the peaks in the longitudinal conductance σ_{xx}. In a number of experiments [1–7] a remarkable result has been obtained: the width $\Delta\nu$ of the peaks shrinks as $T \to 0$ according to a power law $\Delta\nu \propto T^{\kappa}$. The exponent $\kappa \simeq 0.4$ was found in Refs. [1, 2] to be universal; neither the Landau level index nor the electron mobility is relevant at low temperatures. The measurements have been performed down to temperatures as low as a few tens of millikelvins, thus giving a definite indication that extended electron states exist at only one energy within the broadened Landau level. Other states should be localized. Although the question as to the nature of the localization still remains unresolved, various computer simulations [8–12] strongly support this concept yielding the power-law divergence of the localization length $\xi(E) \propto |E|^{-\gamma}$, $\gamma \simeq 2.3$, as the electron energy E approaches the Landau level center ($E = 0$). Recently, the same value of γ has been directly measured by studying how $\Delta\nu$ scales with the sample size in the low-T limit [4].

The conventional explanation of the scaling dependence $\Delta\nu \propto T^{\kappa}$ is as follows [8, 13]. It is assumed that at a finite temperature there exists a phase-coherence length L_ϕ which is shorter the higher T is. One believes that if $L_\phi \ll \xi(E_F)$, E_F being the Fermi energy, the localization is destroyed and the electron system exhibits metallic behavior. Similar to the theory of weak localization [14, 15], L_ϕ is expressed in terms of the diffusion coefficient D and the phase-breaking time τ_ϕ: $L_\phi \sim (D\tau_\phi)^{1/2}$. The time τ_ϕ is made to be proportional to T^{-p} with the exponent p which depends on the inelastic-scattering mechanism. These arguments lead to the conclusion that the width of the conducting energy band vanishes with decreasing T as $T^{p/2\gamma}$, so that $\kappa = p/2\gamma$ (to describe the experimental data in this way, one has to admit that $p \simeq 2$).

Although such an approach looks very attractive, introducing the phase-breaking time to account for the temperature-induced delocalization at $\sigma_{xx} \sim e^2/h$ is not obvious. There is no generally accepted theory for τ_ϕ in the quantum Hall regime. Here we would like to suggest an explanation of the scaling behavior $\Delta\nu(T)$ in terms of the strong localization (approaching a peak from the region where $\sigma_{xx} \ll e^2/h$). We start with the notion that the only possible mechanism of transport in the strongly localized electron system is hopping. It is known that predominant in the low-temperature limit is variable range hopping. In this regime, due to the existence of the Coulomb gap, the temperature dependence of σ_{xx} should have the form [16, 17]

$$\sigma_{xx} = \sigma_0 e^{-(T_0/T)^{1/2}}, \tag{1}$$

where

$$T_0(\nu) = C\frac{e^2}{\varepsilon\xi(\nu)}, \tag{2}$$

$\xi(\nu)$ is the localization radius of the states on the Fermi level for a given ν, ε is the dielectric constant, and $C \simeq 6$ in two dimensions [18]. This temperature dependence was observed in the middle of the Hall plateaus [19, 20]. Note that Ono [21] also derived Eq. (1) (with a different expression for T_0) assuming a finite density of states at the Fermi level and using unperturbed wave functions of isolated impurities $\psi(\rho) \propto e^{-\rho^2/4\lambda^2}$, where λ is the magnetic length. It is known [22], however, that tails of wave functions are actually of a simple exponential form $e^{-\rho/\xi}$ due to multiple scattering of a tunneling electron. Together with the Coulomb gap in the density of states this results in Eqs. (1),(2). As mentioned above, the length $\xi(\nu)$ diverges as ν approaches a half-integer ν_0:

$$\xi(\nu) = \xi_0|\nu - \nu_0|^{-\gamma}, \quad \gamma \simeq 2.3. \tag{3}$$

Correspondingly, the value of T_0 tends to zero as $\nu \to \nu_0$. Hence, at a given temperature, there should exist a spe-

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cific value of ν at which the exponential factor in Eq. (1) becomes unity. It is natural to assume that it is the difference between this value and ν_0 that determines the width $\Delta\nu$ of the peak. In this case, solving equation $T_0(\nu) = T$ with the use of the relations (2),(3) immediately yields a power-law dependence of $\Delta\nu$ on T:

$$\Delta\nu = \left(\frac{T}{T_1}\right)^{\kappa} \tag{4}$$

with $\kappa = 1/\gamma$ and

$$T_1 = C\frac{e^2}{\varepsilon\xi_0}. \tag{5}$$

For $\gamma \simeq 2.3$ we arrive at the experimental value $\kappa \simeq 0.4$. As for the characteristic temperature T_1, to our knowledge, it is the first time an explicit expression for T_1 is given. Note that T_1 is of the order of T_0 in the middle of an adjacent plateau. To compare with what is experimentally observed, we should define the elementary length ξ_0 depending on the properties of a random potential. Provided the potential fluctuations are short range, so that their correlation radius is less than or of the order of the magnetic length λ, one may expect that ξ_0 for the lowest Landau levels is $\sim \lambda$. One believes that fluctuations of this kind are realized in InGaAs/InP heterostructures, the experiment on which [1] most clearly confirms the universality of the exponent κ. To find T_1, we extract $\Delta\nu$ from the data presented in Ref. [1] according to the formula $\Delta\rho_{xy}/\Delta\nu = (d\rho_{xy}/dB)_{max}(B/\nu)$. Here $\Delta\rho_{xy}$ and $(d\rho_{xy}/dB)_{max}$ stand for the total change in the Hall resistance ρ_{xy} on passing the transition region and for the maximum value of its derivative with respect to the magnetic field correspondingly. The value of T_1 obtained this way is $\simeq$ 320 K for $\nu = \frac{3}{2}$ (the $N = 0 \downarrow$ Landau level). Substituting then λ for ξ_0 in Eq. (5) we find $C \simeq 2$. This value is only 3 times lower than the rough estimate [18] based on the one-electron picture of hopping. The discrepancy may be related with the dielectric constant enhancement in the critical region of the metal-insulator transition. It should be noted, however, that when the steps at $\nu = \frac{5}{2}$ and $\frac{7}{2}$ (the spin-split $N = 1$ level) are treated in the same way, $T_1 \simeq 22$ K is obtained. This temperature is much smaller than what one could expect according to Eq. (5) with $\xi_0 \sim \lambda$ and $C \simeq 2$. It is worth comparing $T_1 \simeq 22$ K with the measured value of T_0 for hopping at $\nu = 3$ which was found to be 7.8 K for similar samples [20]. This value is of the same order of magnitude as T_1 and also much less than what would be expected from Eq. (2). The fact that both the characteristic temperatures for the spin-split $N = 1$ level are so small probably indicates that the length ξ_0 for this level is in fact much larger than λ. We will return to this puzzle below.

The suggested approach permits us to elucidate yet another interesting phenomenon observed at very low temperatures. It was found in Refs. [4, 23] that the width $\Delta\nu$ of the σ_{xx} peaks grows with increasing current J, i.e., with the increase of the Hall electric field $\mathcal{E}_H$. Let us show that the dependence $\Delta\nu(\mathcal{E}_H)$ can be understood in terms of the theory of hopping in a strong electric field [24, 25]. This theory is based on the fact that there exists a quasi-Fermi level inclined by the electric field $\mathcal{E}$. Zero-temperature hopping with phonon emission then becomes possible and, even though there are no absorption processes, the local Fermi distribution with an effective temperature $\sim e\mathcal{E}\xi$ is formed [24, 25]. On this account, the exponent of the current-voltage characteristics at $T = 0$ may be obtained from that of the Ohmic conductivity by replacing $T \to e\mathcal{E}\xi/2$. If the Ohmic transport obeys the law (1), the zero-temperature conductivity should behave with increasing electric field as

$$\sigma_{xx} = \sigma_0 e^{-(2T_0/e\mathcal{E}_H\xi)^{1/2}}. \tag{6}$$

Similarly to the case of Ohmic conductivity the width of the σ_{xx} peak is found from the equation $2T_0(\xi) = e\mathcal{E}_H\xi$. Solving this equation for ξ we get $\xi = (2Ce/\varepsilon\mathcal{E}_H)^{1/2}$, which yields

$$\Delta\nu = \left(\frac{\mathcal{E}_H}{\mathcal{E}_1}\right)^{\alpha}, \quad \mathcal{E}_1 = 2C\frac{e}{\varepsilon\xi_0^2}, \tag{7}$$

where $\alpha = 1/2\gamma = \kappa/2$. Comparing Eqs. (7) and (4),(5) one can notice that the field $\mathcal{E}_H$ leads to the same broadening of the peak as if there was the temperature

$$T_{eff} = \left(\frac{Ce^3}{2\varepsilon}\right)^{1/2} \mathcal{E}_H^{1/2}. \tag{8}$$

This relation is remarkably universal: it contains only one parameter of the sample, its dielectric constant ε. The sensitivity of $\Delta\nu$ to $\mathcal{E}_H$ may be viewed as due to heating in the critical region of the metal-insulator transition. In this connection note the unusual square-root dependence of T_{eff} on $\mathcal{E}_H$. The increase in $\Delta\nu$ with $\mathcal{E}_H$ was clearly observed in Refs. [4, 23]; however, no treatment in terms of power dependences was presented. Our analysis of the lowest temperature data of both the experiments shows that they can indeed be described by introducing $T_{eff} \propto \mathcal{E}_H^{1/2}$.

The third phenomenon we would like to mention here is the saturation of $\Delta\nu$ with decreasing T or $\mathcal{E}_H$ in small samples. It is experimentally established [4] that a σ_{xx} peak stops narrowing as T lowers down to a characteristic temperature T_2 which depends on the sample size L. To evaluate T_2, we follow Ref. [4] and equate L and the localization length at $\Delta\nu = (T_2/T_1)^{\kappa}$. As a result, T_2 turns out to be $\sim e^2/\varepsilon L$, and the corresponding width $\Delta\nu \sim (\xi_0/L)^{\kappa}$. It has been shown above that $\xi(\nu)$ may be governed by the Hall electric field, too. Therefore, one should expect the saturation with decreasing $\mathcal{E}_H$ at the same value of $\Delta\nu$ if $T \ll T_2$. We find that the characteristic Hall field in which this occurs is $(\mathcal{E}_H)_2 \sim e/\varepsilon L^2$.

Now let us look more closely at the starting point of our theory: the conductivity σ_{xx} on both sides of the peak was claimed to be due to variable range hopping (1). The question is, Can activation to the extended states existing in the middle of the Landau level compete with the variable range hopping? We argue that it cannot. To make sure of this, we first consider a single Landau level of the width Γ which is much smaller than the energy distances to the adjacent levels. For example, such is the $N = 0 \downarrow$ level in the experiments on InGaAs/InP [20] and GaAs/GaAlAs [19, 26] samples. Let us compare the contributions to the conductivity from activation and variable range hopping provided the Fermi level is separated from the center of the Landau level by its width Γ. Note that we consider low-mobility samples that do not display the fractional quantum Hall effect. Therefore, the characteristic Coulomb energy $\sim 0.1e^2/\varepsilon\lambda$ is supposed to be small as compared with Γ. In that case, the contribution from activation is given by $\ln\sigma_{xx} \sim -\Gamma/T$ while that from hopping by $\ln\sigma_{xx} \sim -[(e^2/\varepsilon\lambda)/T]^{1/2}$. It is clear that hopping dominates not only at $T \to 0$, which is usual, but even at T of the order of the Coulomb energy (at which temperature $\Delta\nu \sim 1$). Now it is easy to understand that if the Fermi level is closer to the Landau band center the conditions are still more favorable to hopping because $\xi(\nu)$ grows rapidly with decreasing $|\nu - \nu_0|$. It can then be shown in the same way that hopping with an energy transfer larger than $[TT_0(\nu)]^{1/2}$, the typical transfer according to Eq. (1), is also of no importance. Hence, dominant everywhere inside the peak of the density of states (and outside the σ_{xx} peak) is variable range hopping near the Fermi level. In other words, we conclude that the width $\Gamma[T/(e^2/\varepsilon\lambda)]^\kappa$ of the energy band corresponding to $|\nu - \nu_0| \lesssim \Delta\nu$ is always much greater than T.

Let us turn to the question about the conductivity when the Fermi level lies in the gap between the Landau levels. In wide gaps, some approximately constant "background" in the density of states is observed [26, 27]. However, it is an experimental fact that the fraction of the total density of states corresponding to the gap is small [26, 27]. Therefore, the Fermi level may lie in the gap only if ν is very close to an integer. Since the density of states is small, the average distance between the gap states is much larger than their localization radius. It follows that hopping near the Fermi level cannot compete at high enough T with hopping associated with activation to the states in the peak of the density of states. Such an activation-type conduction has been studied in Refs. [26, 27]. As T goes down, the concentration of electrons activated to the bottom of the peak of the density of states decreases rapidly and hopping near the Fermi level becomes dominating. For $\nu = 2$ it happens at $T \lesssim 1$ K both for InGaAs/InP [20] and GaAs/GaAlAs [19, 26]. Thus, the activation-type conductivity does exist in the wide energy gaps but only at high temperatures and in a narrow range of ν around an integer, for which reason it does not influence our estimate of $\Delta\nu$.

For the Landau levels with large N, the spin-split levels, or for the levels corresponding to mixed states in double quantum wells, the σ_{xx} peaks are observed in the magnetic fields that may not be sufficient to form gaps in the density of states. So the Landau levels overlap while the σ_{xx} peaks may not. In this case, variable range hopping near the Fermi level should be the only mechanism of conduction at any ν and T. This is confirmed by the fact that the conductivity at $\nu = 3, 4, 6, 8, 10, 12$ obeys the law (1) even for the highest temperatures [19, 20]. However, rather small values of T_0 are observed for these narrow gaps, e.g., for the gap which separates the centers of the $N = 1 \uparrow$ and $N = 1 \downarrow$ levels. Our approach enables us to relate this fact with another striking phenomenon reported in Ref. [28]: if the only σ_{xx} peak corresponds to the $N = 1$ level, i.e., its spin splitting is not resolved, the width of the peak follows $T^{\kappa/2}$ instead of T^κ as for each of the $\uparrow$ and $\downarrow$ peaks taken separately. The same phenomenon was observed also for the unsplit $N = 2$ level [29]. Moreover, according to direct measurements [29], the localization length exponent in the latter case is much greater than 2.3.

In our picture, the only thing that may account for the change of the exponent in the dependence $\Delta\nu(T)$ is a stronger divergence of the localization length as compared with Eq. (3). For example, the value of ξ for the $N = 1$ level should behave as

$$\xi(\nu) \sim \xi_0|\nu - 3|^{-2\gamma} \tag{9}$$

[if two levels overlap strongly, the values of ν corresponding to the extended states are close to an integer in contrast to a half-integer in Eq. (3)]. By analogy with the derivation of Eqs. (4),(5), this assumption yields the width of the unsplit level

$$\Delta\nu = (T/T_1')^{1/2\gamma}, \quad T_1' \sim e^2/\varepsilon\xi_0. \tag{10}$$

It follows that the same temperature $e^2/\varepsilon\xi_0$ is characteristic for both the single and unsplit levels [compare Eqs. (5),(10)]. It would be important to verify this result experimentally. As the Fermi level approaches closely the center of any of the two Landau levels, $\xi(\nu)$ must diverge with the usual exponent. Therefore, when $|\nu - 3|$ becomes $\sim E_g/\Gamma \ll 1$, E_g being the energy distance between the centers of the levels, one should expect a crossover from the dependence (10) to that which is similar to (3) but with much larger "elementary length" $\xi_0' = \xi_0(\Gamma/E_g)^\gamma$ resulted from matching in the crossover point. The divergence of $\xi(\nu)$ should take place at $\nu = 3 \pm \delta\nu$, where $\delta\nu \sim E_g/\Gamma$. Thus, our conjecture is that the localization length behaves as follows:

$$\xi \sim \lambda\left[\frac{\Gamma^2}{|E^2 - \frac{1}{4}E_g^2|}\right]^\gamma, \quad E_g \ll \Gamma, \tag{11}$$

where the energy E is reckoned from the middle of the gap. At $E = 0$ we get $\xi \sim \lambda(\Gamma/E_g)^{2\gamma} \gg \lambda$. This settles the puzzle as to the large value of ξ_0 mentioned in the discussion following Eq. (5). If the two σ_{xx} peaks are resolved and the hopping conductivity in the middle between them is observed, the value of T_0 should be strongly reduced in comparison with that for large gaps:

$$T_0 \sim \frac{e^2}{\varepsilon\lambda}\left(\frac{E_g}{\Gamma}\right)^{2\gamma}. \tag{12}$$

This equation also gives the characteristic temperature at which the two peaks merge.

We think that the theory presented in this paper may be valid also for the case of a long-range random potential, but only if $\Delta\nu \ll (\lambda/R)^2$, where $R \gg \lambda$ denotes the correlation radius of the potential fluctuations. In such a potential, the one-particle localization is classical at $|E| \gg E_c \sim \Gamma(\lambda/R)^2$ and, consequently, $\Delta\nu \sim T/\Gamma$ at $T \gg E_c$.

Recently, a remarkable observation of the scaling behavior $\Delta\nu \propto T^\kappa$ in the fractional quantum Hall regime has been reported [3, 30]. According to Ref. [30], the exponent $\kappa \simeq 0.4$ is the same as for the integer quantum Hall effect. This gives an indication that our results may be applicable to the fractional regime, too. If that is the case, the characteristic temperatures T_1 and T_0 contain the fractional charges and so should be much smaller than those for the integer effect. The experimental values of T_1 and T_0 are actually very small [30, 31].

We are not aware of any published results of the direct measurements of the variable-range-hopping conductivity away from the centers of the gaps. To verify this theory, it would be useful to study the temperature dependence of σ_{xx} in the whole range of ν. Provided it is described by Eq. (1), one could try to fit the dependence T_0 on $|\nu - \nu_0|$ by the power law $T_0 \propto |\nu - \nu_0|^\beta$. Our prediction is that the exponent β is equal to κ^{-1} where κ determines the temperature dependence of the width of the σ_{xx} peaks.

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