Neutron Star Crusts

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We calculate properties of neutron star matter at subnuclear densities using an improved nuclear Hamiltonian. Nuclei disappear and the matter becomes uniform at a density of about $0.6n_s$, where $n_s \approx 0.16\ \mathrm{fm}^{-3}$ is the saturation density of nuclear matter. As a consequence the mass of matter in the crusts of neutron stars is only about half as large as previously estimated. In about half of that crustal mass, nuclear matter occurs in shapes very different from the roughly spherical nuclei familiar at lower densities. The thinner crust and the unusual nuclear shapes have important consequences for theories of the rotational and thermal evolution of neutron stars, especially theories of glitches.

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Above a density of about $3\times10^{11}\ \mathrm{g\,cm^{-3}}$ matter in neutron stars consists of a matrix of nuclei immersed in a sea of neutrons and a roughly uniform sea of electrons. When the density of matter approaches that of nuclear matter, the nuclei merge to form a uniform liquid of neutrons, protons, and electrons. In this Letter we calculate the equation of state of matter in this region, and consider how, and at what density, the transition from roughly spherical nuclei at low densities to the uniform matter takes place.

Long ago it was pointed out [1] that when nuclei in dense matter occupy more than half of space, it is energetically favorable for them to turn inside out, since in that way the surface and Coulomb energies can be reduced. This idea was developed further, especially in the context of stellar collapse, and it was demonstrated that as the fraction of space, u, occupied by nuclear matter and the mass density increase there is a series of transitions. With increasing density, nuclear matter, at first approximately spherical drops, becomes rodlike structures ("spaghetti"), and then platelike ones ("lasagna"). For u greater than about 0.5, the nuclear shapes are similar to those for lower fillings, but with the role of nuclear matter and neutrons interchanged [2–4]. This picture, which was first proposed on the basis of a liquid-drop model of the nucleus [2–4], has been confirmed and elaborated on by calculations using the differential Thomas-Fermi approximation [5, 6].

We have calculated the properties of matter using a compressible liquid-drop model [7] whose basic ingredients are the energy densities of nuclear matter and neutron matter, together with the interfacial energy between nuclear and neutron matter. To ensure that the properties of nuclear and neutron matter are treated consistently, we use for the energies of bulk nuclear and neutron matter the same function of the neutron and proton densities. The interfacial energy is calculated with the same microscopic Hamiltonian as is used for obtaining bulk properties. We have also calculated curvature contributions to the interfacial energy. These play a larger role in neutron stars than in stellar collapse since the interface between nuclear matter and neutron matter is more diffuse than the surfaces of laboratory nuclei.

The key ingredient in the calculations is the microscopic interaction Hamiltonian. The free energy of uniform-density nuclear and neutron matter has been calculated by Friedman and Pandharipande [8], who employed the microscopic V14 two- and TNI three-body nucleon forces, used hypernetted chain techniques, and considered a range of densities and temperatures. The microscopic interaction we favor, here called FPS, has been obtained by fitting [9] a Skyrme-like energy-density functional to their values [10]. The assumption basic to the Skyrme model, that the effective interaction has the spatial character of a (possibly density dependent) two-body delta function plus derivatives, then leads to a specification of the Hamiltonian even for a system with spatially varying density. As a stringent test of this assumption we have calculated [11] in the Hartree-Fock approximation the ground-state energies of a group of eight doubly closed-shell spherical nuclei ranging from ^{16}O to ^{208}Pb. The rms deviations from experiment of the energy per nucleon for the three forces Skyrme 1′ [12], SKM [13], and FPS are, respectively, 151, 94, and 42 keV. Thus our FPS Skyrme interaction reproduces remarkably well, with no adjustments, the ground-state energies of stable nuclei. This somewhat unexpected result suggests that the deduced terms in the Hamiltonian proportional to density gradients, which are needed to describe the nuclear surface, are not unrealistic, and that they can also be used to explore the properties of inhomogeneous matter with very small proton fractions, in the density range up to nuclear saturation density. The new numerical results we present are those obtained with the FPS interaction, although the 1′ results agree quite closely with them.

To investigate the nonspherical phases of neutron star matter we have used a compressible liquid-drop model based on the form described earlier [7]. The sample of matter considered is a Wigner-Seitz cell of appropriate shape (spherical, cylindrical, or planar) containing a dense nucleus surrounded by a nucleon vapor and enough electrons to be electrically neutral. For matter closer to symmetrical nuclear matter the more elaborate Thomas-Fermi calculations [5, 6] lead to qualitative conclusions similar to those based on the simple model. We expect a similar result to apply here, so the simple model should be adequate for initial investigations [14]. The Hamiltonian density of the Skyrme interaction chosen gives directly the volume or bulk energy of the nucleons in the cell. The interface energy and its curvature correction are calculated [15] in the Hartree-Fock approximation. In the electron contribution to the total energy, screening has been included, also exchange and surface diffuseness corrections to the Coulomb energy. At a given temperature T and density n, with the assumption of beta equilibrium and zero neutrino density, thermodynamic equilibrium determines all of the other dimensions and properties of the cell and of the matter. It also determines which of the geometries (spheres, cylinders, or plates, the former two as nuclei or as bubbles) is energetically favored.

In Fig. 1 are shown energy differences from the two-fluid phase for the various geometries, as a function of baryon density, for the FPS and SKM equations of state. The one-fluid curve for each equation of state thus represents the bulk energy to be gained by a phase separation, and the curves for each specified geometry give the additional surface plus Coulomb energy required when the phases are separated. The intersections of the curves signal approximately the phase changes. The phase transition between one nuclear shape and another is first order, and for the FPS force the change from spherical to cylindrical shape occurs at $n \approx 0.064$ fm^{-3}, as Fig. 1 shows, with a density jump of approximately 0.0001 fm^{-3}. A particularly interesting prediction of our calculations is that the dissolution of nuclei, and the adoption of a uniform fluid form, occurs at a density $n \approx 0.096$ fm^{-3}, which is only 60% of the density of symmetrical nuclear matter. (The possibility that the phase boundary occurs at a density well below n_s, even without the invocation of nonspherical phases, was suggested some time ago [7].)

The results shown in Fig. 1 illustrate the considerable difference in bulk properties between FPS and SKM (the interaction used by Bonche and Vautherin [16] in their dense-matter explorations) for this very neutron-rich matter. One sees that for SKM, the energy differences due to surface and Coulomb energies for the three-, two-, and one-dimensional nuclear phases are larger than, but of the same order as, those for FPS. The one-fluid matter becomes energetically favorable at densities much too small for the filling fraction to approach 0.5, the value needed for the bubble phases to be calculable. From a detailed examination of the pure neutron results, it appears that the adjustment made in SKM to fit the Siemens-Pandharipande neutron matter was made for the chemical potential of the proton, not that of the neutron. For the phase equilibrium involved, however, it is the neutron chemical potential that is important. In ordinary Skyrme forces such as this one, there is insufficient flexibility to fit both quantities, a deficiency that is overcome in the

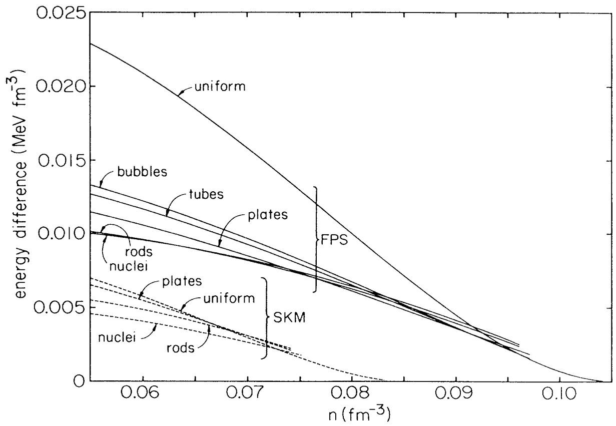

FIG. 1. Energies per unit volume as a function of density for the one-fluid phase, and the three-, two-, and one-dimensional nucleus phases, with (for FPS) the bubble (inverted structure) versions of the first two, after subtraction of the energy of the two-fluid phase, neglecting Coulomb and interface effects. The two nuclear interactions illustrated are SKM [6] and the version of FPS [8, 9] described in the text.

Skyrme version of FPS.

For examining the consequences of these new phases, a brief note of their properties may be useful. We give the radii of the squared-off proton distribution and, in parentheses, of the Wigner-Seitz cell [7]; in square brackets we give the mean proton and neutron densities, in fm^{-3}: 3D nuclei at the phase boundary, 8.7 fm (19.2 fm) [0.0021, 0.062]; 2D nuclei, 5.9 fm (15.2 fm) [0.0023, 0.068]; 1D nuclei, 3.1 fm (11.2 fm) (these are half of the diameter) [0.0030, 0.082]; 2D bubbles, 10.0 fm (13.0 fm) [0.0033, 0.088]; 3D bubbles at the boundary with uniform matter, 10.3 fm (13.4 fm) [0.0035, 0.092]. Except where noted, these are values at the midpoints of the density range where that phase is the most stable one.

Shown in Fig. 2 is a density profile of the crust of a neutron star obtained using these FPS results in the TOV equations [17]. (The model has a central density of 0.726 fm^{-3}, i.e., $4.5n_s$, and other properties listed in Table I.) Because of the rapid decrease of the density in the crust region, the crustal mass, defined as the matter exterior to the uniform fluid, depends strongly on the density at which the transition to the uniform-fluid phase occurs. Our conclusion that the transition density is only about $0.6n_s$ implies that the amount of crustal matter in neutron stars is considerably less than that calculated [18] on the assumption that nuclei dissolve at approximately nuclear matter density, as found by Baym, Bethe, and Pethick (BBP) [1]. To explore the extent of the effect, we compare the FPS results with those of the other interactions we have mentioned. To do this cleanly it is convenient to consider models with the same mass and radius. (We give later the scaling properties of the crust when those quantities are changed.) To achieve this objective, the equation of state of BBP [1], continued be-

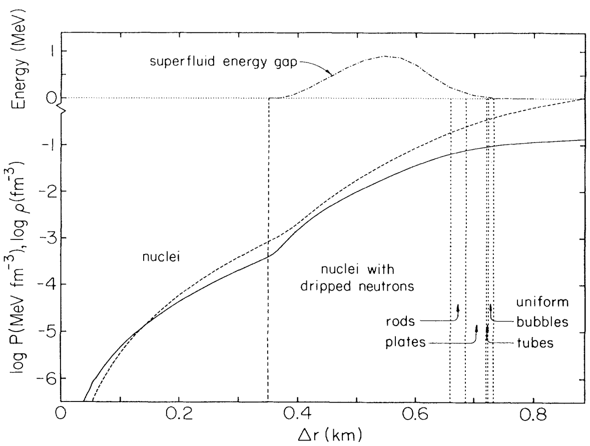

FIG. 2. Profile of a neutron star crust as given by FPS [8, 9]. The distance (in km) is measured from the surface. The solid line is density ρ/m_n, in fm^{-3}, and the dashed line is pressure, in $\mathrm{MeV\,fm}^{-3}$, plotted logarithmically. Vertical lines indicate the phase boundaries described in the text. At the top is shown the superfluid energy gap [22].

low neutron drip by the results of Baym, Pethick, and Sutherland [19], has been supplemented at high densities by FPS. The radii and crust masses of the star models for this composite model, for FPS, and for SKM, all of which have had central densities adjusted to give masses of $1.445 M_\odot$, are given in Table I. The crust transition densities used are 0.14 fm^{-3} for BBP [1], 0.0725 fm^{-3} for SKM (see Fig. 1), and 0.0957 fm^{-3} for the present work with FPS (see Fig. 1).

As can be seen from Table I, the crustal mass of the neutron star models given by the FPS interaction is about a factor of 2 smaller than that of BBP. This is also the case for the mass of "dripped" neutrons surrounding nuclei in the crust. Another surprising result is that, for the FPS interaction, matter with nuclei in nonspherical shapes makes up half the mass of the crust.

In a number of theories of glitches in the periods of neutron stars the magnitudes of contributions to the moment of inertia play an important role, and some of these are given in Table I. Again the results for the FPS interaction are smaller than the BBP ones by a factor of 2. It remains to be seen whether models which invoke pinning, by nuclei, of vortices in a superfluid neutron liquid are viable when the moments of inertia of the superfluid neutrons are so small. The phenomenological requirements on $\Delta I_d/I$, the fraction of dripped neutrons in the crust, obtained by Alpar *et al.* [20], are larger than those given by FPS by almost a factor of 2, in line with star models resembling BBP.

The crust properties we have presented, which were obtained by solving the full TOV equations [17], can be scaled for use with other neutron star models that give different total masses and radii. The crust is thin and contains little of the total mass, so that the TOV pressure

TABLE I. Radii, mass fractions in the crust, moments of inertia I, and their fractions in the crust, of a neutron star of mass $M = 1.445 M_\odot$. The row headed ΔM_c is the mass of all of the material exterior to the fluid core. ΔM_d is the part of it that consists of dripped neutrons. ΔM_n is the total mass that is in the form of solid matter with nonspherical nuclei, and ΔM_{dn} is the part of that material that consists of dripped neutrons. The subscripts apply in the same way to the fractions of the moment of inertia I. The three interactions used are a BBP composite [1], SKM [6], and the version of FPS [8, 9] described in the text. The radius units are km, the mass units are $M_\odot$, and the units of I are $M_\odot\,\mathrm{km}^2$. In parentheses are the fractional amounts.

Force	BBP	SKM	FPS
R	10.49	10.78	10.79
ΔM_c	0.0299 (2.07%)	0.0122 (0.84%)	0.0125 (0.86%)
ΔM_d	0.0242 (1.67%)	0.0103 (0.71%)	0.0084 (0.58%)
ΔM_n	...	...	0.0062 (0.43%)
ΔM_{dn}	...	...	0.0051 (0.35%)
I	61.56	60.89	62.57
ΔI_c	2.74 (4.45%)	1.21 (1.99%)	1.22 (1.94%)
ΔI_d	2.22 (3.60%)	1.02 (1.68%)	0.82 (1.32%)
ΔI_n	...	...	0.59 (0.94%)
ΔI_{dn}	...	...	0.48 (0.77%)

equation can be approximated as

$$\frac{\partial P}{\partial r} \simeq \frac{\rho G M \Lambda}{R^2}, \tag{1}$$

where $\Lambda = (1 - 2GM/Rc^2)^{-1}$ is the redshift factor. The crust mass can therefore be written as

$$\Delta M_{\mathrm{crust}} \simeq 4\pi R^2 \int_0^{\rho_B} \rho(r)\, dr \simeq \frac{4\pi R^4}{GM\Lambda} P_B, \tag{2}$$

where ρ_B and P_B are the density and pressure at the phase boundary defining the crust. Thus for models with different interiors, and different masses and radii, the crustal mass ratios given here scale like $R^4/\Lambda M^2$. Also, the amount of matter exterior to a given density is proportional to the pressure at that density. This provides an easy way to estimate the effect on the crust mass of moving its assumed boundary. From the general relativistic equations [21] a similar approximation may be obtained for the moment of inertia of the crust and its constituent parts:

$$\Delta I_{\mathrm{crust}} \simeq \frac{2}{3} \Delta M_{\mathrm{crust}} R^2 \frac{1 - 2GI/R^3c^2}{1 - 2GM/Rc^2}. \tag{3}$$

The effects of the nonspherical nuclei on microscopic properties of crustal matter have yet to be investigated. They could well influence significantly pinning of superfluid neutron vortices to nuclei. The superfluid energy gap appropriate to the dripped neutron density, as calculated by Wambach, Ainsworth, and Pines [22], does overlap the nonspherical phases as shown in Fig. 2.

Neutrino generation processes, which are important for understanding cooling of neutron stars, need to be reinvestigated without the usual assumption of spherical nuclei. On the basis of standard treatments of the neutrino-antineutrino pair bremsstrahlung one might expect this process to be affected by the unusual nuclear shapes. However, recent work has shown that if one allows for electron-lattice interactions to all orders in perturbation theory, bremsstrahlung by electrons moving in a perfect lattice is strongly suppressed compared with earlier estimates, and this result does not depend on the shape of the nuclei [23]. Another possibility is that some version of the direct URCA process might be allowed in spaghetti or lasagna nuclei as a consequence of protons having a continuous spectrum at the Fermi surface, and of the fact that umklapp scattering processes from the lattice might allow momentum conservation to be satisfied.

Our calculations demonstrate that nuclear models that give similar results for laboratory nuclei do not necessarily do so for the much more neutron-rich conditions in neutron stars. This provides a strong stimulus for laboratory studies of very neutron-rich nuclei, including those that will become accessible with the development of radioactive-beam facilities. Data on such nuclei will be important in refining nuclear models that can be applied to neutron star conditions.

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