New Look at the Solar Neutrino Problem

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We present a graphical and statistical comparison of observations of solar neutrinos with theoretical predictions for the Mikheyev-Smirnov-Wolfenstein effect and a class of "nonstandard" solar "models." The method used to compare the results makes the level of agreement or nonagreement between theory and observation more readily apparent. Included in our analysis are the standard-solar-model flux uncertainties. We also present predictions for Solar Neutrino Observatory charged-current rates.

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The solar neutrino problem has now been with us for over twenty years. For most of this period, its existence relied solely on the data taken at the Homestake mine by Davis and his colleagues [1,2]. However, in the past two years three new experiments, a real-time water detection experiment (Kamiokande) [3] and two radiochemical experiments involving gallium (SAGE and GALLEX) [4,5], have come on line to help resolve the issue. Kamiokande and SAGE provided support for the idea that fewer neutrinos are reaching the Earth than is predicted from the standard solar model [1], and have prompted analyses to investigate which models might best account for the observations [6–10]. Even on the basis of the Homestake and Kamiokande data alone, as we shall graphically display, it seems unlikely that an astrophysical solution of the solar neutrino problem is possible.

However, there are problems with assessing how good or bad a fit any solution of the solar neutrino problem provides to the data. The data are still somewhat sparse and noisy so that statistical analysis often yields ambiguous results. In addition, merely demonstrating that a model can account for the signal does not necessarily illuminate how good a "fit" it provides to the data. For example, the most popular proposal, Mikheyev-Smirnov-Wolfenstein (MSW) neutrino mixing in the Sun, yields predictions which can be in agreement with the average signal from both Homestake and Kamiokande. However, since this model involves two free parameters, the mass squared difference (Δm^2) and the mixing angle ($\sin^2 2\theta$), to fit two data points, the averaged Homestake and Kamiokande signals, perhaps one should be surprised if this model did not fit the data. In fact, this criticism is overstated because, as we shall show, the MSW predictions are quite restrictive. But from the standard mass-mixing-angle plots of confidence regions [1,6,7,11,12], it is difficult to discern this. In addition, comparisons of model predictions to the observations have in general not incorporated the residual uncertainties inherent in the predicted neutrino flux from the Sun.

To better address these questions and to help lay groundwork for interpreting the recent announcement by the GALLEX Collaboration of their Ga result and the future results of the Solar Neutrino Observatory (SNO) [13] heavy-water experiments, we present here a new way of comparing the models to the data—one which explicitly displays the regions allowed by the models in experiment rate-rate space. We consider the MSW mechanism and an ideal "nonstandard model" (NSM), described later. In our work the uncertainties inherent in the solar neutrino production rates are explicitly taken into account by Monte Carlo analysis. Predictions are displayed for Homestake, Kamiokande, SNO, and the gallium experiments normalized to the central value of the standard-solar-model rate [i.e., 7.9 solar neutrino units (SNU) for Homestake, 132 SNU for gallium]. Finally, because the resolution of the solar neutrino problem must ultimately rely on an unambiguous signature (which may require detailed measurements of the neutrino spectrum [1,13–15]), and not on a subtle statistical analysis, we do not present such a statistical analysis here. We believe that the figures we present speak adequately for themselves. We do not discuss models here which predict time variation in the solar neutrino signal, since in that case a method which presents the time correlations of the data and predictions is necessary. Also, certain of the qualitative features of the MSW predictions which we calculate may be characteristic of other neutrino models, and probably also apply to vacuum oscillations, for example [15].

Our presentation grows out of our recent detailed analysis of model predictions and the data [6], in which the problems and ambiguities of the canonical analyses became clearer. In order to assess, for example, how "natural" the observed neutrino signals are in the context of the MSW mechanism, we came to recognize that one should not concentrate on the MSW parameter space which agrees with the signal, but rather with the range in signal space spanned by the locus of all points in MSW parameter space. This is the basis of the present analysis.

We calculate the solar neutrino signal in each detector in the manner discussed in [6]. Here we present a brief overview of the procedure, including improvements made for the present analysis. The complete set of neutrino spectra [1] (rather than just the ^{8}B, ^{7}Be, and *pep* spectra considered in [6]) are numerically propagated through the Sun to the detectors, where they are convolved with interaction cross sections and detector response functions to yield the predicted signals. In adding in the very-low-energy neutrinos we have made certain simplifying assumptions. In calculating the flux of *pp* (to which only Ga is sensitive) we sampled the spectrum at five energies between 0.24 and 0.40 MeV. For the CNO neutrinos we approximate the spectra by appropriate linear combinations of neutrinos 0.5, 1, and 1.5 MeV. We did not incorporate the possibility of double resonances [12], in part because of the computational difficulty in our algorithm, but more importantly because this affects only the very-low-energy neutrinos for a small range of the larger Δm^2 which we consider. Because we do not present the MSW predictions explicitly for the gallium experiments (for reasons described below) and these are the only ones which are sensitive to these neutrinos, it is not necessary to include this correction explicitly here.

In addition we have calculated the variations in the predicted signal coming from the uncertainty in the solar-model input parameters. For a given nonstandard neutrino or solar model we made use of the results of full Monte Carlo solutions of solar models [1,11] which suggest that variations in the ten input parameters (six cross-section factors, $L_\odot$, $R_\odot$, $t_\odot$, and Z/X as described in detail in Chap. 7 of [1]) can be chosen from independent Gaussian distributions with appropriate mean and variance. The neutrino fluxes were calculated from the power-law expressions described in section 7.4 of [1] and the predicted rates in a given pair of detectors were found for each parameter choice, and 100000 runs were performed, choosing the input parameters by Monte Carlo methods applied to Gaussian distributions. From these we formed a histogram in rate-rate space from which a 68%-confidence-level contour was extracted.

We then plot the locus of 68%-confidence-level regions for all points in the appropriate parameter space. [The parameter space we considered for MSW is $10^{-8} \le \Delta m^2 \le 10^{-5}$ (eV)^2 and $\sin^2(2\theta) \in (0.005,1)$. Note that for $\Delta m^2 > 10^{-5}$ (eV)^2 most of the range is ruled out by the energy distribution in Kamiokande.] In the end, we can plot the predicted Homestake signal versus the Kamiokande signal, or Ga signal, etc., for comparison with observation. This provides not only a graphical appreciation of the agreement or disagreement of the predictions with the data, but also illustrates the range of predictions spanned by the "solutions." This latter point allows a better appreciation of the issues of "naturalness." If, for example, a solution could accommodate any Homestake versus Kamiokande result, it would produce a space-filling curve on our projections.

We compare these predictions, in our figures, to the averaged data, where they exist. As described in [6], averaging the Homestake data is a nontrivial task and at least three different possibilities seem viable. We present here the data averaged over the full 20-yr observing period with each data point either weighted by the quoted (symmetrized) error bars or unweighted. We also present the unweighted averaged data for the 3-yr period in which both Homestake and Kamiokande were running.

Figure 1 presents our results for the Homestake-Kamiokande combination. Figure 1(a) displays the prediction of the standard solar model, and the observed averaged signals, and gives the essence of the solar neutrino problem as it now stands. Figure 1(b) displays our results for the MSW and NSM predictions. We stress that this figure encodes in compressed form the results of substantial numerical analysis.

A few words are in order about our nonstandard solar model. Astrophysical effects inside the Sun might reduce either the ^{8}B or both the ^{8}B and ^{7}Be fluxes (e.g., by reducing the central core temperature, on which both of these reactions have a sensitive dependence). We have optimistically imagined here for exploration purposes that some NSM might exist in which this occurs, while all other observables remain in agreement with observation. In fact, of course, *no such real models exist*, nor are we claiming that the results presented here result from any fully consistent solar-model run. All that we have done is

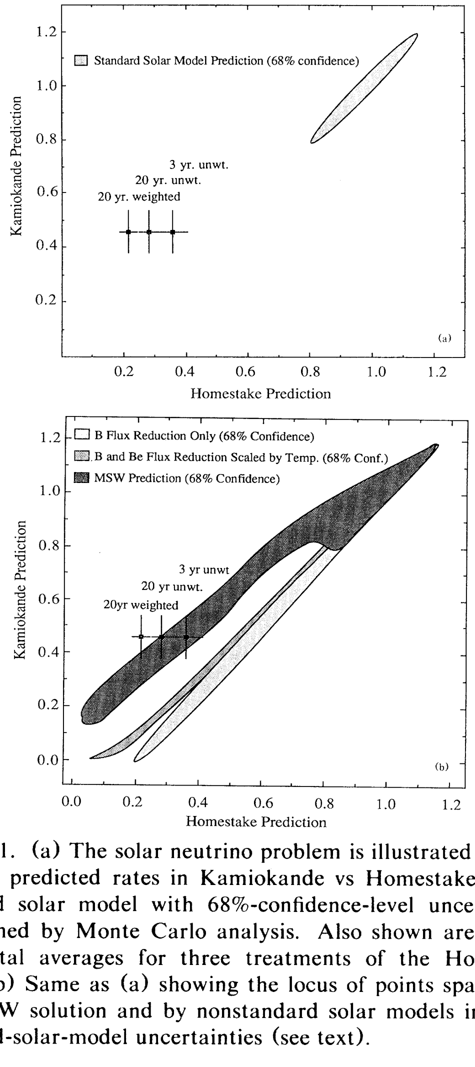

FIG. 1. (a) The solar neutrino problem is illustrated by plotting the predicted rates in Kamiokande vs Homestake for the standard solar model with 68%-confidence-level uncertainties determined by Monte Carlo analysis. Also shown are the experimental averages for three treatments of the Homestake data. (b) Same as (a) showing the locus of points spanned by the MSW solution and by nonstandard solar models including standard-solar-model uncertainties (see text).

allow the ^{8}B flux to vary arbitrarily from 0 to 100% of its predicted level, and have also considered the possibility where the ^{8}B and ^{7}Be fluxes both vary, in proportion to their dependence upon the core temperature (note that more complicated possibilities may be required by the data; see below). For each reduction factor, we have allowed the input parameters to vary using our Monte Carlo algorithm in a manner appropriate to the standard solar model. This allows some idea of the possible variability of the predictions for each ^{8}B flux value. Of course, since we have no explicit model associated with each ^{8}B flux reduction, we cannot be certain that the actual uncertainties are those associated with the standard solar model, but they are probably no smaller.

As can be seen from Fig. 1, even such NSMs provide a poor fit to the data. How poor depends upon the method used to average the Homestake signal, and on the presumed accuracy of the assumed error bars. On the other hand, the MSW mechanism fares well. The actual locus of points in Homestake-Kamiokande space allowed by this model, even incorporating standard-solar-model uncertainties, is quite limited, and moreover overlaps fairly well with the set of different ways of averaging the data. Again, however, one might note that if the 20-yr weighted average is utilized, even the best fit is only marginal at the 68% confidence level. We make no attempt to display which region of mass-mixing-angle space is associated with the best fits, both because this has been discussed in the past [1,6,7,11,12], and because the significance level associated with the best fits is marginal. Note, however, as has been pointed out in the past, that the data suggest that it is the lower-energy neutrinos from the Sun that are being "removed." Specifically, it seems as if Be neutrinos are being removed more efficiently than B neutrinos. It is this fact which is most difficult for NSMs (of the type envisaged here) to accommodate.

We next proceed to the predicted signals for the Ga experiments. In Fig. 2 we present the locus of Ga predictions for our NSM examples plotted against Kamiokande. We also include the data points for the Kamiokande, SAGE (2-yr average), and GALLEX results. We do not present in this figure the region corresponding to MSW predictions. This region is very nearly a space-filling curve, implying that MSW has little predictive power in this case. Also, a large area of the plane can be filled by a relatively small region of mass-mixing-angle parameter space. For example, the GALLEX-Kamiokande result lies in a region that corresponds to two very small ranges: $\Delta m^2 \sim 10^{-5}$ eV^2 and $\sin^2(2\theta) < 10^{-2}$ and $\Delta m^2 \sim 10^{-5}$ eV^2 and $\sin^2(2\theta) \sim 0.5$, as indicated by the GALLEX Collaboration [5]. (Including the Homestake result does not alter this conclusion.) Thus, while the MSW model can almost always provide a good fit to the Ga data, standard mass-mixing-angle plots are needed to determine the extent of the allowed parameter space [16]. This figure does indicate, however, that NSMs are disfavored by SAGE and have only minimal overlap with the GALLEX results at the 1σ level. If the first year SAGE result had been confirmed, then a neutrino-based solution of the solar neutrino problem would have been firmly indicated. As it now stands Ga provides no extra support beyond that already available from the other experiments. But it also should be stressed that it does not provide any evidence against this possibility. Note, for instance, that while the NSM rate dips down into the 1σ GALLEX range, it does so at the expense of lowering the predicted Kamiokande rate.

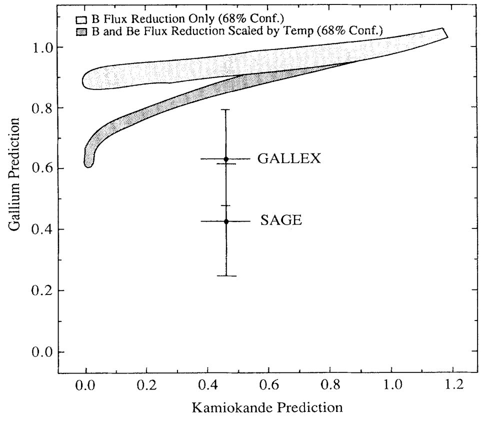

FIG. 2. The locus of points spanned by nonstandard solar models including standard-solar-model uncertainties for gallium vs Kamiokande. Also shown are the average rates quoted by SAGE, Kamiokande, and GALLEX. Errors shown come from adding statistical and systematic errors in quadrature.

The next solar neutrino experiment which is likely to report results is the SNO heavy-water experiment [13]. In Figs. 3(a) and 3(b) we present the predictions for the SNO experiment plotted against both Homestake and Kamiokande. The measured Homestake and Kamiokande (20-yr-weighted) average rates are plotted as hatched areas. Here we consider only the high-rate deuterium absorption cross section: $\nu_e + d \rightarrow p + p + e^-$. This is expected to give the largest counting rate (up to $\sim 10^4$ events/yr are predicted for the standard solar model) and can be separated from the signal from scattering off of electrons by angular cuts. This signal is also independent of issues of neutron capture associated with the neutral-current deuterium dissociation reaction. In "modeling" the SNO detector we assumed an electron recoil energy threshold of 5 MeV, with 100% efficiency for detecting electrons above this energy. Because both SNO and Kamiokande are sensitive only to ^{8}B neutrinos the NSM prediction, including uncertainties, is a thin line at almost 45° passing through the origin. The MSW prediction for Kamiokande-SNO looks somewhat similar to that for Kamiokande-Homestake, with the Kamiokande signal systematically larger than the SNO signal. This is because the SNO charged-current process we have examined is sensitive only to ν_e. Like Homestake, the SNO signal goes to zero while the Kamiokande signal does not,

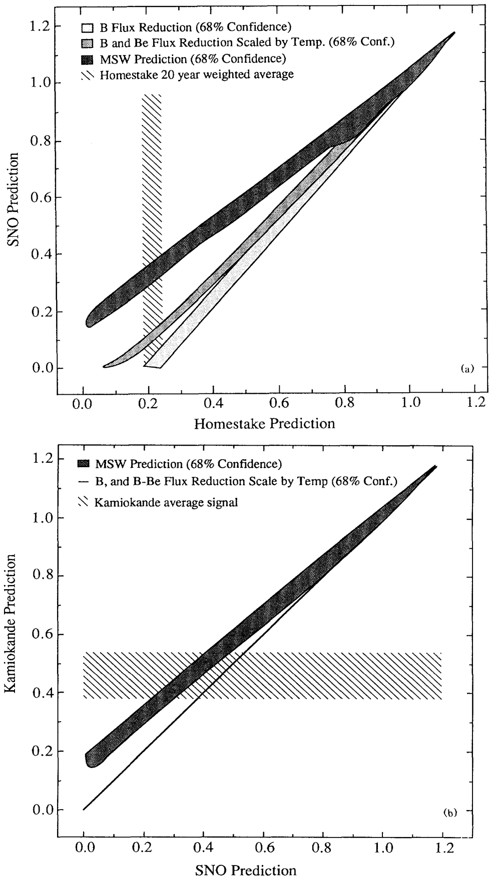

FIG. 3. (a) Same as Fig. 1(b) for SNO vs Homestake. Also shown is the region which would be consistent with the Homestake 20-yr weighted average rate. (b) Same as (a) for Kamiokande vs SNO, displaying the region which would be consistent with the Kamiokande signal.

due to its sensitivity to ν_μ and ν_τ. Thus SNO provides another possible check of the inconsistency of NSM explanations of the solar neutrino problem. As Fig. 3(a) shows, however, the high statistics obtainable by SNO will be useful, and necessary, in order to distinguish between the NSM and MSW predictions.

To conclude, the existing data argue against an astrophysical solution of the solar neutrino problem, *if the Homestake-Kamiokande combination is accurate.* The locus of MSW predictions is restrictive for this combination, and falls very close to the observed rates. Even with the GALLEX result, neutrino-based solutions remain statistically favored. MSW predictions fall within 1σ of all results, while NSM predictions are consistent at 1σ with the GALLEX result but not with the Homestake-Kamiokande combination. For NSMs to be clearly viable, one of these results would likely have to be incorrect, *or an astrophysical mechanism to suppress Be neutrinos more than B neutrinos would be required.* If the Ga data were to converge on the first SAGE result, a neutrino-based solution would be indicated, but the MSW parameter range would not be strongly constrained. If the SAGE result continues to approach the GALLEX value, the MSW space is strongly restricted (at the edge of the allowed parameter space), but the evidence against NSM solutions is weakened. SNO, designed to have high statistics, can solidify our knowledge, check for consistency, and may further slightly constrain neutrino-based models. Moreover, with up to $\sim 10^4$ events per year, SNO may begin deconvolving the neutrino spectrum from the observations, which, when combined with neutral-current observations, could help definitively confirm and, if so, pinpoint the parameters of a neutrino-based solution of the solar neutrino problem.

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