Vibration-Induced Size Separation in Granular Media: The Convection Connection

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We have investigated the rise of a single large glass bead through a vibrated cylindrical column of smaller particles and found that vibration-induced size separation in this geometry arises from convective processes rather than from local rearrangements as had been proposed previously. A convection cycle, rising in the middle and dropping in a thin stream along the walls of the cell, is responsible for all particle motion. Particles larger than the width of the thin downward convection zone are carried to the top of the column and then trapped, resulting in size segregation. For a variety of accelerations, the position of the rising particle can be scaled onto a single curve.

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Grain, gravel, and many powders are common examples of granular materials. They are aggregates of large solid particles which, in the absence of cohesive forces, are held together only by gravity. Granular materials are ubiquitous and their dynamics, such as flow properties and settling behavior, are of central importance to many industrial processes. Nevertheless, the highly complex, disordered structure, and nonlinear internal friction of these materials have so far prevented a comprehensive understanding of their properties [1].

One particularly important issue concerns the mixing properties of granular media consisting of several components which differ in size, density, or shape [2]. Industrially, control of component segregation during either the mixing process or subsequent handling of granular material is crucial. In contrast to ordinary fluids and gases, where particles are distributed according to their density, larger particles are found to rise and segregate at the top of a mixture when it is shaken [3–7]. Recently this problem has been modeled by large-scale computer simulations [8,9] in which granular mixtures were vibrated vertically. These simulations supported earlier reports [4] which had proposed that the upward motion was due to the smaller particles filling the voids generated underneath the larger ones during each shaking event. In this paper we present experimental evidence that the size separation of grains in vibrated granular media can arise instead from a fundamentally different mechanism, namely, convection [10,11].

The experiments used a 35 mm diam vertical Pyrex cylinder, open on the top, that was carefully leveled on a Vibration Test Systems VG100 vibration exciter. The cylinder was then filled to a height h_f with $d=2$ mm diam monodisperse spherical glass beads into which we placed one or more larger spherical beads of the same composition and density as the smaller ones. In order to follow the motion of a single bead, we dyed it with ink so that its size, elastic constants, and density were left unperturbed. This mixture was then subjected to controlled, vertical shakes ("taps") separated by 1 s waiting periods to allow for complete relaxation of the material between taps. Each tap was generated by feeding one cycle of a 30 Hz sine wave into the exciter. A PCB Piezotronics model 303A03 accelerometer monitored the tapping acceleration, a. The bead packings were not sufficiently transparent to allow direct optical measurement of a dyed tracer bead's location unless its position was close to the surface or the cylinder wall. To locate beads deeper inside the column, we stopped the run and carefully removed all material above the tracer bead. Each run was started with a freshly prepared random packing (below as well as above the tracer beads), as we found that partial removal of the material and subsequent continuation of the run significantly disturbed the packing and altered the bead velocity.

Figure 1 shows the results of a typical experiment in which we track the depth, Δ, below the top surface of a single large sphere placed in a background of $d=2$ mm spheres and vibrated at an rms acceleration $a=7g$ ($g=9.8$ m/s^2). Large beads of diameter D up to 25 mm

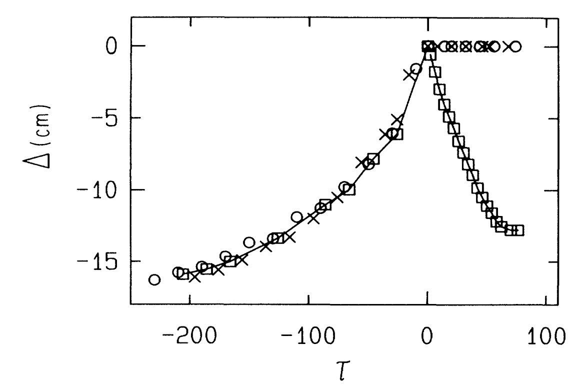

FIG. 1. The depth, Δ, of single $D=19$ mm ($\times$), $D=6$ mm ($\circ$), and $D=2$ mm ($\square$) spheres placed in the background of $d=2$ mm spheres as a function of tap number τ. The particle reaches the top surface ($\Delta=0$) at $\tau=0$. The rms acceleration during each tap was $a=7g$. After reaching the top surface, the 19 and 6 mm beads remain there whereas the 2 mm bead immediately moves to one edge of the container and starts to descend along the wall.

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were studied; $D=19$ mm and $D=6$ mm are shown in Fig. 1. Δ is measured from the top of a tracer bead to the column surface, h_f, and τ is the tap number measured with respect to the time the bead emerges at the top surface. For comparison, we also show Δ for a 2 mm tracer bead. All three beads rise at approximately the same rate. After reaching the surface the larger beads remain at the top, while the 2 mm bead immediately moves to one edge of the container and descends along the wall. This is a manifestation of a convection roll in the system that rises in the center and falls in a thin sheet at the boundaries.

To visualize the convection cycle we place one large bead together with a single layer of dyed 2 mm background beads between otherwise identical, but undyed 2 mm beads. Figures 2(a)–2(c) are schematic drawings illustrating the experimentally observed bead motion. These drawings are cross sections; the actual patterns exhibit cylindrical symmetry around the vertical column axis. Figure 2(a) shows the initial configuration before the first tap. Immediately after trapping begins, a ring of dyed beads along the wall moves toward the bottom of the cylinder [Fig. 2(b)]. Individual beads in this outside ring move coherently over many taps and bead diameters, resulting in a highly symmetric "lock-step" motion down the cylinder wall. Beads reaching the bottom of the cylinder turn inward and begin to move up in the central region of the tube [Fig. 2(c)]. Meanwhile, the inner core of the dyed layer, together with the single large bead, travels to the surface [Fig. 2(b)]. After reaching the top, the small background beads move towards the outer walls and begin to descend in the same fashion as did the outer portion of the initial layer. We have verified that this behavior is independent of the presence of large beads; the convection still occurs if all beads are the same size. In its cross section the overall flow pattern resembles that obtained by Taguchi [12], Gallas, Herrmann, and Sokolowski [13], and Thompson [14] in computer simulations of two-dimensional, rectangular convection cells and also those seen experimentally in two-dimensional geometries [10,11].

In Fig. 1 the speed of the bead motion increases with the height of the bead in the column. Also the speed of the downward-moving layer near the wall is faster than the upward-moving plug near the center of the container. This nonlinear behavior does not depend on the ratio $\Phi=D/d$ of tracer bead diameter, D, to background bead diameter, d. However, by comparing various filling heights, we find that the rise velocity depends on the distance Δ to the top surface, rather than on the height h, above the bottom. This is shown in the inset in Fig. 3 where Δ is plotted versus τ, the number of taps remaining to reach the column surface. Within our experimental accuracy, all data for a given applied acceleration a follow a unique profile, independent of size ratios Φ, initial starting points h_i, and cylinder filling heights h_f.

In the main part of Fig. 3 we show that the data for the different accelerations can be scaled in such a way so as to fall on a single curve. In this plot the depth Δ has been scaled by the acceleration a: $\tilde{\Delta}=\Delta\Gamma_0/(a-\Gamma_0)$, where $\Gamma_0\approx 1.2g$ is the onset acceleration where convection rolls start to appear [15]. Such scaling indicates that the large beads slow down as the acceleration approaches Γ_0 and stop moving when this threshold acceleration is reached. This is consistent with our picture that the upward

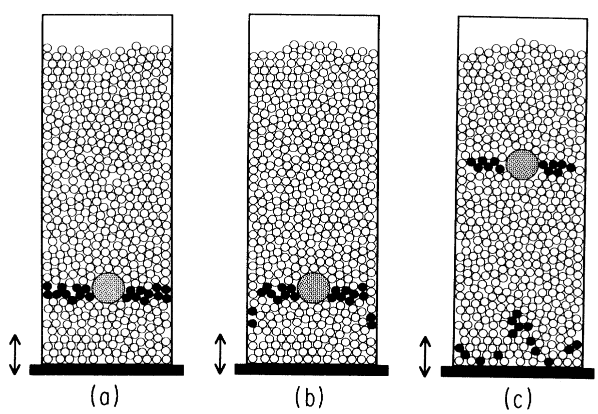

FIG. 2. A schematic diagram of the cross section of the cylinder. (a) The initial configuration before the onset of tapping. A large bead rests in a layer of colored small beads. This layer is placed in a background of small beads that are identical except for their color to small beads in the layer. (b) After just one or two taps, the colored beads against the wall start to move downward. (c) After more taps, both the large bead and the small colored beads at the center move up in the cylinder. The beads near the walls which have reached the bottom of the cylinder move inwards and start to rise.

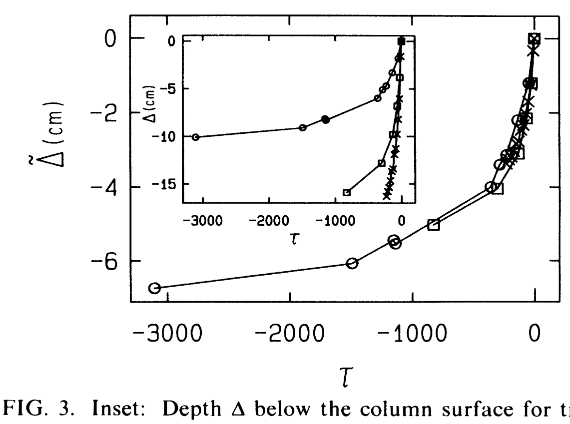

FIG. 3. Inset: Depth Δ below the column surface for tracer beads as function of tap number τ for three rms accelerations [$a=3g$ (○), $5g$ (□), and $7g$ (×)]. For a fixed acceleration, all the data follow a common curve, which is independent of the bead size, the initial starting height of the bead, and the column height. The main figure shows $\tilde{\Delta}=\Delta\Gamma_0/(a-\Gamma_0)$, the scaled depth, vs τ for all the accelerations shown in the inset. $\Gamma_0=1.2g$.

motion of the large particles is produced by the convection rolls within the cell. This form for the scaling also indicates that the velocity of the particle at the top surface should be proportional to $a-\Gamma_0$. Since all these data were taken with identical boundary conditions, the scaling does not indicate how to include any of the frictional effects of the boundary.

Size segregation occurs in our experiments only in the sense that beads of sufficiently large size, once swept up with background beads to the top of the column, are unable to follow the convection cycle through the very narrow outer region of downward motion and are thus trapped at the top surface. Depending on its size, the larger particle may be able to penetrate the top layer of the material for a certain distance before it is thrown back into the upward-moving part of the roll. Thus we might expect a smearing of the boundary between the segregated species depending on Φ. We are presently investigating this dependence. As we discuss below, when we suppress the convection no relative particle motion occurs and larger beads remain at their initial positions even after extended tapping.

These findings are in contrast to recent simulations by Jullien, Meakin, and Pavlovitch [9] of size separation under vertical vibrations. These authors found that local avalanching underneath the larger beads produces the upward motion of large particles. As a consequence of this mechanism, larger beads in the simulations rise to the top with constant velocity. This occurs as long as $\Phi=D/d$ is larger than a threshold value Φ_c. Below Φ_c a stratified steady state results in which larger beads stop rising once they attain an equilibrium height determined by Φ. Based on an angle of repose $\Theta_r=54°$ for the background beads, Jullien *et al.* obtained $\Phi_c=2.8$. If a smaller value of Θ_r is used (closer to the experimental value $\Theta_r\approx 30°$ for glass spheres), we expect the value of Φ_c to be reduced. Our data, on the other hand, demonstrate that, as part of the convection cycle, upward motion reaching the top surface of the column continues even for $\Phi=1$. At least for the experimental configuration used here, it appears that convection rather than "local avalanching" (or some other internal rearrangement process during tapping) [8,9] is the important mechanism for size segregation.

Boundaries play the crucial role in initiating and sustaining any relative particle movement in granular materials. In this regard, these materials behave very differently from ordinary fluids where buoyancy forces can drive the convection. Convection appears directly linked to the interaction of beads with the cylinder walls. For smooth, slippery walls convection was greatly reduced. Conversely, cylinders roughened on the inside with a uniform coating of 0.5 mm glass beads consistently showed strong, symmetric, convection cycles (the data shown in Figs. 1 and 3 were taken in cylinders prepared in this way). New Pyrex cylinders formed symmetric convection rolls, but prolonged use resulted in asymmetric roll formation. We suspect that the polishing action of the vibrated bead pack gradually lowered the frictional interaction between the beads and wall. When the 35 mesh coating was applied to only a thin vertical strip along the wall, with the rest of the cylinder left smooth, we found a very asymmetric convection pattern, with downward movement of the outer layer occurring only near the rough strip as shown schematically in Fig. 4(a).

Computer simulations have not yet considered the role of the boundary conditions in sufficient detail. The absence of any sign of convective behavior in the size segregation simulations by Rosato *et al.* [8] and Jullien *et al.* [9] may be related to their use of periodic boundary conditions. Indeed, an earlier simulation by Haff and Werner [16] observed segregation only when slanted walls were implemented but not when frictionless, vertical wall boundary conditions were used. The recent 2D simulations by Taguchi [12] and Gallas, Herrmann, and Sokolowski [13] clearly indicate that strong inelastic interactions between beads and walls are necessary to produce convection cycles. This is also consistent with the experimental results of Clément, Durand, and Rajchenbach [11] on convection in a two-dimensional layer. They found that high friction between the beads as well as between the beads and the boundaries was necessary to produce convection cycles.

As a final demonstration that convection drives size segregation, we have designed a container in which the large particles descend to its bottom rather than rise to its

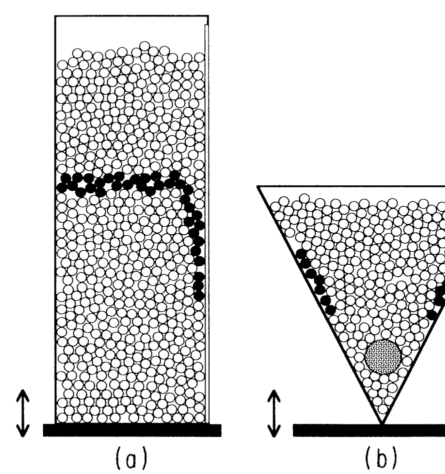

FIG. 4. (a) A cross section of the configuration after several taps when the right side of a cylinder has been coated with rough sand. The particles near the rough part of the wall move quickly toward the bottom of the cylinder while the rest of the particles remain much less affected. (b) Cross section of the configuration in a conical cell after a number of taps. A large bead follows the convection roll through the center of the cone to its bottom, but then cannot follow the smaller colored beads back up along the walls in the thin region of the convection roll.

top during vibration. Using a container with outwardly slanting walls (such as in a conical or spherical geometry) we find that the convection moves upward along the walls and moves downward in a wider flow in the middle of the container as illustrated in Fig. 4(b). The boundary motion appears to be wider than what occurs in the case of vertical walls. In this situation, the large bead is entrained in the flow and moves downward in the middle of the container but then cannot fit into the thinner stream moving upward along the wall. The large beads are consequently trapped at the bottom rather than at the top of the system as we have seen with vertical walls.

This study raises a number of intriguing questions regarding three-dimensional convection patterns in granular media. It is not yet known how the width of the cylinders influences the pattern. (In quasi-two-dimensional systems we observe convection rolls even in systems with a width 200 times the bead diameter [17].) Also, further work is required to elucidate the detailed three-dimensional shape of the convection rolls inside the bulk of the material. While we can trace individual grains in relatively narrow containers, we do not, in general, have probes of the interior of large-scale bulk motion. Previous work on granular materials using radioactive tracer beads [7] or x rays [18] has been limited in terms of resolution or imaging depth. Finally, although it is clear that the boundary conditions at the walls are important for creating convection, the precise nature of the interplay between frictional forces and the shape of the walls is not understood. Our experiments show that if the walls slant outwards, there is motion upward along the wall, whereas if the walls are vertical (and exert sufficiently strong friction forces) the grain motion is downward at that point. Consequently, there must be some wall angle that may depend on the details of this interplay at which convection rolls are not initiated. Although our results do not rule out the possibility that there are situations where local grain rearrangements might cause segregation, they do show that there is a fundamentally different mechanism for size segregation which is based on convection. This clearly indicates a deficiency in the present understanding of these two phenomena. Far from being separate and unrelated we find that convection can, in fact, drive segregation.

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