Inflation at the Electroweak Scale

Lloyd Knox[(1)] and Michael S. Turner[(1),(2),(3)]
[(1)]*Department of Physics, The University of Chicago, Chicago, Illinois 60637-1433*
[(2)]*Department of Astronomy & Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433*
[(3)]*NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500*
(Received 25 August 1992)

We present a model for slow-rollover inflation where the vacuum energy that drives inflation is of the order of G_F^{-2}; unlike most models, the conversion of vacuum energy to radiation ("reheating") is moderately efficient. The scalar field responsible for inflation is a standard-model singlet, develops a vacuum expectation value of 4×10^6 GeV, has a mass of about 1 GeV, and can play a role in electroweak phenomena. We also discuss models where the energy scale of inflation is somewhat larger, but still well below the unification scale.

PACS numbers: 98.80.Cq, 12.15.Ji

Over the past decade cosmologists have come to realize that elementary-particle physics plays a very important role in cosmology: Microphysical events that took place during the earliest moments of the Universe ($t \ll 10^{-5}$ sec) and involved very high energies ($E \gg 1$ GeV) likely hold the key to understanding some of the most puzzling features of the Universe today. For example, baryon-number, *C*, and *CP* violating interactions occurring early on can explain the net baryon number of the Universe (baryogenesis [1]); the ubiquitous dark matter may be comprised of relic elementary particles (particle dark matter [2]); an early period of rapid expansion may account for the smoothness and spatial flatness of the Universe (inflation [3]); and a variety of early Universe scenarios have been proposed to explain the origin of the density inhomogeneities necessary to seed the formation of structure in the Universe (inflation, cosmic strings [4], and textures [5]).

Until recently it appeared that the "input microphysics" for these intriguing speculations involved energies of the order of 10^{14} GeV or larger (grand-unification scale or larger), well beyond the "reach" of terrestrial experiments. However, scenarios for baryogenesis based upon physics at the electroweak scale have been put forth [6], and here we propose a simple model for inflation at the electroweak scale. Early Universe scenarios based upon electroweak-scale physics are particularly appealing since the underlying physics may be tested in the near future, e.g., at LEP (CERN), at the Tevatron (Fermilab), at HERA (DESY), or at the Superconducting Super Collider.

Historically, inflation [7] developed from an attempt to solve the monopole problem associated with grand unified theories (GUTs) (extreme overproduction of magnetic monopoles during the GUT phase transition [8]), and thus involved unification-scale energies. Further, because the baryon asymmetry of the Universe must be produced *after* inflation and most scenarios for baryogenesis involve superheavy particles and unification-scale physics, it seemed necessary that inflation involve a very high energy scale. Indeed, in essentially all models of inflation the vacuum energy that drives inflation is of the order of $(10^{14}\ \mathrm{GeV})^4$ [9]. Moreover, in some models of inflation—chaotic inflation [10], inflation based upon a simple supergravity model [11], and extended inflation [12]—the energy scale of inflation is set by requiring density perturbations of an appropriate size, and in these models that energy scale *must* be of the order of the unification scale.

In this Letter we discuss a simple model where the scale of inflation can be as small as the electroweak scale (≈ 1 TeV). We begin the description by reviewing the requirements that a "successful" model of inflation must satisfy [3,13].

(1) Sufficient inflation to solve the horizon and flatness problems. This corresponds to $N \sim 30 + \ln[T_{\rm RH}/(1\ \mathrm{TeV})]$ *e*-foldings of the cosmic-scale factor during inflation, where $T_{\rm RH}$ is the temperature at the beginning of the postinflation, radiation-dominated epoch. When the energy scale of inflation is smaller, the required amount of inflation is less.

(2) Density perturbations of appropriate size: $\delta\rho/\rho \approx 10^{-5}$ (the most difficult requirement to satisfy). Density perturbations must be large enough to initiate structure formation and small enough to be consistent with the smoothness of the cosmic background radiation (CBR); the recent detection of CBR temperature anisotropies on angular scales larger than about 10° by the differential microwave radiometers (DMR) on Cosmic Background Explorer (COBE) [14] sets precisely the amplitude of the density perturbations.

(3) Sufficiently high reheat temperature. The Universe must be radiation dominated by the epoch of primordial nucleosynthesis so that nucleosynthesis proceeds in the usual way ($T_{\rm RH} \gtrsim 10$ MeV), and hot enough after inflation for baryogenesis to take place, as any preinflation baryon asymmetry is diluted exponentially by the enormous entropy release associated with reheating. While it was thought that baryogenesis required temperatures in excess of 10^{10} GeV or so, interesting models now exist where baryogenesis occurs at the electroweak scale [6] and temperatures as low as 1 GeV [15,16].

(4) The abundance of unwanted, massive relics such as monopoles, gravitinos, and oscillating scalar fields produced after inflation must be very small. In order that such nonrelativistic relics do not contribute too much mass density today, their energy density after inflation must be less than $[(10^{-8}\text{ GeV})/T_{\rm RH}]$ that of radiation. This is easier to accomplish when the energy scale of inflation is lower: The constraint is less stringent, and many dangerous relics are too heavy to be produced. For example, when the scale of inflation is very low, GUT symmetry breaking must occur before inflation, automatically solving the monopole problem.

(5) An integral—better yet, a testable—part of a sensible particle-physics model.

We denote the scalar field responsible for inflation by ϕ; as is well appreciated, in slow-rollover inflation ϕ must be very weakly coupled in order to satisfy the density-perturbation constraint [3]. At the energy scale of interest, ϕ must be a gauge singlet of the effective Lagrangian [17]. For simplicity, we take its scalar potential to be of the Coleman-Weinberg type [18], where the symmetry-breaking minimum is generated by radiative corrections,

$$V(\phi)=B\sigma^4/2+B\phi^4[\ln(\phi^2/\sigma^2)-\tfrac{1}{2}]\,; \tag{1}$$

other simple polynomial potentials can also be used (e.g., $V=V_0-\alpha\phi^4+\beta\phi^5$ [19]). Here σ is the global minimum of the potential and B is a dimensionless coupling whose value must be about 10^{-15}–10^{-14} to achieve density perturbations of the appropriate size. (This is nothing new; *all* models of inflation have such a small coupling constant whose fundamental understanding is still lacking [3].) Coleman-Weinberg potentials are very flat near $\phi=0$, $V\simeq\mathcal{M}^4-b\phi^4$, where $\mathcal{M}^4=B\sigma^4/2$, $b=|\ln(\phi^2/\sigma^2)|B$, and for this reason have been used often in models of inflation [20].

If, for the moment, we ignore the coupling of ϕ to other fields, the equation of motion for ϕ in the expanding Universe is

$$\ddot\phi+3H\dot\phi+V'=0\,; \tag{2}$$

we have assumed that ϕ is homogeneous (at least over a region the size of the Hubble radius). During inflation ϕ "rolls" slowly—but inevitably—toward $\phi=\sigma$; as it does its potential energy drives a nearly constant expansion rate,

$$H^2=\frac{8\pi V(\phi)}{3m_{\rm Pl}^2}\simeq\frac{4\pi B\sigma^4}{3m_{\rm Pl}^2}\,, \tag{3}$$

where $m_{\rm Pl}\equiv G^{-1/2}=1.22\times10^{19}$ GeV is the Planck mass. During the slow-roll phase, when ϕ is near the origin ($\phi\lesssim\phi_e$ [21]), the $\ddot\phi$ term can be neglected so that $\dot\phi\simeq-V'/3H$. Using this approximation, it follows that during the time it takes the scalar field to evolve from ϕ to the minimum of its potential the cosmic-scale factor grows by $N(\phi)$ e-foldings:

$$N(\phi)\simeq\frac{8\pi}{m_{\rm Pl}^2}\int_\phi^{\phi_e}\frac{V(\phi)d\phi}{-V'}\simeq\frac{\pi}{2|\ln(\phi^2/\sigma^2)|}\frac{\sigma^4}{m_{\rm Pl}^2\phi^2}\,, \tag{4}$$

where $|\ln(\phi^2/\sigma^2)|\approx60$ is approximately constant during the slow roll. In order to achieve the 30 or so e-foldings required the initial value of the scalar field must be less than $\sigma^2/30m_{\rm Pl}\simeq10^{-14}\sigma[\mathcal{M}/(1\text{ TeV})]$; this is the least attractive feature of our model.

During the slow-roll phase density fluctuations arise due to quantum fluctuations in the scalar field ϕ. The amplitude of the perturbation on a given scale λ, when that scale crosses inside the horizon, is roughly [3]

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm hor,\lambda}\sim\left(\frac{H^3}{V'}\right)_{N_\lambda}\sim\sqrt{B}N_\lambda^{3/2}\,, \tag{5}$$

where the subscript N_λ indicates that the quantity is to be evaluated when the scale of interest crossed outside the horizon during inflation, which occurs $N_\lambda\simeq21+\ln[T_{\rm RH}/(1\text{ TeV})]+\ln[\lambda/(1\text{ Mpc})]$ e-foldings before the end of inflation. To achieve $\delta\rho/\rho\approx10^{-5}$, B must be of order 10^{-15}.

The quadrupole temperature anisotropy detected by the COBE DMR [14] allows us to be more precise. Expanding the CBR temperature on the sky in spherical harmonics, the quadrupole anisotropy is related to $a_2^2\equiv\sum_m\langle a_{2m}^2\rangle$ (the average, over all observers in the Universe, of the sum of the $l=2$ amplitudes squared) and the inflationary potential:

$$\left(\frac{\Delta T}{T}\right)_Q^2=\frac{a_2^2}{4\pi}=\frac{32\pi}{45}\frac{V^3}{V'^2m_{\rm Pl}^6}\approx\frac{2|\ln(\phi^2/\sigma^2)|B}{45\pi^2}N_\lambda^3\,. \tag{6}$$

Setting $N_\lambda\sim30$, the scale of relevance for the quadrupole anisotropy, and taking $(\Delta T/T)_Q\simeq6\times10^{-6}$ [14], we find that $B=6\times10^{-15}$. This result is relatively insensitive to the scale of inflation—for $\sigma=10^{16}$ GeV, $B\simeq3\times10^{-15}$—but very sensitive to the value of $(\Delta T/T)_Q$, which is probably uncertain by a factor of 2.

One last remark about density perturbations: From Eq. (5) we see that the perturbations are not quite scale invariant, $(\delta\rho/\rho)_{\rm hor,\lambda}\propto N_\lambda^{3/2}$. Expanding $(\delta\rho/\rho)_{\rm hor,\lambda}$ about the mean of the galaxy scale (1 Mpc) and the present horizon scale (10^4 Mpc) we find that $(\delta\rho/\rho)_{\rm hor,\lambda}\propto\lambda^{0.06}$ (corresponding to a power spectrum $|\delta_k|^2\propto k^n$ with $n=0.88$). This has the effect of depressing perturbations on small scales relative to large scales by about a factor of 2, and may be important, as most numerical simulations indicate that an exactly scale-invariant spectrum of density perturbations normalized to the COBE DMR quadrupole has too much power on small scales [22].

Quantum fluctuations during inflation also give rise to a spectrum of gravitational waves [23]; these gravitational waves cross the horizon after inflation with an amplitude of the order of $H/m_{\rm Pl}\sim2\times10^{-32}[\mathcal{M}/(1\text{ TeV})]^2$, orders of magnitude smaller than in models where the scale of inflation is comparable to the unification scale—and far too small to be detected.

Finally, consider reheating, the conversion of the vacuum energy to thermal radiation. After its slow roll, the ϕ field begins to oscillate about the minimum of its potential, and the vacuum energy that drives inflation is converted into coherent scalar-field oscillations (a condensate of nonrelativistic ϕ particles). Reheating takes place when the ϕ particles decay into light fields, which, through their decays and interactions, eventually produce a thermal bath of radiation [3,24]. During the period of coherent ϕ oscillations the Universe is matter dominated and the energy density trapped in the ϕ field decreases as the cube of the scale factor. The reheat temperature is determined by the decay time of the scalar-field oscillations, which is given by the inverse of the decay width Γ of the ϕ. If $\Gamma \lesssim H$, the coherent-oscillation phase is relatively long and the reheat temperature $T_{\rm RH} \simeq (m_{\rm Pl}\Gamma)^{1/2} < \mathcal{M}$, corresponding to less than 100% conversion of vacuum energy to radiation. On the other hand, if $\Gamma \gtrsim H$, ϕ oscillations decay rapidly, and $T_{\rm RH} \simeq \mathcal{M}$, corresponding to 100% conversion of vacuum energy to radiation. Next we discuss why reheating is typically very inefficient in slow-rollover inflation, and how it becomes more efficient as the scale of inflation is decreased.

Suppose ϕ couples to a light, Majorana fermion with Yukawa coupling g; its decay width $\Gamma = g^2 m_\phi/4\pi$, where $m_\phi^2 = V''(\sigma) = 8\sqrt{2B}\,\mathcal{M}^2 \simeq (1\ \text{GeV}^2)[\mathcal{M}/(1\ \text{TeV})]^2$. The condition for efficient reheating is

$$\frac{\Gamma}{H} = \left(\frac{3g^4}{8\pi^3}\right)^{1/2} \frac{m_{\rm Pl}}{\sigma} \simeq \left(\frac{g}{2\times 10^{-6}}\right)^2 \frac{1\ \text{TeV}}{\mathcal{M}} \gtrsim 1\,. \quad (7)$$

The larger the scale of inflation, the larger the value of g required for efficient reheating; for $\mathcal{M} = 10^{14}$ GeV, good reheating requires $g \gtrsim 0.5$.

Next, consider the other constraints to g. In order not to spoil the flatness of $V(\phi)$, the radiative corrections due to the fermion that couples to ϕ must be small: This requires $g^4 \ll B$ or $g \ll 3\times 10^{-4}$. The fermion acquires a mass of order $m_f \sim g\sigma$, which must be less than half the mass of the ϕ. This provides the stricter constraint, $g \lesssim \sqrt{2B}$, and illustrates how reheating and density perturbations work at cross purposes: Reheating is better for a larger value of B, but density perturbations require a very small value for B.

By saturating the bound $g \lesssim \sqrt{2B} \sim 10^{-7}$, we can express the maximum achievable reheat temperature as a function of the scale of inflation:

$$\frac{T_{\rm RH}(\max)}{\mathcal{M}} \sim \frac{(\Gamma m_{\rm Pl})^{1/2}}{\mathcal{M}} \simeq B^{5/8}\left(\frac{m_{\rm Pl}}{\mathcal{M}}\right)^{1/2} \approx 0.1\left(\frac{1\ \text{TeV}}{\mathcal{M}}\right)^{1/2}. \quad (8)$$

For $\mathcal{M} \sim 1$ TeV, $T_{\rm RH}(\max)$ is of order 100 GeV, and grows only as $\sqrt{\mathcal{M}}$—for $\mathcal{M} \sim 10^{14}$ GeV, $T_{\rm RH}(\max) \sim 10^7$ GeV. Other modes of reheating are possible; e.g., $\phi \to 2\chi$ (χ is another scalar field), through interaction terms of the form $\mathcal{L}_{\rm int} = \beta\phi\chi^2$ or $\lambda\phi^2\chi^2$. Up to factors of order unity, Eq. (8) also applies [25,26].

Let us squarely address the least attractive feature of our model, the small initial value of ϕ required for sufficient inflation, $\phi_i \lesssim \sigma^2/30m_{\rm Pl} \simeq 10^{-14}\sigma[\mathcal{M}/(1\ \text{TeV})]$. Many models of slow-rollover inflation require a small initial value for ϕ [27]; the very small value required here traces directly to the very low energy scale of inflation: For comparison, taking $\mathcal{M} \sim 10^{14}$ GeV, $\phi_i \lesssim 10^{-3}\sigma$. We note that this problem can be mitigated by degrees by simply increasing the scale of inflation.

In order to achieve inflation in our model there must be regions of the Universe where the value of the ϕ field is very small; such regions will undergo inflation. In regions where the value of the ϕ field is not small, there will be no inflation. After inflation, the regions where ϕ was sufficiently small have grown exponentially in size—and with plausible assumptions about the distribution of the initial value of ϕ the inflated regions should occupy most of the physical volume of the Universe [28].

Such a small initial value for ϕ is not spoiled by the quantum fluctuations in ϕ, which are of the order of $H/2\pi \sim 2\times 10^{-7}\sigma^2/m_{\rm Pl} \sim 10^{-19}\sigma[\mathcal{M}/(1\ \text{TeV})]$. Thermal fluctuations will spoil such localization: $\langle\phi^2\rangle_T^{1/2} \sim T \simeq 1$ TeV $\sim 10^{-4}\sigma$. However, it can be argued that ϕ is so weakly coupled it is not in thermal contact with the Universe; indeed, this argument has been used for other models of inflation [11].

Another way of insuring that the small initial value of the ϕ field is not spoiled by thermal fluctuations is to arrange that inflation begin "cold," $T \ll 1$ TeV. There are plausible ways that this might occur. If the Universe, or a small portion of it, were so negatively curved that it became curvature dominated early on, say, at a temperature $T_{\rm CD}$, then the temperature when inflation begins is $T_{\rm inflate} \sim (1\ \text{TeV}^2)/T_{\rm CD}$, which can easily be small enough to render the thermal fluctuations impotent. (Within the spirit of "generic" initial conditions, one would expect the curvature radius at the Planck epoch to be of the order of the Hubble radius, in which case $T_{\rm CD} \sim m_{\rm Pl}$ and $T_{\rm inflate} \sim 10^{-13}$ GeV.) Or, the Universe can become matter dominated long before inflation, e.g., by monopoles produced at the GUT phase transition, or other massive relics produced copiously in the early Universe. And, of course, it is not necessary that the Universe have any radiation in it prior to inflation: It could have begun cold.

Finally, we comment briefly on the phenomenology of our model. Because ϕ is very weakly coupled, it must be an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ singlet; however, it can *indirectly* influence electroweak physics. With $\mathcal{M} \sim 1$ TeV, the vacuum expectation value (VEV) of ϕ is $\sigma \sim 4\times 10^6$ GeV, which can induce a negative mass squared for the Higgs field (call it ψ) that does lead to electroweak symmetry breaking, through a coupling $\lambda\psi^2\phi^2$ [29]. A negative mass squared of order 1 TeV requires $\lambda \sim 10^{-7}$. Since the radiative corrections to the $V(\phi)$ due to ψ are of order $\lambda^2/4\pi^2 \sim 10^{-15} \sim B$, they are about the right size to account for the ϕ field's symmetry-breaking potential. The VEV of ϕ can give rise to particle masses, e.g., right-handed neutrinos; in this case reheating can take place by ϕ decays into right-handed neutrinos and their subsequent decays into light leptons. If the scale of inflation is raised slightly, $\mathcal{M} \sim 200$ TeV, the mass of the ϕ particle is of order several hundred GeV. In this case, reheating can take place through ϕ decays into electroweak Higgs and their subsequent decays into the particles of the standard model.

In sum, we have presented a simple model of slow-rollover inflation where the vacuum energy that drives inflation can be as small as the electroweak scale—orders of magnitude smaller than in models previously discussed. Inflation at an energy scale far below the unification scale has a number of attractive features: reheating is more efficient; the monopole problem is more easily solved; and last, but not least, such a model is potentially testable.

We thank J. Harvey, B. Mertens, S. Raby, J. Rosner, X. Shi, and M. Worah for useful discussions. This work was supported in part by the DOE (at Chicago and Fermilab) and by the NASA through NAGW-2381 (at Fermilab).

[1] E. W. Kolb and M. S. Turner, Annu. Rev. Nucl. Part. Sci. **33**, 645 (1984).
[2] See, e.g., M. S. Turner, Phys. Scr. **T36**, 167 (1991).
[3] For a textbook discussion of inflation see, e.g., E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), Chap. 8; A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, 1990).
[4] See, e.g., A. Vilenkin, Phys. Rep. **121**, 263 (1985).
[5] See, e.g., N. Turok, Phys. Rev. Lett. **63**, 2625 (1989).
[6] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B **245**, 561 (1990); **263**, 86 (1991); L. McLerran *et al.*, *ibid.* **256**, 451 (1991); N. Turok and J. Zadrozny, Nucl. Phys. **B358**, 471 (1991); A. Dolgov, Phys. Rep. (to be published).
[7] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[8] J. Preskill, Phys. Rev. Lett. **43**, 1365 (1979); Ya. B. Zel'dovich and M. Y. Khlopov, Phys. Lett. **79B**, 239 (1978).
[9] See, e.g., S.-Y. Pi, Phys. Rev. Lett. **52**, 1725 (1984); Q. Shafi and A. Vilenkin, *ibid.*, **52**, 691 (1984); K. Freese, J. Frieman, and A. Olinto, *ibid.* **65**, 3233 (1990); A. D. Linde, Phys. Lett. **129B**, 177 (1983); R. Holman, P. Ramond, and G. G. Ross, *ibid.* **137B**, 343 (1984).
[10] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[11] Holman, Ramond, and Ross (Ref. [9]).
[12] D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).
[13] P. J. Steinhardt and M. S. Turner, Phys. Rev. D **29**, 2162 (1984).
[14] G. F. Smoot *et al.*, Astrophys. J. **396**, L1 (1992).
[15] S. Dimopoulos and L. Hall, Phys. Lett. B **196**, 135 (1987).
[16] J. Cline and S. Raby, Phys. Rev. D **43**, 1781 (1991).
[17] If ϕ couples to a gauge field with strength g, radiative corrections due to gauge-boson loops lead to $B \sim g^4$, making $B \gg 10^{-14}$ unless $g \lesssim 10^{-4}$.
[18] S. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[19] This potential has its global minimum at $\sigma = (5V_0/\alpha)^{1/4}$, where $\beta = 4\alpha/5\sigma$ and density perturbations of the appropriate amplitude imply $\alpha = 3 \times 10^{-13}$. Such a nonrenormalizable potential could arise in the effective Lagrangian at low energies.
[20] See, e.g., A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); Shafi and Vilenkin (Ref. [9]); Pi (Ref. [9]).
[21] The end of the slow-roll epoch occurs when $|V''| \approx 9H^2$; for our model, $\phi_e/\sigma \simeq [\pi/|\ln(\phi^2/\sigma^2)|]^{1/2}(\sigma/m_{\rm Pl}) \simeq 10^{-13}$.
[22] J. Gelb *et al.*, Astrophys. J. (to be published).
[23] V. A. Rubakov, M. Sazhin, and A. Veryaskin, Phys. Lett. **115B**, 189 (1982); R. Fabbri and A. Pollock, *ibid.* **B125**, 445 (1983); L. Abbott and M. Wise, Nucl. Phys. **B244**, 541 (1984).
[24] A. Albrecht *et al.*, Phys. Rev. Lett. **48**, 1437 (1982); L. Abbott and M. Wise, Phys. Lett. **117B**, 29 (1982); A. D. Linde and A. Dolgov, *ibid.* **116B**, 329 (1982).
[25] Some aspects of reheating addressed here have been discussed previously by S. Barr and G. Segre, Phys. Rev. Lett. **62**, 2781 (1989).
[26] There are possibilities for improving the reheating efficiency. The strict constraint to the coupling of the field into which the ϕ decays can be circumvented if there are two contributions to its mass, of opposite sign, so that its coupling to the ϕ can exceed this bound. Further, the highest temperature reached after inflation occurs just when the ϕ particles begin to decay, $T_{\rm max} \simeq (T_{\rm RH}\mathcal{M})^{1/2}$. However, as the temperature falls from $T_{\rm max}$ to $T_{\rm RH}$ the entropy per comoving volume increases by $(T_{\rm max}/T_{\rm RH})^5$. In spite of the entropy production it may be possible to take advantage of this fact.
[27] All models of inflation that employ a symmetry-breaking potential require a small initial value for ϕ; the issue of initial conditions for $(\phi, \dot{\phi})$ has recently been addressed by D. Goldwirth and T. Piran, Phys. Rep. **214**, 223 (1992).
[28] It has been shown that models of inflation like ours are eternal; i.e., once the Universe becomes vacuum dominated *anywhere,* due to the role of quantum fluctuations most of the physical volume of the Universe will forever be inflating; see, e.g., A Vilenkin, Phys. Rev. D **27**, 2848 (1983), or Linde, *Particle Physics and Inflationary Cosmology* (Ref. [3]). Viewed in this context, the small initial value required for ϕ is irrelevant.
[29] In this regard we have followed the models of Shafi and Vilenkin [9] and Pi [9] where the VEV of the ϕ was used to induce GUT symmetry breaking.