How Fast Does Information Leak Out from a Black Hole?

Jacob D. Bekenstein

Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106
and The Racah Institute of Physics, Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel[(a)]

(Received 19 January 1993)

Hawking's radiance departs from blackbody form due to the mode dependence of the barrier penetration factors. Thus it is not the maximal entropy radiation for given energy. To check whether this entropic deficiency is consistent with the possibility that the radiance may carry information about the quantum state in the far past, we compare estimates of the actual entropy emission rate with the maximal possible one for the same power. Standard quantum communication theory then shows that the permitted information outflow rate can be as large as the rate of black hole entropy decrease. The initial information may thus gradually leak out during the evaporation.

PACS numbers: 97.60.Lf, 04.60.+n, 05.70.−a, 89.70.+c

Following his theoretical discovery of the black hole radiance that bears his name, Hawking noted [1] that such radiation seems to contradict accepted quantum physics. If a black hole forms from matter prepared in a pure state, and then radiates away its mass in ostensibly thermal radiation, one is left with a high-entropy mixed state of radiation. This contradicts the quantum dogma that a pure state will always remain pure under unitary evolution. A related contradiction follows from the interpretation of black hole entropy as the measure of the information hidden in the black hole about the ways it might have been formed [2]. Since fully thermal radiation is incapable of conveying detailed information about its source, that hidden information remains sequestered as the black hole radiates, and when the hole finally evaporates away, the information is lost forever. These two contradictions are facets of the black hole information-loss paradox.

Among the reactions to the paradox, three have received much attention (for reviews see Refs. [3] and [4]). First is Hawking's proposal to accept the loss of information and the transmutation of pure into mixed state as an inevitable consequence of the merging of gravity with quantum physics [1]. However, specific schemes for accomplishing this have been found to be incompatible with locality or conservation of energy [4,5]. A second point of view [1,6] holds that black hole evaporation leaves a remnant of Planck dimensions which retains all the information in question. However, the bound on specific entropy or information [7], or considerations from quantum gravity [8], tell us that an object of Planck mass and dimension can hold only a few bits of information; thus the posited remnants cannot fit the information bill of a large evaporating black hole. Attempts to circumvent these limits rely on remnants which look small but contain a large space within [4,9]. This approach has yet to demonstrate that such remnants are stable. Yet a third view [10] is that by exploiting subtle correlations in the radiation, the information manages to leak back out from the incipient black hole in the course of the evaporation (the leak cannot be left to the late stages of evaporation without incurring the problems accompanying remnants [4,6]). For information leak throughout the evaporation to be a reasonable resolution of the paradox, it must be shown that an information flow of the appropriate magnitude can come out of the black hole's near environs. A step in this direction is taken in the present paper.

Lately these three viewpoints have been widely examined by means of the $1+1$ dimensions dilaton-gravity model of an evaporating black hole proposed by Callan, Giddings, Harvey, and Strominger [3,11]. Whatever the final outcome of this program, it is still unclear how it bears on the realistic case of $(1+3)$-dimensional black holes. Therefore, any new model independent approach which can address the $(1+3)$-dimensional case would be of great conceptual help. We here employ a thermodynamic argument (which in fact makes it virtually model independent) to show that for the $(1+3)$-dimensional Schwarzschild black hole, an outflow of information of the required magnitude to resolve the information problem is permitted in principle. We do not explore here specific mechanisms for information leak.

Each mode of the Hawking radiance is thermal by virtue of the exponential distribution of the number of quanta emitted [see Eq. (2) below] [12]. And the modes are uncorrelated [13]. Nevertheless the spectrum emerging at large distances is distorted from Planckian form. This is usually explained by the mode dependence of the curvature and angular momentum barrier penetration factor $\Gamma_{sjmp}(\omega)$, where s stands for the particle species, j and m for the angular momentum quantum numbers, and p for the polarization, with $\Gamma < 1$ in general. For a Schwarzschild black hole of mass M, inverse temperature $\beta_{\rm bh} = 8\pi GM/\hbar$ and entropy $S_{\rm bh}$, the average energy in a mode is (henceforth we set $c=1$)

$$\varepsilon_{sjmp}(\beta_{\rm bh},\omega) = \frac{\hbar\omega\Gamma_{sjmp}(\omega)}{e^{\beta_{\rm bh}\hbar\omega} \pm 1}, \tag{1}$$

where henceforth the upper (lower) sign corresponds to fermions (bosons).

An analogy can be drawn between this distortion from blackbody character and that in the radiation from a star

0031-9007/93/70(24)/3680(4)$06.00

which, generally, is far from blackbody because it emerges from layers at different temperatures. Much is learned about a star's atmosphere (composition, pressure, rotation, magnetic fields) from the departure of its spectrum from blackbody, e.g., spectral lines. Physically, the information conveying ability of stellar radiation depends on its being entropy deficient as compared to blackbody radiation. Likewise, compared with blackbody radiation with the same power (but, of course, different inverse temperature $\beta_{\rm bh}$), Hawking radiance is subentropic. This can be seen by ascribing each mode an effective temperature such that the corresponding mode in Planckian radiation at that temperature has identical mean occupation number. Then because $\Gamma_{sjmp}(\omega)$ decreases with ω, the effective temperature does so too. By transferring energy from high to low frequencies (mode temperatures) one can thus distort the Hawking radiance and increase its entropy. The entropic deficiency is consistent with the possibility that Hawking radiance may be carrying information about the state of the quantum fields in the far past, i.e., just the information that is supposed to be lost. This would, of course, be impossible if the radiance were exactly blackbody.

But how to quantify the information outflux, if any? Let us look at the question in the light of quantum communication theory (for reviews see Ref. [14]). We adapt Lebedev and Levitin's pioneering thermodynamic approach [15], and measure information in natural units (nits); 1 nit $=\log_2 e$ bits. To this end we consider the entropy of Hawking's radiance, whose emission rate we denote by $\dot{S}$, as entropy (uncertainty about the state) of the noise which is adulterating the signal conveying the information. The radiance power, $\dot{E}$, will be interpreted as the sum of noise and signal powers. With this scenario the maximum rate at which information can be recovered from the radiation by a suitable detector is, according to Ref. [15], $\dot{I}_{\rm max}\equiv\dot{H}-\dot{S}$ where $\dot{H}$ is the maximum entropy rate possible for the actual power $\dot{E}$ under the boundary conditions of the system. The rationale of this result is that any deficit from the maximum possible entropy (at given energy) implies that the radiation is partially ordered and can thus convey a quantity of information equal to the entropy deficit. (Actually, if the noise is correlated with the signal, as may well be the case in the Hawking radiance, $\dot{I}_{\rm max}$ will be larger [14]; in this case our arguments below are actually strengthened.)

For convenience we shall use the notation $i\equiv\{sjmp\}$. The probability distribution for the black hole to spontaneously emit n quanta in mode $\{i,\omega\}$ is given by [12]

$$p_{\rm sp}(n)=(1\pm e^{-\gamma_i})^{\mp 1}e^{-\gamma_i n}, \tag{2}$$

where $\gamma_i(\beta_{\rm bh},\omega)$ is defined by

$$\frac{1}{e^{\gamma_i}\pm 1}=\frac{\Gamma_i(\omega)}{e^{\beta_{\rm bh}\hbar\omega}+1}. \tag{3}$$

Substituting this in Shannon's formula gives the entropy in the given mode

$$\sigma_i(\beta_{\rm bh},\omega)=\pm\ln(1\pm e^{-\gamma_i})+\frac{\gamma_i}{e^{\gamma_i}\pm 1}. \tag{4}$$

We may also reexpress Eq. (1) as

$$\varepsilon_i(\beta_{\rm bh},\omega)=\frac{\hbar\omega}{e^{\gamma_i}\pm 1}. \tag{5}$$

The entropy outflux rate and the power may now be expressed as

$$\dot{S}=\sum_i\int_0^\infty\sigma_i(\beta_{\rm bh},\omega)\frac{d\omega}{2\pi}, \tag{6}$$

$$\dot{E}=\sum_i\int_0^\infty\varepsilon_i(\beta_{\rm bh},\omega)\frac{d\omega}{2\pi}, \tag{7}$$

where $d\omega/2\pi$ is the rate at which modes of type i emanate from the hole.

Page [16] has calculated numerically the contributions of various particle species to $\dot{E}$ and $\dot{S}$, and states the results in terms of the dimensionless ratios $\mu\equiv\dot{E}(GM)^2\times\hbar^{-1}$ and $\nu\equiv\dot{S}/\beta_{\rm bh}\dot{E}$. For each species of light neutrinos (antineutrinos) he finds $\mu=4.090\times10^{-5}$ and $\nu=1.639$ with modes having $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ being the overwhelming contributors. For photons $\mu=3.371\times10^{-5}$ and $\nu=1.500$ with modes having $j=1,2,3$ making the dominant contribution. And for gravitons $\mu=3.84\times10^{-6}$ and $\nu=1.348$ with modes having $j=2,3$ contributing overwhelmingly. If the black hole emits three species of neutrinos and antineutrinos (each with a single helicity), photons and gravitons, the overall numbers are $\mu=2.829\times10^{-4}$ and $\nu=1.619$ (this last value involves the individual ν's weighted by the μ's).

We now have to compare $\dot{S}$ with the entropy rate $\dot{H}$ of the maximally entropic (blackbody) distribution whose power $\dot{E}_{\rm eff}$ equals $\dot{E}$. In practice we shall compare Page's $\dot{S}$ (restricted to the angular momentum modes he found to dominate the power) with the entropy flow $\dot{H}$ in the *same* modes of a blackbody distribution with inverse temperature $\beta_{\rm eff}$ determined by the equality $\dot{E}_{\rm eff}=\dot{E}$ over all the mentioned modes. By not including angular modes which contribute little to $\dot{E}$ we are surely constricting the available phase space and underestimating $\dot{H}$. However, by our procedure we are simultaneously overestimating $\dot{H}$ because blackbody radiation populating a finite number of angular momentum modes assigns substantial weight to modes with $\omega\to0$; for nonzero orbital angular momentum these correspond to arbitrarily large impact parameter, and are thus not related to the black hole. These spurious modes broaden the phase space and so artificially increase $\dot{H}$. It is unclear which effect is the stronger, and we shall ignore the issue henceforth.

To calculate the blackbody quantities we replace for each mode $\gamma_i\to\beta_{\rm eff}\hbar\omega$ in Eq. (4) and Eq. (5). The integral of the logarithmic term in σ_i of Eq. (6) can be combined with the other term by integration by parts. Using

$$\int_0^\infty \frac{x\,dx}{e^x \pm 1} = \frac{\pi^2(3\mp 1)}{24} \quad (8)$$

one can cast the results in the form

$$\frac{\dot{H}}{2\beta_{\rm eff}} = \dot{E}_{\rm eff} = \frac{\pi}{12\hbar\beta_{\rm eff}^2}\sum_i g_i\,, \quad (9)$$

where $g_i = 1$ or $\frac{1}{2}$ for a boson or fermion mode, respectively. The modes that carry significant energy account for $\sum_{\rm bos} g_i = 54$ and $\sum_{\rm fer} g_i = 36$ (three neutrino species). Equating $\dot{E}_{\rm eff}$ with Page's $\dot{E}$ gives

$$\beta_{\rm eff} = 11.48\beta_{\rm bh} \text{ and } \dot{H} = 22.97\beta_{\rm bh}\dot{E}\,. \quad (10)$$

Recalling that $\nu = 1.619$ and $\dot{S}_{\rm bh} = \beta_{\rm bh}\dot{M} = -\beta_{\rm bh}\dot{E}$ we conclude that

$$\dot{I}_{\rm max} = \dot{H} - \dot{S} = 21.35|\dot{S}_{\rm bh}|\,. \quad (11)$$

Although the above figure for $\dot{I}_{\rm max}$ may be an overestimate, it is so large as to suggest that an information leak of sufficient magnitude to resolve the information paradox is allowed. For example, if $\dot{I}$, the actual information outflow rate, amounts to $|\dot{S}_{\rm bh}|$ throughout the course of evaporation of a massive black hole down to $M \approx 1\times 10^{14}$ g (when the emission of massive particles becomes important and most of the initial black hole entropy has disappeared [16]), the total outgoing information equals the information originally lost in the hole's formation. This would resolve one facet of the information paradox. And if $\dot{I} = 1.619|\dot{S}_{\rm bh}|$ throughout, the total outflowing information will equal that widely regarded as lost because of the mixed thermal state of the Hawking radiance. This would support the view of some [10] that the radiance is in a pure state all along, and thus resolve the other facet of the paradox.

The view is possible [G. Horowitz (private communication)] that the information flow in Eq. (11), since it is related to the barrier's effects [the $\Gamma_i(\omega)$], merely represents information about the barrier, not the sequestered information. We evaluate this possibility by imagining how $\dot{I}_{\rm max}$ would change if, Einstein's equations notwithstanding, we could do away with the barrier. Specifically, we shall calculate $\dot{I}_{\rm max}$ for a reference stylized black hole which has $\Gamma_i(\omega) = 0$ for all modes that contribute negligibly to $\dot{E}$ of the real hole, and $\Gamma_i(\omega)$'s as large as physically permitted for the modes that do contribute significantly. Furthermore, since we wish to compare entropies, which makes sense only at fixed energy, we shall require that the reference black hole's power $\dot{E}'$ equal $\dot{E}$ of the actual black hole; this requires a particular choice for its inverse temperature, $\beta'_{\rm bh}$ different from the real hole's $\beta_{\rm bh}$.

For fermion modes no reason is known to prevent $\Gamma_i(\omega)$ from approaching unity. For bosons the story is different. This may be appreciated by treating the black hole's interaction with radiation by means of Einstein A and B coefficients [17]. Let $A_i(\omega)$ be the spontaneous emission coefficient for mode i with frequency ω and $B_{i\downarrow}(\omega)$ the corresponding stimulated emission coefficient. This means that if n quanta are incident in mode i, a mean number $A_i + B_{i\downarrow}n = A_i(n+1)$ will be emitted in the same mode. The second equality stems from the Einstein relation $A_i = B_{i\downarrow}$, and has the expected dependence on $n+1$. In addition, a mean number of quanta $B_{i\uparrow}n$ will be absorbed, where $B_{i\uparrow}(\omega)$ is the Einstein absorption coefficient. The Einstein relation $g_2B_{i\downarrow}(\omega) = g_1B_{i\uparrow}(\omega)$ is also valid here with the degeneracy factors g_2 and g_1 replaced by the corresponding numbers of black hole internal states $\exp[S_{\rm bh}(M)]$ and $\exp[S_{\rm bh}(M-\hbar\omega)]$ [12].

Because $\hbar\omega \ll M$, $g_2/g_1 \approx e^{-\beta_{\rm bh}\hbar\omega}$. Combining the above relations with the requirement that A_i reproduce the mean number of quanta implied by Eq. (1), we obtain, in particular,

$$\Gamma_i(\omega) = (1 - e^{-\beta_{\rm bh}\hbar\omega})B_{i\uparrow}\,. \quad (12)$$

This says that the "barrier penetration factor" $\Gamma_i(\omega)$ is not the same thing as the absorption coefficient because of stimulated emission effects [17]. Now $B_{i\uparrow}$ may be interpreted as the probability that a single incident quantum is absorbed, and must therefore lie in the range [0,1]. It follows that $\Gamma_i(\omega) \le 1 - e^{-\beta_{\rm bh}\hbar\omega}$. This is a prediction which seems to be consistent with all known numerical and analytic calculations of the $\Gamma_i(\omega)$ [16,18]. In view of it we shall take for our reference black hole $\Gamma_i(\omega) = 1 - e^{-\beta_{\rm bh}\hbar\omega}$ for boson modes and $\Gamma_i(\omega) = 1$ for fermion ones if the corresponding modes contribute significantly to $\dot{E}$ of the real black hole, and $\Gamma_i(\omega) = 0$ if they do not.

We shall first compute $\dot{E}'$ and determine $\beta'_{\rm bh}$. From the assumed $\Gamma_i(\omega) = 1 - e^{-\beta_{\rm eff}\hbar\omega}$ for boson modes it follows from Eqs. (3) and (5) that $\varepsilon_i(\beta'_{\rm bh},\omega) = \hbar\omega e^{-\beta'_{\rm bh}\hbar\omega}$. For fermions $\Gamma_i(\omega) = 1$ implies that $\varepsilon_i(\beta'_{\rm bh},\omega)$ looks like that in Eq. (5) with $\gamma_i \to \beta'_{\rm bh}\hbar\omega$. Thus Eq. (7) gives

$$\dot{E}' = (2\pi\hbar\beta'^2_{\rm bh})^{-1}\sum_{\rm bos} g_i + \pi(12\hbar\beta'^2_{\rm bh})^{-1}\sum_{\rm fer} g_i\,. \quad (13)$$

We require this $\dot{E}'$ to equal $\dot{E}$ which itself equals $\dot{E}_{\rm eff}$. Referring to Eq. (9) and the cited $\sum_i g_i$ we obtain

$$\beta'_{\rm bh} = 0.8745\beta_{\rm eff} = 10.04\beta_{\rm bh}\,. \quad (14)$$

Next we compute the reference black hole's entropy radiation rate $\dot{S}'$. Comparing the mentioned ε_i for bosons with Eq. (5) tells us that $e^{\gamma_i} = 1 + e^{\beta'_{\rm bh}\hbar\omega}$. It then follows from Eq. (4) that $\sigma_i(\beta'_{\rm bh},\omega) = \ln(1 + e^{-\beta'_{\rm bh}\hbar\omega}) + e^{-\beta'_{\rm bh}\hbar\omega} \times \ln(1 + e^{\beta'_{\rm bh}\hbar\omega})$. The corresponding expression for fermions is obtained from Eq. (4) by letting $\gamma_i \to \beta'_{\rm bh}\hbar\omega$. After integration by parts, Eq. (6) gives

$$\dot{S}' = \frac{1}{\hbar\beta'_{\rm bh}}\left[\left(\frac{\pi}{24} + \frac{\ln 2}{\pi}\right)\sum_{\rm bos} g_i + \frac{\pi}{6}\sum_{\rm fer} g_i\right] = \frac{37.83}{\hbar\beta'_{\rm bh}}\,. \quad (15)$$

Recalling Eqs. (10) and (14), and the value $\mu = 2.829 \times 10^{-4}$, we obtain the information outflux rate corrected for effects of the barrier,

$$(\dot{I}_{\rm max})_{\rm corr} = \dot{H} - \dot{S}' = 1.88|\dot{S}_{\rm bh}| = 0.187|\dot{S}'_{\rm bh}|\,, \quad (16)$$

where $\dot{S}'_{bh} = -\beta'_{bh}\dot{E}' = 10.04\dot{S}_{bh}$.

Because the barrier is not actually a detachable feature, it is unclear from our schematic picture of the removal of its effects whether $(\dot{I}_{max})_{corr}$ should be compared with $|\dot{S}_{bh}|$ or with $|\dot{S}'_{bh}|$ (real and reference black holes radiate with the same power). However, it is significant that $(\dot{I}_{max})_{corr}$ falls between the two values and is of the same order as either. Thus our earlier impression from Eq. (11) that the departure of Hawking radiance from blackbody is enough to permit an interestingly large information outflux stands.

Let us sum up. According to communication theory the entropy deficiency of Hawking's radiance allows it to convey information about its source in an amount of order of the initial black hole entropy. This black hole entropy was expected to be large because of the vast loss of information about the hole's interior incurred upon the "tracing out of interior field degrees of freedom," the step which is widely thought to introduce entropy into black hole physics. Now just as in Szilard's famous discussion of Maxwell's demon where acquisition of information about the location of the molecule in the box was tantamount to the "gas" having less entropy than expected, so here, the gradual information outflux is tantamount to the black hole entropy becoming gradually less and less than originally expected. And indeed, the black hole entropy, as measured by the horizon area, decreases. However, the radiation's density matrix arises from the same tracing out whose effects, as gauged by the magnitude of the black hole entropy, are gradually undone. This suggests that in the end the radiation entropy will vanish. At least in an approximate sense the pure state may be reconstituted.

It would be surprising if nature has not taken advantage of this information window to obviate the information paradox. There remains the task of identifying the mechanism of information leak. The prominent part played by stimulated emission in deforming the blackbody spectrum makes processes associated with it likely culprits.

I thank Gary Horowitz, Don Page, and Andy Strominger for informative conversations, and Jim Hartle for hospitality in Santa Barbara.

[a]Permanent address. Electronic address: bekenste@vms.huji.ac.il

[1] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976); Commun. Math. Phys. **87**, 395 (1982).
[2] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[3] J. Harvey and A. Strominger, Report No. hep-th/9209055 (unpublished); S. Giddings, Report No. hep-th/9209113 (unpublished).
[4] J. Preskill, Report No. hep-th/9209058 (unpublished).
[5] T. Banks, M. E. Peskin, and L. Susskind, Nucl. Phys. **B244**, 125 (1984).
[6] C. Carlitz and R. Willey, Phys. Rev. D **36**, 2336 (1987); Y. Aharonov, A. Casher, and S. Nussinov, Phys. Lett. B **191**, 51 (1987).
[7] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981); J. D. Bekenstein and M. Schiffer, Phys. Rev. D **39**, 1109 (1989).
[8] S. Giddings, Phys. Rev. D **46**, 1347 (1992).
[9] L. Susskind and L. Thorlacius, Nucl. Phys. **B382**, 123 (1992); T. Banks, A. Dabholkar, M. R. Douglas, and M. O'Loughlin, Phys. Rev. D **45**, 3607 (1992); T. Banks, M. O'Loughlin, and A. Strominger, *ibid.* **47**, 4476 (1993).
[10] D. Page, Phys. Rev. Lett. **44**, 301 (1980); G. 't Hooft, Nucl. Phys. **B256**, 727 (1985); **B335**, 138 (1990); C. Carlitz and R. Willey, Phys. Rev. D **36**, 2327 (1987); C. Holzhey and F. Wilczek, Nucl. Phys. **B380**, 447 (1992); F. Wilczek, Report No. hep-th/9302096 (unpublished).
[11] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, R1005 (1992).
[12] J. D. Bekenstein, Phys. Rev. D **12**, 3077 (1975).
[13] L. Parker, Phys. Rev. D **12**, 1519 (1975); R. M. Wald, Commun. Math. Phys. **45**, 9 (1975).
[14] Y. Yamamoto and H. A. Haus, Rev. Mod. Phys. **58**, 1001 (1986); J. D. Bekenstein and M. Schiffer, Int. J. Mod. Phys. C **1**, 355 (1990).
[15] D. S. Lebedev and L. B. Levitin, Dokl. Akad. Nauk SSSR **149**, 1299 (1963) [Sov. Phys. Dokl. **8**, 377 (1963)].
[16] D. N. Page, Phys. Rev. D **13**, 198 (1976); **14**, 3260 (1976).
[17] J. D. Bekenstein and A. Meisels, Phys. Rev. D **15**, 2775 (1977); J. D. Bekenstein, in *To Fulfill a Vision*, edited by Y. Ne'eman (Addison-Wesley, Reading, MA, 1981).
[18] S. A. Teukolsky and W. H. Press, Astrophys. J. **193**, 443 (1974); A. A. Starobinsky and S. M. Churilov, Zh. Eksp. Teor. Fiz. **65**, 3 (1974) [Sov. Phys. JETP **38**, 1 (1974)]; N. Sanchez, Phys. Rev. D **18**, 1030 (1978).