Supercurrent Drag via the Coulomb Interaction

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We investigate the supercurrent drag effect due to the Coulomb interaction between two spatially separated superconductors. The supercurrent for a given wire/layer is shown to depend on the superfluid velocity in the *other* wire/layer. The magnitude of this effect is calculated. This supercurrent drag effect should be observable in experiments.

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The discovery of the high T_c superconductors a few years ago led to a renewed interest in layered superconductors [1]. The interaction between the superconductivity which exists on each separate superconducting layer is of particular concern. For example, it has been emphasized that, instead of vortex lines, one should visualize vortex "pancakes" on each layer [2]. Understanding the interaction between pancakes on different layers is thus an important problem. Similarly, there have been numerous discussions, if anyon superconductivity is realized on each layer, as to how the anyon states on different layers are correlated with each other (see, e.g., Ref. [3]).

To illustrate the interaction between the layers, we here consider the special case of a two layer/wire system. We shall consider uniform superflows in each layer/wire, and consider how the Coulomb interaction between the layers/wires is affected due to the superflows. This Coulomb interaction is via virtual fluctuations into higher eigenstates, and is in some sense a generalization of the van der Waals forces. We shall show that this interaction will lead to a supercurrent appearing on a layer/wire due to the superfluid velocity on the *other* layer/wire, even though there is *no* tunneling of electrons from one to the other.

This supercurrent drag effect itself is not new. It has been discussed for ^{3}He-^{4}He mixture, if both species become superfluid [4], and in neutron stars, where the neutrons and the protons are believed to condense into a superfluid mixture [5]. It is also implicit in the work in ^{3}He$(-A)$, where the supercurrent for the $S_z = +1$ spin pairs, say, depends on the superfluid velocities of *both* the $S_z = +1$ and the $S_z = -1$ pairs [6, 7]. This supercurrent drag will result whenever there is an appropriate Fermi liquid interaction between the two species. We believe, however, this is the first time that it has been discussed for two spatially *separated* electronic superconductors. This current is *different* from that produced by magnetic induction. We shall evaluate this current, and we believe that this current can be measured in a realistic experiment.

We first consider two identical, infinitely long superconducting wires separated by a distance D at zero temperature. For simplicity we assume that the radius of the wires a is smaller than both the coherence length ξ and the penetration depth, so that variations along the radial directions within the wires can be ignored. It is easy to derive the plasma mode(s) spectrum by writing down the equations for the conservation of charge and the equations of motion for the superfluids [8–10], analogous to what was done in the normal state [11]. The dispersions in the presence of superfluid velocities v_1 and v_2 on the wires 1 and 2, respectively, are determined by the determinantal equation

$$\begin{vmatrix} (\omega - qv_1)^2 - s^2q^2 & -X \\ -X & (\omega - qv_2)^2 - s^2q^2 \end{vmatrix} = 0, \tag{1}$$

where s is the plasma mode velocity of each wire if there were neither superflow nor interwire Coulomb interaction, $s^2 = s_0^2 + 4\pi n_0 e^2 K_0(0)/m$. Here s_0 is related to the compressibility of the electrons without the intrawire Coulomb interaction, and is given by $v_F/\sqrt{3}$ (v_F) if the wire is thick (thin) compared with the inverse of Fermi wave vector, and n_0 the electron density per unit length of the wire. The logarithmic divergence of the Bessel function K_0 at zero argument is to be cut off due to the finite thickness of the wire. We shall not indicate this cutoff explicitly here. $X = 4\pi n_0 e^2 q^2 K_0(|qD|)/m$ is from the Coulomb interaction between the wires.

Equation (1) determines the dispersion of two ($\pm$) plasma modes, corresponding to whether the charge densities of the two wires oscillate in or out of phase. The frequency of these modes is affected by the superfluid velocities. The zero point energy of the system, given by $\sum_q \frac{1}{2}[\hbar\omega_+(q) + \hbar\omega_-(q)]$, is thus a function of the velocities. We find, apart from a constant (independent of the velocities) in which we shall not be interested, per unit length

$$\triangle E_{12} = -\kappa_0(v_1 - v_2)^2, \tag{2}$$

where $\kappa_0 = +\hbar n_0^2 e^4/16\pi m^2 s^5 D^2$. Equation (2) is correct to the lowest (second) order in the interaction between the wires (i.e., to order X^2). The sign of Eq. (2) is due to the fact that the relative velocity lowers the out-of-phase mode frequency (ω_-) more than it raises the in-phase mode (ω_+). The dependence of κ_0 on the distance D follows from the fact that the frequencies for both modes are *linear* in q and thus the sum over q involves

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$\int dq|q|[K_0(qD)]^2 \propto 1/D^2$ [12].

The above calculation can be generalized to finite but small temperatures, where the relevant quantity is the free energy of the system, evaluated by considering the thermal occupations of the plasma modes. We find that this finite temperature correction to the free energy *decreases* the effective coupling coefficient κ_0 in equation (2), as well as renormalizing the superfluid densities of each wire. To lowest order the fractional correction to κ_0 is of order $[k_BT/(\hbar s/D)]^2$ as expected since s/D is the characteristic plasma frequency which would be affected by the presence of the relative velocities. This temperature correction is typically small [13].

In the above we have computed the free energy of the system by simply considering the "vacuum fluctuations" or the thermal occupation of the plasmon modes. Alternatively one can consider the change in the free energy via perturbation theory in the interaction *between* the wires. In second order perturbation the interaction energy can be expressed in terms of the density-density correlation functions for each wire, which we have obtained in the random phase approximation. The results are identical to those presented above. The sign of Eq. (2) [and (3) below] is in accordance with a general result derived by Rojo and Leggett [3], that, within second order perturbation theory, the two wires/layers have a lower interaction energy if they are in the time-reversed state ($v_1 = -v_2$) than if they are in the same state ($v_1 = v_2$). Here the superflow explicitly breaks parity and thus allows a bulk rather than an edge effect [3].

A similar calculation can be carried out for two $2d$ layers separated by a distance D. The analysis is much more involved because, instead of two *linear* modes in $1d$, the two modes now acquire qualitatively different dispersions, with $\omega_- \propto q$ and $\omega_+ \propto q^{1/2}$, respectively, in the the long-wavelength limit . This is due to the appearance of a new length scale, q_{TF}^{-1} for the Thomas-Fermi screening in the spectrum. We find that, for a wide range of reasonable parameters, the interaction energy per unit area is, at zero temperature, well approximated by

$$\triangle E_{12}^L = -\kappa_0^L(\mathbf{v}_1 - \mathbf{v}_2)^2, \tag{3}$$

where $\kappa_0^L \approx \hbar/48\pi\sqrt{2}v_FD^3$. We shall refer the readers to Ref. [10] for the details in obtaining this result. Equations (2) and (3) indicate that the force between the wires/layers is affected by the presence of superfluid velocities (and hence current). We shall study this force elsewhere, and in this paper concentrate on the supercurrent drag effect.

To understand the physical significance of the velocity dependence of this interaction energy, we recall that the superfluid velocity is $(\hbar\nabla\chi - 2e\mathbf{A}/c)/2m$, where χ is the phase of the order parameter, and that the supercurrent (at point $\mathbf{r}$) is related to the derivative of the free energy with respect to the vector potential $\mathbf{A}$ (at point $\mathbf{r}$). Using $F = \rho_s(v_1^2+v_2^2)/2 - \kappa_0(v_1-v_2)^2$, we obtain, for example, the current on the second wire

$$I_2 = \frac{e}{m}(\rho_{22}v_2 + \rho_{21}v_1), \tag{4}$$

where $\rho_{22} = \rho_s - 2\kappa_0$, $\rho_{21} = 2\kappa_0$. Thus the supercurrent of the second wire depends on the superfluid velocity on itself *as well as that of the first wire.* As in previous literature [4,5], we shall call this the "supercurrent drag."

To make the physics more transparent we first ignore the coupling to the vector potential, i.e., simply assume $v_2 = \hbar\nabla\chi_2/2m$, etc. Now consider the current in the second wire, with initially $I_2 = 0$ ($v_2 = 0$), and with the current of the first wire (and hence v_1) slowly increasing from zero (or consider bringing in the first wire with a current from infinity to the second wire). *If* v_2 remains zero, i.e., there is *no change in the phase gradient*, then a current in this wire will start to flow. Notice that the second wire is no longer at its minimum free energy state. In the minimum free energy state, v_2 ($\nabla\chi_2$) must achieve the value so that $\partial F/\partial v_2 = 0$ [$\partial F/\partial(\nabla\chi_2) = 0$], and hence by definition $I_2 = 0$. Superconductivity is essential in the above argument, so that the wire can be trapped in a metastable state. The energy barrier associated with the superconductivity prevents (apart from phase slips by thermal/quantum fluctuations) v_2 from acquiring a finite value so as to achieve $I_2 = 0$. For a drag current much smaller than the (mean field) critical current, this barrier height ΔF is of the order of $F_c\pi a^2\xi$ where F_c is the condensation energy density. For a superconducting wire with $\xi \sim 500$ Å and $a \sim 100$ Å, this barrier is $\sim 10^2k_BT_c$ except near T_c. The rate of thermal phase slips, given by $\Omega e^{-\Delta F/k_BT}$ where Ω is a prefactor estimated by McCumber and Halperin [14], is found to be negligible except very near T_c. The rate of quantum phase slips can be estimated by replacing k_BT in the above expression by $\hbar/\tau$ where τ is of the order of $\sim \hbar/k_BT_c$ [15]. We find that this rate is negligible in laboratory time scales as well.

In a recent paper, Rojo and Mahan [16] considered the Coulomb interaction between two wires in the *normal* state, and obtained an expression similar to Eq. (2) for the interaction energy, with $v_{1,2}$ replaced by $\delta k_{1,2}/m$, where $\delta k_{1,2}$ are the shifts in the occupation numbers, i.e., the "momentum states" are occupied for $-k_F + \delta k_{1,2} < k < k_F + \delta k_{1,2}$ for wires 1 and 2, respectively. They have calculated, for $T = 0$, the current in wire 2 *for the state* $\delta k_2 = 0$, and show that it is finite if $\delta k_1 \neq 0$. However, as argued above, we do not think that in *equilibrium* I_2 can be nonzero. Imperfections of the wire or coupling to the environment will in general lead to rearrangement in quasiparticle occupations in $\mathbf{k}$ space (scattering), eventually with $\partial F/\partial k_2 = 0$ so that the current I_2 is identically zero.

We also note that if wire 2 is a superconductor but an open circuit, no current or voltage should develop. This is because the superconductor does not have to go through any energy barrier to develop a phase gradient (and hence v_2) so that $I_2 = 0$. This should be contrasted with the current drag by scattering (a dissipative process) which is operative at finite temperature for *normal* wires [17].

In the rest of this paper we shall discuss the feasibility of detecting this drag current experimentally. We shall consider the configuration of Fig. 1, with two identical loops, length Λ, width D_1 separated by a distance D, and $\Lambda \gg D \sim D_1$. (This is chosen in order to have a current which has a significant contribution from the Coulomb drag yet not dominated by the magnetic induction, see below.) We assume that wire 1 carries a finite total phase winding around the wire, whereas wire 2 does not. We shall find the current I_2 flowing in wire 2. Notice that I_1 in general is finite, but I_2 will be zero if there were no (magnetic or Coulomb) interaction between the wires. We shall simply consider $T = 0$, where the Coulomb drag effect is most significant.

For this purpose we first notice that since the electrons are charged, the superfluid velocities are related not only to the phase gradient but also to the vector potential $\mathbf{A}$. For a given experimental arrangement the current should be determined by solving the Maxwell equation $\nabla \times (\nabla \times \mathbf{A}) = 4\pi \mathbf{J}/c$ self-consistently together with the constitutive equations [Eq. (4)]. This in general is a complicated mathematical problem. To gain more physical insight, we shall first take a few (somewhat drastic) approximations and confine ourselves to orders of magnitude. We anticipate that the current I_2 will be much smaller than I_1. Therefore the vector potential for any point on the loop 2 is roughly given by that generated by loop 1. Since this vector potential $\mathbf{A}$ is nonuniform over loop 2, a nonuniform phase gradient $(\nabla\chi_2)$ will develop so that the current is a *constant* along the loop, and with the constraint that $\oint_{c_2} \nabla\chi_2 \cdot dl_2 = 0$ (assuming that there are no phase slips). For an estimate of v_2 in (4), we shall thus take the average of $-e\mathbf{A}/mc$ projected along the wire (more precisely, along the direction indicated by the arrows in Fig. 1) as generated by the first wire [18]:

$$v_2 \approx -\frac{e}{mc}\frac{I_1}{c}\left[\ln\frac{(D+D_1)^2}{D(D+2D_1)}\right]. \tag{5}$$

This, when substituted in the first term of Eq. (4), gives

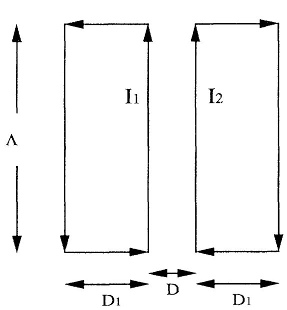

FIG. 1. A setup examined in the text.

the contribution to the current on wire 2 due to *magnetic induction.* The effect in which we shall be interested is contained in the second term of Eq. (4). For an estimate of this term, we notice that the Coulomb drag effect on different arms of the loop is trying to reinforce/cancel each other. We shall simply take an effective coupling, κ_{eff}, to be the average of contributions on the different arms of loop 2 as calculated in (2):

$$\kappa_{eff} \approx \frac{1}{2}\frac{\hbar n_0^2 e^4}{16\pi m^2 s^5}\left[\frac{1}{D^2} - \frac{2}{(D+D_1)^2} + \frac{1}{(2D_1+D)^2}\right]. \tag{6}$$

The current on wire 1 is given by an expression that is similar to (4) with indices 1, 2 interchanged. Anticipating that the Coulomb and magnetic interaction with wire 2 will only give a small correction to the physical quantities on this wire 1 we approximate v_1 by the original (noninteracting) value, and shall simply take $I_1 = n_0 e v_1$ [18]. We can now find the ratio of the magnitude of the current produced by the "Coulomb drag" to that generated by the magnetic induction (approximated by $\rho_s v_2$). This ratio is approximately given by

$$r \approx \frac{3^{5/2}}{32\pi}\left[\frac{1}{k_F a_B}\frac{1}{(k_F D)^2}\frac{1}{(v_F/c)^2}\right]\frac{g_\kappa(D_1/D)}{g_{ind}(D_1/D)}, \tag{7}$$

where the dimensionless functions g_κ and g_{ind} are related to the geometric factors for the Coulomb and magnetic induction, respectively, $g_\kappa(x) \equiv [1 - 2/(1+x)^2 + 1/(1+2x)^2]$, $g_{ind}(x) \equiv \ln[(1+x)^2/(1+2x)]$. $a_B \equiv \hbar/me^2$ is the Bohr radius. In obtaining (7) we have assumed that each wire has a radius $\gg k_F^{-1}$, and have approximated the plasma velocity s by $s_0 = v_F/\sqrt{3}$. Because of the smallness of v_F/c, though for reasonable distances $k_F D \gg 1$, the ratio r can still be of order (or even larger than) 1 (choosing $x \sim 1$), and thus the current produced by the Coulomb interaction is comparable to that induced by the magnetic (Biot-Savart) interaction. Notice also that since $\rho_{22}, \rho_{21} > 0$ in Eq. (4), we see from (5) and (6) that the Coulomb drag current and the induction current are *opposite* to each other.

We shall now estimate I_2^κ, the part of the current produced by the Coulomb drag effect. With the same approximations as outlined above in obtaining (7), we find

$$I_2^\kappa \approx \frac{3^{3/2}}{16\pi^2}\frac{a^2}{(k_F a_B)^2 D^2} g_\kappa(D_1/D) I_1 . \tag{8}$$

In obtaining Eq. (8) we have assumed that the current is uniform over the cross section of the wire. If we take $a \sim 100$ Å and $D_1 \sim D \sim 1000$ Å, then $I_2^\kappa \approx 10^{-3} I_1$. In this regard we also note that this order of magnitude for I_2^κ should *not* be altered if wire 1 (but not wire 2) is in the normal state [19]. Thus the supercurrent produced by the Coulomb drag may readily be observable.

We have calculated the current rigorously in the configuration of Fig. 1 (assuming $\Lambda \to \infty$ and $\kappa/\rho_s \ll 1$) and verified that the above features are essentially correct, with some minor corrections: (i) the induction current involves the product of v_2 above with not the original superfluid density ρ_s but a quantity reduced by the Coulomb drag; (ii) because the problem has to be solved self-consistently, the two effects strictly speaking do not simply add; and therefore (iii) the overall magnitudes of both the induction/drag currents are corrected by terms that correspond to self- and mutual-drag/induction. These corrections are of order κ/ρ_s, $(e^2\tilde{\rho}_s/m^2c^2)\ln(D_1^2/a^2)$, or $(e^2\tilde{\rho}_s/m^2c^2)g_{ind}(D_1/D)$ relative to the ones presented above. Here $\tilde{\rho}_s$ is a reduced superfluid density [cf. (i)]. If both κ/ρ_s and $(e^2\rho_s/m^2c^2) \sim (a^2/\lambda_L^2)$ are small numbers (here λ_L is the London penetration depth of the corresponding *bulk* superconductor), the above estimates will be essentially unchanged.

A similar design may also be possible for the "two-dimensional" case, with the "wires" in Fig. 1 replaced by "sheets" (of thickness d) extending out of the plane of the paper. The ratio r is given by an expression similiar to Eq. (7) (up to some numerical factors), with $g_\kappa^L(x) \equiv 1 - 2/(1+x)^3 + 1/(1+2x)^3$, and with the quantity in the square brackets replaced by $1/(k_F d)(v_F/c)^2(k_F D)^3$. Because of the much more rapid decrease of the Coulomb drag effect with the distance D, achieving $r \gg 1$ [in the modified Eq. (7)] may not be so simple.

It is also interesting to estimate ρ_{12} for the oxide layered superconductors. Typically we find $\rho_{12}/\rho_{11} \sim 10^{-2}$ for neighboring planes. This, however, may be a significant correction in physical situations where the velocities in the neighboring planes differ significantly.

In summary we have considered the Coulomb *supercurrent* drag effect. This effect is distinct from, and possibly more important than, the magnetic induction effect, and can be observed experimentally.

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[12] Therefore the maximum wave vector of interest to us is $q_{max} \sim 1/D$, so that $qa \ll 1$ is well satisfied if $a \ll D$, and the wires can be treated as one dimensional.
[13] The above is applicable so long as $k_B T \ll$ the gap of superconductors. At higher temperatures corrections due to quasiparticle excitations have to be taken into account.

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[18] More precisely this value has to be corrected by the self-induction of the wire, see below.
[19] The supercurrent on this second wire can be shown to be the "nonquasiparticle" response of the system [in the language of superfluid Fermi liquid, see, e.g., A. J. Leggett, Phys. Rev. **140**, A1869 (1965)] due to the change in the quasiparticle energies in this wire via the Coulomb interaction with the current-carrying first wire. This shift in quasiparticle energies can be shown to be insensitive to the superconducting transition of the first wire.