Electron-Electron Scattering in Quantum Wires and Its Possible Suppression Due to Spin Effects

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A microscopic picture of electron-electron pair scattering in single mode quantum wires is introduced which includes electron spin. A new source of excess noise for hot carriers is presented. We show that zero magnetic field spin splitting in quantum wires can lead to a dramatic spin-subband dependence of electron-electron scattering, including the possibility of strong suppression. As a consequence extremely long electron coherence lengths and new spin-related phenomena are predicted. Since electron bands in III-V semiconductor quantum wires are in general spin split in zero applied magnetic field, these new transport effects are of general importance.

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We show that electron-electron pair scattering in quantum wires is fundamentally different from two-dimensional (2D) or three-dimensional (3D) systems and that it is essential to include the electron spin in the analysis. We show that spin splitting of the electron bands causes a spin-subband dependence of electron pair scattering rates, and may cause a dramatic reduction of electron-electron scattering for hot electrons in one of the two spin subbands. We show that electron pair scattering can cause fluctuations of electron spin, energy, and wave number, and therefore is expected to contribute excess noise to electrical current. We expect that spin related effects in quantum wire transport, as demonstrated in the present work, will become important in mesoscopic transport experiments.

Electron-electron pair scattering is for many conditions the strongest scattering mechanism, limiting the electron lifetime and the phase coherence length. More generally, electron-electron interactions cause or contribute to such diverse phenomena as superconductivity, Wigner crystallization, magnetic ordering, and heavy fermion effects. In two-dimensional and three-dimensional systems (but not in 1D) electron pair scattering contributes to thermalization of hot electrons. Although it does not contribute directly to diffusive transport, it enters the collision integral of the Boltzmann equation and thus in 3D and 2D (not in 1D) contributes to the establishment of diffusive transport. The general properties of electron pair scattering in 3D have been investigated extensively [1], but detailed quantitative information for experimental semiconductor structures has only become available recently in 2D [2–4], and for 1D [5]. The weak localization regime, where impurity scattering dominates, has been intensively investigated, but little is known about the properties of electron-electron scattering in high mobility quantum wires. We assume in the present work that electrons in a quantum wire form an ordinary Landau liquid as supported by Ref. [6] and that disorder effects are negligible. The present work concerns single mode quantum wires, with a single (or a few) transverse modes per spin orientation, i.e., wires with widths of the order of 100 Å.

First, we show that electron-electron pair scattering is a phase breaking scattering process even in a single-mode quantum wire. Figure 1(a) demonstrates such a process: a spin-up electron at $(\mathbf{p},\uparrow)$ scatters with a spin-down electron at $(\mathbf{k},\downarrow)$, resulting in a hole at $(\mathbf{k},\downarrow)$, an

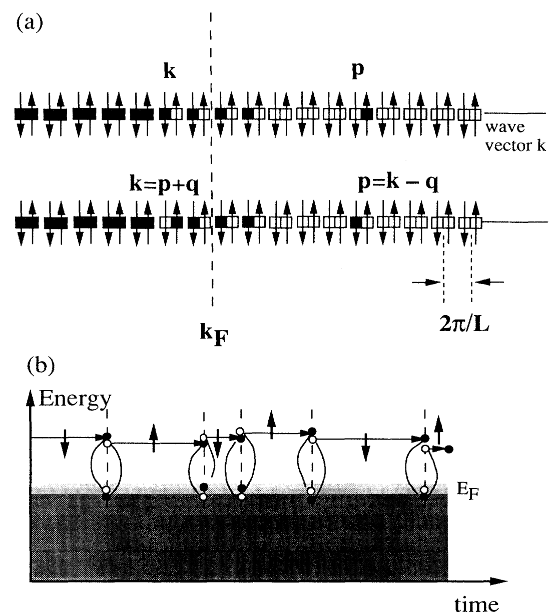

FIG. 1. Microscopic picture of typical electron-electron pair scattering process in a single mode quantum wire. (a) Spin-up electron at $(\mathbf{p},\uparrow)$ scatters with spin-down electron at $(\mathbf{k},\downarrow)$. Pair scattering of electrons in the same spin subband is forbidden in 1D. (b) Electron pair scattering in a quantum wire causes fluctuations of the electron spin. If the bands are spin split, electron energy and wave vector fluctuate as well, giving rise to a new source of excess noise for a current of hot carriers.

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electron at $(\mathbf{k},\uparrow)$, and an electron at $(\mathbf{p},\downarrow)$. Electron pair scattering in a single-mode quantum wire can only occur for pairs of electrons in opposite spin subbands, while it is forbidden for pairs of electrons in the same spin subband due to energy and momentum conservation and the Pauli principle. Similarly, it was shown in Ref. [4] that in a 2D electron gas scattering for pairs of electrons in the same spin subband is typically 50% weaker than for opposite spin subbands, although not totally forbidden as in 1D.

Figure 1(b) shows the resulting picture for a non-equilibrium electron propagating in a quantum wire. Electron pair scattering flips electrons between the two different spin subbands. Therefore, pair scattering leads to fluctuations of electron spin. In general, the two spin subbands in a quantum wire will be split in energy, and the spin states will be mixed. As Fig. 1(b) demonstrates, pair scattering in the presence of spin-subband splitting causes fluctuations of electron energy and wave number in addition to the spin fluctuations of a propagating electron, and therefore should be experimentally important in quantum wire devices as a new contribution to current dependent excess noise.

In bulk III-V semiconductors, bands are spin split at zero applied magnetic field in all directions except [100] due to the lack of inversion symmetry (see [7]). Spin splitting has terms proportional to k and k^3; typical bulk values are shown in the inset of Fig. 2. The equivalent magnetic fields which would have to be applied externally to produce a similar splitting at a Fermi energy around 20 meV are quite large. In quantum wells and quantum wires, terms in addition to the bulk terms are expected [8, 9]. Spin splittings for 2D systems have recently been measured [10–12]. For the rest of this work, we will show results taking the conduction band structure equal to that of bulk GaAs. We keep in mind that the precise value of the splitting and the spin mixing will vary for different types of quantum wires, although there are always two spin subbands in a single-mode wire. We will not discuss sample dependent details further in the present Letter, and we will simply label the two subbands as spin-up and spin-down.

The essence of our results can be explained with Fig. 2. We consider a pair scattering process, where an electron in the spin-up subband at $(\mathbf{p},\uparrow)$ scatters with a spin-down electron at $(\mathbf{k},\downarrow)$. Once $\mathbf{k}$ and $\mathbf{p}$ are selected, the final states $(\mathbf{k}-\mathbf{q},\downarrow)$ and $(\mathbf{p}+\mathbf{q},\uparrow)$ are determined by energy and momentum conservation. (In the absence of spin splitting, or when the subbands are parallel, $\mathbf{k}-\mathbf{q}=\mathbf{p}$.) The probability for this process is given by the product of the square of the Coulomb matrix element multiplied by the thermal factor $f_{k,\downarrow}\,(1-f_{k-q,\downarrow})\,(1-f_{p+q,\uparrow})$, where

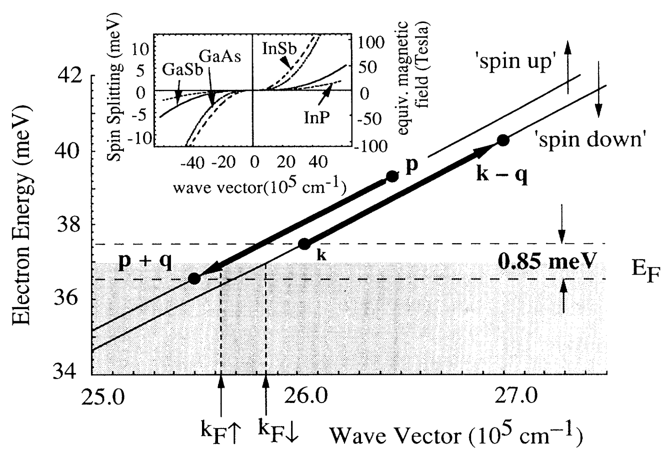

FIG. 2. Schematic diagram of electron-electron pair scattering process in a quantum wire with spin splitting. An electron $(\mathbf{p},\uparrow)$ is scattered by electron $(\mathbf{k},\downarrow)$. Diagram shows typical spin-splitting of the conduction band expected in a quantum wire along GaAs [110] near the Fermi energy in zero applied magnetic field. Note that spin composition of the bands is mixed and dependent on the details of the wire. For temperatures low compared to the energy separation of states $(\mathbf{p}+\mathbf{q},\uparrow)$ and $(\mathbf{k},\downarrow)$ (here approximately 0.85 meV as indicated), the population factors entering the scattering probability will lead to a dramatic suppression of forward pair scattering for electrons in the spin-up subband and to an enhancement of the scattering rate for the spin-down subband. Inset shows typical values for spin splitting in the bulk.

the f_k's are Fermi-Dirac occupation factors. Clearly, spin splitting strongly reduces the thermal occupation probability factor for this scattering process. As a consequence, forward (k near $+k_F$) scattering is strongly suppressed for one particular spin orientation (here spin-up), while there is a small increase for the other spin orientation (here labeled spin-down). The strong spin-subband dependence of the scattering probability relies on the strong k dependence of the spin splitting (bulk terms are proportional to k and k^3). It can be easily seen that spin-subband-dependent scattering rates are not expected for k-independent splittings. Furthermore, for scattering processes with $k \approx -k_F$ and $q \approx -2k_F$ pair scattering rates are almost independent of the subband. These facts weaken the spin-subband dependence of the total pair scattering rates, but detailed calculations outlined below show that in many circumstances strong spin-subband dependence prevails.

To confirm this surprising result quantitatively, we calculate the scattering rates. The total scattering rate for an electron at wave vector p,σ is expressed as

$$\left.\frac{1}{\tau_{ee}}\right|_{p,\sigma} = \frac{2\pi}{\hbar}\sum_{k,q} f_{k,\sigma'}\,(1-f_{k-q,\sigma'})\,(1-f_{p+q,\sigma})\left|\frac{\langle k-q,\sigma';p+q,\sigma\,|V|\,k,\sigma';p,\sigma\rangle}{\epsilon(q,(E_{p,\sigma}-E_{p+q,\sigma})/\hbar)}\right|^2 \times\,\delta\left(E_{p+q,\sigma}+E_{k-q,\sigma'}-E_{p,\sigma}-E_{k,\sigma'}\right), \qquad (1)$$

where $\langle k-q,\sigma';p+q,\sigma|V|k,\sigma';p,\sigma\rangle = e^2F^{1D}_{ijkl}(q\times w)/L\epsilon_0\epsilon_r$ is the 1D Coulomb interaction matrix element. $F^{1D}_{ijkl}(q\times w)$ is the 1D Coulomb form factor consisting of a four-dimensional integral involving the wave functions and the Bessel function K_0, which we determine by numerical integration assuming a wire with a square cross section. The dielectric function $\epsilon(q,(E_{p,\sigma}-E_{p+q,\sigma})/\hbar)$ takes account of dynamic screening. For the present calculation we integrate the finite temperature Ehrenreich expression for the polarizability numerically for the two spin-split conduction bands. We assume that the quantum wire electron band structure is described by the bulk $k\cdot p$ dispersion. The integrals are calculated numerically using adaptive multipoint Gauss-Kronrod integration.

The details of an experimental quantum wire will affect the band dispersion, spin composition of the bands, the dielectric function, and the matrix elements. The essential point of the present Letter is the prediction of a large difference in the electron scattering rates for the two spin subbands. The effects discussed in the present Letter are a consequence of the band splitting, the Pauli principle, Fermi occupation factors, and energy and momentum conservation for electron pair scattering. They are expected for many variations of the band structure, spin mixing and details of the wave functions in different types of wires.

Figure 3 compares the excess energy and spin-subband dependence of differential pair scattering rates in a GaAs quantum wire for electrons in the spin-up and spin-down subbands for a wire assumed to have the conduction band structure of GaAs along [110]. The carrier concentration is 1.6×10^6 cm^{-1}, temperature is $T=1.4$ K, and we assume a square wire of width 100 Å and infinite confinement potential. Because of the exponential character of the Fermi population factors, the forward pair scattering rates for the spin-up subband are many orders of magnitude lower compared to the spin-down subband. As expected, Fig. 3(b) shows that the spin-subband dependence does not occur for electrons scattering with partners at $k\approx -k_F$. Figures 3(a) and 3(b) clearly show that for electrons with excess energies more than 1 meV, the total scattering rate is substantially larger for a hot electron in the spin-down subband. We have investigated many combinations of spin splitting strength, temperature, and excess energy, and details will be published separately. Constructing quantum wires with specific spin splitting, carrier concentration, and choosing particular temperature and excess energy will allow us to tune the spin dependence of the electron pair scattering rates. Further it can be seen from Fig. 3 that the total scattering rates also show some spin dependence for equilibrium electrons near the Fermi level ($\Delta=0$), although the spin subband dependence is not strong.

Figure 4 shows the total electron pair scattering rates calculated by numerically integrating Eq. (1) over $-\infty<k<+\infty$, and the corresponding scattering lengths. The

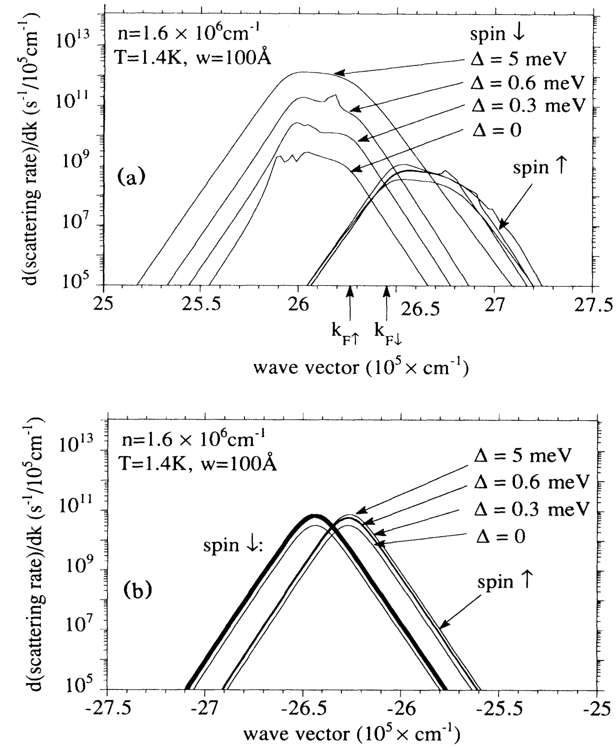

FIG. 3. Differential scattering rates for electrons in the spin-up and spin-down subbands of a quantum wire with spin splitting. (a) As a consequence of the different Fermi population factors electron-electron scattering with partners near $+k_F$ is substantially lower for one particular spin orientation (here spin up), while scattering for the opposite spin orientation (here spin down) is enhanced. (Numerical anomalies at $q=0$, where no dephasing takes place, are eliminated from the figure.) (b) For scattering with partners near $-k_F$ the scattering rates are essentially independent of the spin subband.

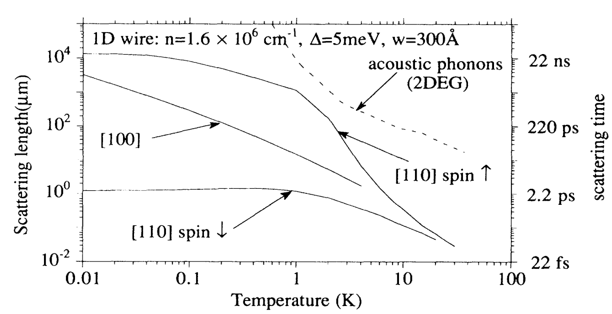

FIG. 4. Total electron-electron pair scattering rates determined by numerical integration of Eq. (1) over $-\infty<k<+\infty$. Results are shown for a quantum wire, with conduction subband dispersions assumed to be those of bulk GaAs oriented along the [100] and [110] crystal orientations. Because of spin splitting the total scattering rates are strongly suppressed for one of the two spin subbands (here spin-up), while they are increased for the opposite spin subband. Dashed line shows acoustic phonon scattering for a 2DEG from Ref. [13].

spin dependence of the forward scattering ($k \approx +k_F$) causes a strong spin subband dependence of the total rates. For millikelvin temperatures scattering lengths in excess of millimeters are predicted for one of the two spin subbands, while at helium temperatures lengths around 20 μm are predicted. To observe such long scattering lengths, other competing scattering mechanisms have to be sufficiently weak. Impurity scattering and interface roughness scattering can be reduced by improvements in fabrication techniques. In Ref. [5] it was estimated that remote ionized impurity scattering can also be reduced sufficiently. The dashed line in Fig. 4 shows the strength of acoustic phonon scattering in 2D from Ref. [13]; comparable data for 1D are not yet available. Stronger spin splitting due to choice of a different material, or in-built electric fields, may lead to stronger suppression of scattering and longer coherence lengths. Figure 4 also shows that the spin-subband dependence of the scattering rates disappears above a temperature larger than the typical splitting energy of here 0.85 meV, indicated in Fig. 2, although this temperature may be much increased for materials with higher spin splitting.

We will now comment on the significance and on experimental predictions. We have introduced a microscopic picture for electron pair scattering in single mode quantum wires and calculated scattering rates. Such work is essential to understand microscopic details of transport, or other details such as spin relaxation, which have recently attracted attention in 2D [14]. We also demonstrated a new source of excess (i.e., current induced) noise. The predicted spin-subband dependence of electron pair scattering rates leads to the prediction of a range of novel spin-subband-dependent transport properties. The present work demonstrates that electron spin can have even more dramatic effects in quantum wires than in 2D. Investigations of mesoscopic transport have progressed to the point where very detailed electronic spectroscopy of quantum dots coupled to quantum wire electron waveguides can be performed (see, e.g., Ref. [15]). Spin-subband dependence may allow high resolution experiments of magnetic sublevels in quantum dots, and it may lead to electron spin polarization effects in hot electron transport.

In summary, we have introduced a microscopic picture of electron pair scattering for quantum wires, which includes spin. We demonstrated it to be a source of phase breaking and excess noise. We show that spin splitting leads to unequal forward and total pair scattering rates for electrons in the spin-up and spin-down subbands of a quantum wire. We predict the possibility of strong reduction of pair scattering for one of the two spin subbands, and very long spin-subband-dependent coherence lengths. Several other spin-related effects may arise as a consequence of spin-subband-dependent electron pair scattering rates.

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