Electron-Electron Interactions and Spontaneous Spin Polarization in Quantum Hall Edge States

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We study the effects of electron-electron interactions on the ground state of integral quantum Hall edge states in the Hartree-Fock approximation. We find that, in the absence of Zeeman splitting, the outermost edge state undergoes a spontaneous transition between spin-unpolarized and spin-polarized ground states at a sample-dependent critical value of the bulk filling $\nu_{\rm bulk}^c \sim 4$. The abrupt appearance, in the spin-polarized state, of a sizable (about a magnetic length) separation between edge states of opposite spin should make the transition accessible to a range of magnetotransport experiments.

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Edge states in the integral [1–3] and fractional [2–5] quantum Hall regimes have been the subject of intense study in the past few years because of their importance to magnetotransport in a broad range of mesoscopic and macroscopic systems. While most theoretical studies have developed a noninteracting picture of edge states, attention has recently shifted to the effects of electron-electron interactions on edge-state properties [3–6]. Electron spin has played little role in theories of edge states, which have considered two limits: the integer regime, where the two spin states are taken to be degenerate, and the fractional regime, where the system is generally taken to be fully spin polarized [7].

In this Letter, we use the Hartree-Fock approximation to study the effects of electron-electron interactions on the ground state of quantum Hall edge states in the integer regime. We show that, in the absence of Zeeman splitting, the outermost edge state undergoes a spontaneous transition between spin-unpolarized and spin-polarized ground states at a sample-dependent critical value $\nu_{\rm bulk}^c$ of the bulk Landau level filling. We find that $\nu_{\rm bulk}^c$ is about 4 for a typical sample, and is a slowly varying function of sample parameters. The spin-polarized ground state has a spatial separation between edge states of opposite spin that is on the order of the magnetic length and is only weakly dependent on the Zeeman energy. The abrupt appearance of this large separation should make the transition observable in experiments that measure inter-edge-state equilibration, Aharonov-Bohm oscillations in quantum dots, or quantized conductance through point contacts.

We begin by considering a wide quantum wire along the y axis in strict two dimensions with a uniform magnetic field $\mathbf{B}$ in the z direction. Taking $\mathbf{A}=Bx\hat{\mathbf{y}}$, we write the Hartree-Fock single-particle wave functions as $\psi_{nX}^{\sigma}(x,y)=\exp(-iXy/l^2)\phi_{nX}^{\sigma}(x)$, where $l=|\hbar c/eB|^{1/2}$ is the magnetic length and $\phi_{nX}^{\sigma}(x)$ is an eigenfunction of the Hamiltonian $\mathcal{H}=\mathcal{H}_0+V_c(x)+V_H(x)+\mathcal{H}_{\rm ex}^{\sigma}+\sigma\varepsilon_Z$ with eigenvalue $\varepsilon_n^{\sigma}(X)$. Here $\mathcal{H}_0=[p_x^2+\hbar^2 l^{-4}(x-X)^2]/2m$, $V_c(x)$ is the bare confining potential that defines the wire, and $V_H(x)$ is the Hartree potential

$$V_H(x)=-2\frac{e^2}{\epsilon l}\int d\xi'\,\tilde{n}(\xi')\ln\left|\frac{x}{l}-\xi'\right|, \tag{1}$$

where $\tilde{n}(x/l)=l^2\sum_{n\sigma X}\nu_n^{\sigma}(X)|\phi_{nX}^{\sigma}(x)|^2$ is the (scaled) two-dimensional electron density, and $\nu_n^{\sigma}(X)$ is the X-dependent filling factor for each Landau level and spin state. Self-consistency requires that $\nu_n^{\sigma}(X)=f(\varepsilon_n^{\sigma}(X))$, where $f(\varepsilon)$ is the Fermi function. The spin-dependent terms in the Hamiltonian are the exchange operator $\mathcal{H}_{\rm ex}^{\sigma}$ and the Zeeman splitting (with $\varepsilon_Z=\frac{1}{2}g\mu_B B$), which both conserve X.

We want to focus on an edge region and, if possible, formulate the Hartree-Fock approximation in a way that depends only weakly on details external to this edge. To this end, we decompose the charge density into a uniform slab of density $\nu_{\rm bulk}/2\pi$ and dipolar distributions of charge concentrated near the edges of the slab: $\tilde{n}=\tilde{n}_{\rm slab}+\Delta\tilde{n}^L+\Delta\tilde{n}^R$, with

$$\tilde{n}_{\rm slab}(\xi)\equiv(\nu_{\rm bulk}/2\pi)\Theta(\xi-x_L/l)\Theta(x_R/l-\xi)\,.$$

Here $\Theta(\xi)$ is the unit step function, and we use the definitions $x_R\equiv(2\pi/\nu_{\rm bulk})l\int_0^{\infty}d\xi\,\tilde{n}(\xi)$, $\Delta\tilde{n}^R(\xi)\equiv\Theta(\xi)\times[\tilde{n}(\xi)-\tilde{n}_{\rm slab}(\xi)]$ for the right-hand edge and equivalent definitions for the left-hand edge. We can now divide V_H into a part V_H^L that depends on $\Delta\tilde{n}^L(\xi)$ and is smoothly varying near x_R, and a part

$$V_H^R(x)=-2\frac{e^2}{\epsilon l}\int d\xi'\,\Delta\tilde{n}^R(\xi')\ln\left|\frac{x}{l}-\xi'\right| + \frac{\nu_{\rm bulk}}{\pi}\frac{e^2}{\epsilon l}\frac{x-x_R}{l}\left(\ln\left|\frac{x-x_R}{l}\right|-1\right) \tag{2}$$

that is independent of $\Delta\tilde{n}^L(\xi)$ and can vary rapidly near x_R. The separation of V_H and the spatial dependence of V_H^L and V_H^R are shown schematically in Fig. 1. We now lump V_H^L with V_c into an effective confining potential $V_c^{\rm eff}=V_c+V_H^L$, *which is nearly independent of local rearrangements of charge near* x_R, if the wire is wide and the density near the center is uniform. Our approach is to apply the Hartree-Fock approximation treating $V_c^{\rm eff}$ as a truly fixed confining potential and V_H^R as the effective

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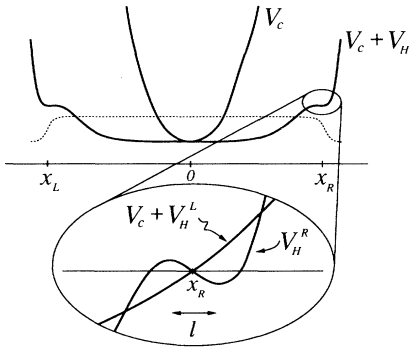

FIG. 1. Potential and density profiles for a wide quantum wire. The bare confining potential V_c and the total self-consistent potential V_c+V_H are shown as solid lines, and the electron density as a dashed line. The blowup of the region near x_R shows the separation of the self-consistent potential into V_H^R and the effective confining potential $V_c+V_H^L$.

Hartree potential.

To illustrate this, we solve the case $\nu_{\text{bulk}}=2$ explicitly in the strong field limit, where we can ignore mixing between Landau levels. We shift the origin to x_R for notational simplicity and consider a spin-polarized trial state with integral Landau level fillings: $\nu_0^\sigma(X;\Delta X)=\Theta(-X-\sigma\Delta X/2)$. The single-particle energies corresponding to the trial solution depend on spin and on the width parameter ΔX:

$$\varepsilon_0^\sigma(X;\Delta X)=\varepsilon_0^{\text{eff}}(X)+\varepsilon_H(X;\Delta X)+\varepsilon_{\text{ex}}^\sigma(X;\Delta X)+\sigma\varepsilon_Z\,,$$

where

$$\Delta\tilde{n}^R(\xi;\Delta X)=\sum_\sigma \tilde{n}_1\left(\xi+\frac{\sigma}{2l}\Delta X\right)-\frac{1}{\pi}\Theta(-\xi)\,, \tag{3}$$

$$\varepsilon_c^{\text{eff}}(X)=\pi^{-1/2}\int d\xi\, e^{-(\xi-X/l)^2}V_c^{\text{eff}}(l\xi)\,, \tag{4}$$

$$\varepsilon_H(X;\Delta X)=\pi^{-1/2}\int d\xi\, e^{-(\xi-X/l)^2}V_H^R(l\xi;\Delta X)\,, \tag{5}$$

$$\varepsilon_{\text{ex}}^\sigma(X;\Delta X)=-\frac{e^2}{2\pi\epsilon l}\int_0^{[X+(1/2)\sigma\Delta X]/l} d\xi\, e^{-\xi^2}K_0(\xi^2)\,. \tag{6}$$

Here $\varepsilon_{\text{ex}}^\sigma(X;0)$ is the single-particle exchange energy for $\nu_0^\sigma=\Theta(-X)$ filling [defined so that $\varepsilon_{\text{ex}}^\sigma(0;0)=0$], $\tilde{n}_1(\xi)=[1-\text{Erf}(\xi)]/4\pi$ is the density for the same filling [where $\text{Erf}(z)$ is the error function], and $K_0(z)$ is the modified Bessel function.

We want to know under what circumstances a trial state of this form can be a fully self-consistent solution of the Hartree-Fock equations. It is easy to see that for very hard confining potentials effects of the confining potential will dominate electron-electron interactions near the edge and our integer filling trial state (with $\Delta X=0$ for $\varepsilon_Z=0$) will satisfy the Hartree-Fock equations. As the confining potential is softened, the ground state continues to have integer filling of the form $\nu_0^\sigma(X;\Delta X)$ until eventually, for very soft potential, it gives way to a solution with fractional filling. We shall focus on the regime where integer filling obtains, and study the behavior of the value ΔX^* that renders the trial solution self-consistent in this regime. We emphasize that the integer-filling states we find are true self-consistent solutions of the Hartree-Fock equations, representing true local minima in the full space of Hartree-Fock wave functions, and are not merely variational solutions in the restricted subspace of states with integer filling.

Within the integer-filling regime we find ΔX^* by minimizing the energy

$$E(\Delta X)=\sum_{\sigma X}\nu_0^\sigma(X;\Delta X)\{\varepsilon_0^\sigma(X;\Delta X)-\tfrac{1}{2}[\varepsilon_H(X;\Delta X)+\varepsilon_{\text{ex}}^\sigma(X;\Delta X)]\}\,,$$

then checking for self-consistency by calculating $\varepsilon_0^\sigma(X;\Delta X)$ explicitly [8]. In our case the energy can be written in the simple form

$$E(\Delta X)=E(0)+\frac{1}{2\pi l^2}\left\{-\varepsilon_Z\Delta X+\int_0^{\Delta X/2}dX[\varepsilon_c^{\text{eff}}(X)-\varepsilon_c^{\text{eff}}(-X)+\varepsilon_H(2X;0)]\right\}. \tag{7}$$

The value ΔX^* that minimizes the energy can be found numerically for a given V_c^{eff}, which need not be smoothly varying. Alternatively, we can gain insight by expanding $\varepsilon_c^{\text{eff}}(X)=a_1X+a_2X^2+\cdots$ [taking $\varepsilon_c^{\text{eff}}(0)=0$] and writing

$$E(\Delta X)\approx E(0)+\frac{1}{2\pi l}\left\{-\varepsilon_Z\Delta\tilde{X}+\frac{1}{4}\frac{e^3}{\epsilon l}(\alpha-\alpha_c)\Delta\tilde{X}^2+\frac{1}{4!}\frac{e^2}{\epsilon l}(\alpha_3+\beta)\Delta\tilde{X}^4+\cdots\right\}, \tag{8}$$

an expansion in powers of $\Delta\tilde{X}\equiv\Delta X/l$. In this expansion $\alpha=a_1l/(e^2/\epsilon l)$ is the (scaled) slope of $\varepsilon_c^{\text{eff}}$, $\alpha_c\equiv-l\varepsilon_H'(0;0)/(e^2/\epsilon l)=0.404$, $\beta\equiv l^3\varepsilon_H'''(0;0)/(e^2/\epsilon l)=0.318$, and $\alpha_3=\frac{3}{4}l^3a_3/(e^3/\epsilon l)$ (a prime indicates derivative with respect to X). Equation (8) has the form of Landau free energy for a second-order phase transition, with $\Delta\tilde{X}$ as the order parameter and ε_Z as the external field. With ε_Z set to zero, we see that there is a spontaneous spin-polarizing transition when α is reduced below α_c. The exact numerical solution of (7) for $\varepsilon_Z=0$ and $\varepsilon_Z\neq0$ (solid lines) is shown in Fig. 2 for the case $\varepsilon_c^{\text{eff}}=a_1X$ and compared with the approximate result $\Delta X^*/l=[(3/\beta)|\alpha-\alpha_c|]^{1/2}\Theta(\alpha_c-\alpha)$ (dashed line), found by truncating

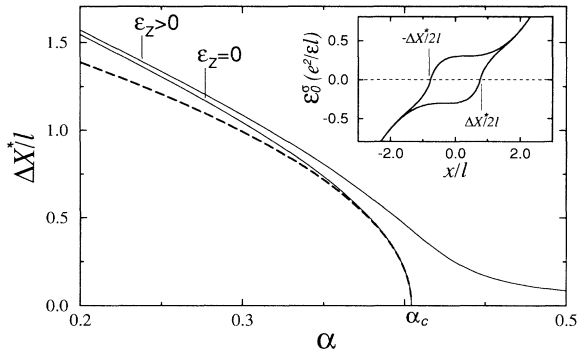

FIG. 2. Equilibrium separation ΔX^* of edge states of opposite spin as a function of slope parameter $\alpha = a_1 l/(e^2/\epsilon l)$, with and without the Zeeman energy appropriate for GaAs at $B=2$ T (solid lines). The dashed line shows Landau theory approximation. Inset: Self-consistent single-particle energies $\varepsilon_0^\sigma(X; \Delta X^*)$ for $\alpha = 0.2 < \alpha_c$ (solid lines); Fermi level (dashed line).

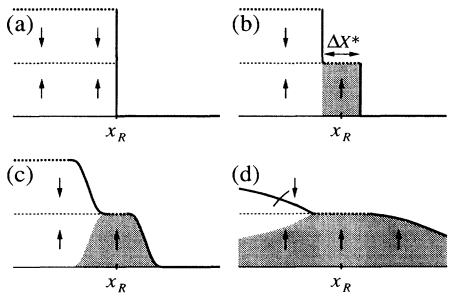

FIG. 3. Evolution of $\nu(X)$ near the edge as the effective confining potential is softened. (a) Unpolarized ground state for steep confining potential ($\alpha > \alpha_c$). (b) Spontaneously polarized integral-filling state for $\alpha < \alpha_c$. (c) Possible ground state with regions of fractional filling. (d) Ground state with fractional filling in the electrostatic regime, with a spin-polarized incompressible region separating wide compressible regions that have states of opposite spin at the Fermi level. Shading indicates net spin polarization.

the series (8) after $\Delta\tilde{X}^4$. For this $\varepsilon_c^{\rm eff}$, our trial solution fails (and fractional filling appears) for $\alpha < \alpha_f = -0.76$. Note that α_f, unlike α_c, depends on the form of $\varepsilon_c^{\rm eff}$.

It is perhaps surprising that although $E(\Delta X)$ in Eq. (7) has been calculated within the Hartree-Fock approximation, it does not depend on the exchange energy. As a consequence, the energy-minimizing separation ΔX^* between edge states of opposite spin is determined by *direct* rather than exchange interactions. The explanation for this is twofold. First, the instability at α_c is itself due to direct interactions. Indeed, the unpolarized $\nu = 2 \rightarrow 0$ solution becomes unstable at the same α_c even in the Hartree approximation, but gives way to a state with fractional filling, rather than to one with a spin polarization. The role of exchange in the Hartree-Fock analysis is to stabilize the spin-polarized integer-filling state relative to states with fractional filling [because exchange favors completely filled or completely empty (spin-split) Landau levels]. Second, since the total exchange energy is the same for any solution with integral filling, exchange does not affect the value of ΔX^* in the integer-filling regime.

In Figs. 3(a)–3(d) we use the total filling factor $\nu(X) \equiv \sum_{n\sigma} \nu_n^\sigma(X)$ to summarize schematically the evolution of the edge states as the parameter α is reduced. For $\alpha > \alpha_c$, we have the situation in Fig. 3(a), where the edge is unpolarized (taking $\varepsilon_Z = 0$). As α is reduced below α_c, the electron system undergoes a second-order transition to the spin-polarized state shown in Fig. 3(b). As α is reduced still further, the ground state acquires regions with fractional filling, as shown in Fig. 3(c). In the limit of very soft potential, the Landau level filling is fractional and the screening is metallic everywhere, except for an incompressible region at $\nu = 1$ due to the exchange energy gap [3]. When corrections to Hartree-Fock are included, the filling will no longer be strictly integral even for very hard confining potentials, but a Fermi surface (i.e., a discontinuity in filling) will still exist for each spin state [6], and the limiting cases of an unpolarized ground state for hard-wall confinement and a polarized ground state for soft confinement will still obtain. Hence the symmetry-breaking transition from a ground state where the Fermi surface positions for the two spin states coincide to one where they are split should remain.

We can make contact with a realistic system and bring out the physics of the spin-polarizing transition by considering the electrostatic model introduced by Gelfand and Halperin [5] for edges in mesa-etched samples. In such a sample, negative surface charge at the mesa wall depletes the semi-infinite electron gas to a distance d from the surface, which typically is large compared to the interelectron spacing, $r_0 = (\pi n_0)^{-1/2}$, where n_0 is the bulk electron density. If the surface charge is modeled by a line charge λ lying in the plane of the electron gas, and if the spacing between the electron gas and the positive donor layer is neglected, then $\lambda = -2en_0 d$ and the classical electrostatic density profile is given by [5]

$$n_{\rm es}(x) = n_0 \frac{2}{\pi} \left\{ \frac{|x/d|^{1/2}}{1+|x/d|} + \tan^{-1}|x/d|^{1/2} \right\} \Theta(-x), \quad (9)$$

where the surface charge lies at $x = d$. To apply our earlier analysis to the outermost edge state in this case, we fix the electronic charge in the higher Landau levels at its classical density, and allow the two spin states in the lowest Landau level to minimize their energy, as discussed above. The appropriate effective confining potential $V_c^{\rm eff}$ is determined by the depletion width d and bulk filling $\nu_{\rm bulk}$. The Hartree-Fock ground state is then determined by the two dimensionless parameters $\nu_{\rm bulk}$ and $\tilde{d} \equiv d/r_0$. For given $\tilde{d}$, there is a critical bulk filling $\nu_{\rm bulk}^c(\tilde{d})$ (> 2) such that for $\nu_{\rm bulk} > \nu_{\rm bulk}^c$ the lowest Landau level is spin unpolarized, while for $\nu_{\rm bulk} < \nu_{\rm bulk}^c$ it is

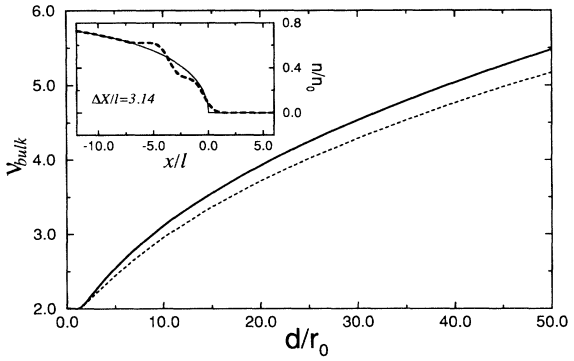

FIG. 4. Solid line shows the critical value of bulk filling factor ν_{bulk} as a function of d/r_0, where d is the depletion width and r_0 is the interparticle spacing; the dashed line shows the value of ν_{bulk} where $\Delta X = l$. Inset: Comparison of classical electrostatic electron density (solid line) to the density given by our approximation (dashed line), with parameters $\nu_{\text{bulk}} = 3.2$ and $d/r_0 = 25$.

spin polarized, with ΔX^* on the order of the magnetic length (see Fig. 4).

Within this model the spin-polarizing transition may be understood as follows. In the electrostatic solution (9), the density gradient $|dn/dx|$ is large near the edge, diverging as $x \to 0^-$, but becomes small when one is further from the edge. If ν_{bulk} is large, the region with $0 < 2\pi l^2 n_{\text{es}}(x) < 2$ is a narrow strip close to the edge, where the density profile is steep. The quantum solution can best approximate this by choosing an abrupt drop in $\nu(X)$ from 2 to 0, as in Fig. 3(a), with no polarization in the outermost edge state. If ν_{bulk} is only slightly greater than 2, however, and if $d \gg l$, then the region with $0 < 2\pi l^2 n_{\text{es}}(x) < 2$ is wide on the scale of l, and the quantum solution mimics the more gradual density profile by developing a spin-polarized ground state, as in Fig. 3(b), 3(c), or 3(d). Note that without exchange, the quantum solution would achieve a gradual density profile through fractional filling. In situations with $\nu_{\text{bulk}} \geq 4$, when the outermost edge state is typically unpolarized, there might nevertheless occur a spontaneous polarization of edge states corresponding to higher Landau levels, as these occur deeper in the sample, where the electrostatic density gradient is smaller.

Since a large spatial separation between edge states of opposite spin appears abruptly when the lowest Landau level becomes spin polarized, and since ν_{bulk} can be varied by changing B, the transition should be detectable in several experiments. In measurements of equilibration between opposite spin states in the outermost edge channel [9], for example, the abrupt increase in separation should strongly reduce the scattering rate and increase the equilibration length. In measurements of Aharonov-Bohm (AB) oscillations in conductance, both in quantum dots [10] and in single point contacts [11], the spatial separation between different spin states will lead to different AB frequencies and to beats in the AB oscillations as a function of magnetic field. In point contacts [2,12], the separation should lead to spin-split conductance steps that depend on the perpendicular and not the in-plane component of B. Of general relevance also are experiments on nonlinear tunneling across incompressible spin-polarized $\nu = 1$ regions in the bulk [13].

In conclusion, we have shown that within the Hartree-Fock approximation the outermost edge state undergoes a spontaneous transition to a spin-polarized ground state at a critical bulk filling factor. The transition leads to a separation between edge channels of opposite spin on the order of the magnetic length that should be observable in a variety of magnetotransport experiments.

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