Measurement of Energy Spectral Density of a Flow in a Rotating Couette System

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The energy spectral density of axial velocity components in a rotating Couette-Taylor system was obtained using a Fourier transform of axial velocity distributions. As this configuration has well defined spatial periodicity, strong peaks appear on the spectrum corresponding to Taylor vortex flow and wavy vortex flow modes and their harmonics. A continuous background shows an exponential decay with wave number, which supports the results of the numerical simulations. A variation in the decay rate at reduced Reynolds number shows a maximum at $R^* \approx 22$. At this Reynolds number, an energy exchange between the first and second harmonics was observed.

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Flow in a rotating Couette system has been extensively used for investigating the passage of flow transition from laminar to turbulence through chaos, since a transition is gradual and nonhysteretic [1]. In a system with the outer cylinder fixed and sufficient height to gap ratio ($\Gamma \geq 20$), the controlling parameter is the Reynolds number R. When increasing R, a flow transition occurs at R_c to Taylor vortex flow (TVF), which forms a stationary axial roll structure and gives rise to a nonzero axial velocity component. At still higher R, R_w, a further transition to wavy vortex flow (WVF) occurs where an azimuthal wave mode sets in. At still higher Reynolds number, two kinds of secondary azimuthal waves (MWV) start to modulate the WVF, leading to a quasiperiodic flow motion. It has been reported that one of these azimuthal modes, called the GS mode [2], modulates the WVF globally and another, the ZS mode [3], is a standing wave and appears strongly near the outflow jet region in a roll pair.

We have experimentally studied this quasiperiodic mode (MWV) using a space-averaged power spectra from data sets of time-dependent velocity fields obtained by the ultrasonic velocity profile measuring method [4], and reported that two azimuthal wave modes of the so-called GS and ZS modes coexist over a wide range of the Reynolds number [5], which support the findings of Coughlin *et al.* [6].

In all of the earlier experimental investigations including our own [7] (except for flow visualization studies), the temporal periodicity has been used to characterize these wavy modes [8]. On the other hand, in the numerical studies such as those of Marcus [9] and Coughlin and Marcus [10], the energy spectrum which was obtained from spatial information was used for investigating these characteristics. Our data set used in the earlier reports is comprised of 128 spatial points and 1024 temporal points. In a separate paper [5], this data set was viewed as 128 time series of 1024 data points and the position-dependent power spectra were used for investigating the temporal characteristics of the MWV regime. In general, however, it is possible to draw out spatial characteristics of a flow field from the same data set. The configuration of the rotating Couette system has a very clear and well defined spatial periodicity in the axial direction so that the Fourier analysis in the space domain directly yields a spectrum of wave numbers. We report here results of an analysis of such data by using the spatial Fourier transform.

The experimental setup and measuring method were described in detail in Refs. [4,5]. The radius ratio of our Couette system is $\eta = R_i/R_o = 0.904$ (R_i is the radius of inner cylinder and R_o is that of outer cylinder) and the aspect ratio is $\Gamma = L/d = 20$ ($d = R_o - R_i$, L is the column height). Only the inner cylinder is rotated. The Reynolds number R is defined as $R = \Omega R_i d/\nu$ (Ω is the angular velocity of the inner cylinder, ν is the kinematic viscosity of the working fluid) and the reduced Reynolds number as $R^* = R/R_c$, where the critical Reynolds number R_c is 134.5 [1]. The liquid used in this experiment was a mixture of water and 30% glycerol. The measuring method was by ultrasound velocity profile monitor [11], which can successively obtain a series of instantaneous velocity profiles. A measuring volume of one data point has a half disk shape of radius 2.5 mm and thickness 0.75 mm with its center located at $r = R_o$. The measurement region with 128 spatial points in between starts at 40 mm from one end of the column and extends to 135 mm. The measurement time was 72–130 msec for a velocity level of a few centimeters per second. The data used in this report are for Reynolds number $9.7 \leq R^* \leq 39.07$ and cover the flow regimes of WVF and MWV.

We measured the velocity distribution of the axial component along the outer wall, $V_z(z,t)$, in a form of V_{ij} ($i = 0$–127, position; $j = 0$–1023, time). A spatial fast Fourier transform was performed on each of the 1024 velocity profiles, yielding 1024 spectra, V_{kj} (complex,

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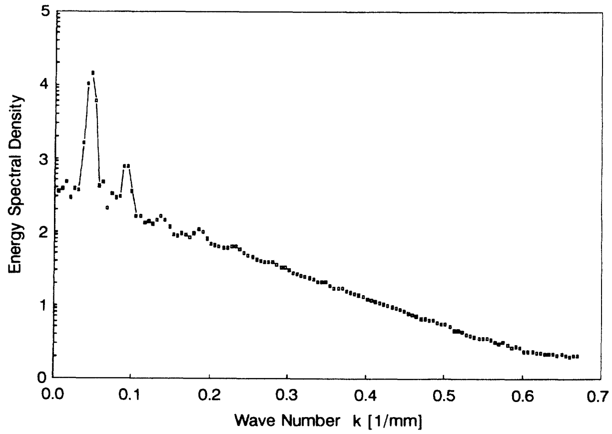

FIG. 1. Energy spectral density. $R^* = 16.40$.

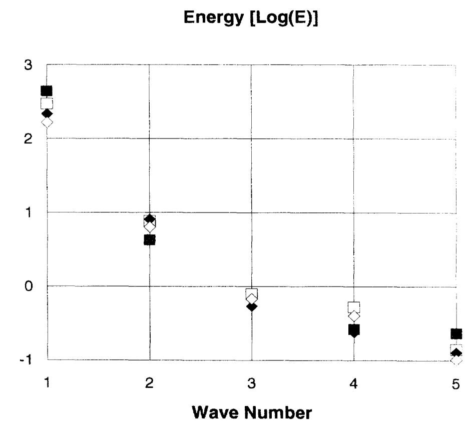

FIG. 2. Energy spectral density (peak area) of fundamental mode (k_0) and its higher harmonics. Solid square, $R^* = 24.8$; open square, 20.7; solid diamond, 18.2; and open diamond, 16.1.

$k = 0$–128, wave number). A cosine window was used at both ends (10% of the total data length) of the profile with an additional 128 points of null data for $i = 128$–255. The Fourier spectrum was computed as $S_{kj} = |V_{kj}|^2$, and averaged over 1024 spectra. A spectrum then represents a one-dimensional energy spectral density in the axial direction as a function of axial wave number. Our measurement is one dimensional and the flow is nonisotropic.

Figure 1 shows the time-averaged energy spectral density obtained at $R^* = 16.40$. Although we obtain time-dependent spectra and some periodicity is observed, we focus our observation here on the time-averaged spectral density. In the lower wave number region, isolated peaks are clearly seen, representing the fundamental mode (k_0) and its harmonics. The confirmation of this fact was established by comparing the corresponding wavelength of the highest peak (lowest wave number, k_0) with the size of the rolls. This wave number k_0 corresponds to the TVF and WVF, since axially they have very similar wavelengths. In this example, up to 4 times higher harmonics ($4k_0$) are observed. This will be discussed later. At higher k, the spectrum is exponential (linear in the log-linear plot) for quite a wide range of k. This agrees with results obtained in numerical simulations of TVF by Marcus [9] and of WVF and MWV by Coughlin and Marcus [10]. Marcus discussed the results, in contrast to Bénard convection, by stating that "Only the $k = 0$ mode is driven directly by the torque . . . and it must lose its energy to other modes by nonlinear interaction. Energy given to the system is passed to higher k modes and no kinetic energy is given by other mechanisms and thus the energy spectrum must be smooth, while in Bénard convection, nonlinear interaction from heat flux gives energy directly to higher modes and thus it is not smooth." At the same time, Marcus used dimensional analysis and showed that indeed the spectrum is exponential. It is likely that this argument must be valid not only for modes which follow the shift-reflect (SR) symmetry used in these simulations but also for modes decomposed generally, as evident from the result of our measurements.

All the modes used in the simulation fulfill a shift-reflect symmetry while our data points are in arbitrary units defined in terms of the resolution of the wave number. If those modes which appeared in the spectra as isolated peaks fulfill the SR symmetry, then they can be compared with the results of simulation directly. The peak areas of the isolated peaks at lower k were estimated by subtracting the exponential background under the peaks (linear in the log-linear plot) and then plotted as a function of k as shown in Fig. 2. It shows a smooth but nonexponential decrease with k. Nevertheless, this also agrees qualitatively with the results of simulations by Marcus and by Coughlin and Marcus, if one scrutinizes their results more carefully. Rather the linear relationship in their plot on a log-linear scale is violated at lower wave numbers, and shows a tendency similar to ours.

In an attempt to scale the variation of the slope of energy spectral density with respect to Reynolds number, the spectrum was fitted to an exponential curve, $\log(S) = -Ak + B$, by a least-squares fitting routine using the part which includes no peaks. The result is plotted in Fig. 3. The variation of the slope (or decay rate) (A) is smooth and gradual. It increases with R^*, reaches a maximum, and then decreases. The decrease at higher R^* appears steeper than the increase at lower R^*. The decrease in the slope of energy spectral density means that the energy is preferentially diluted to modes at higher wave numbers. Our results seem reasonable if one considers that the energy is given into the $k = 0$ mode and then progressively passed to higher wave number mode. However, it is interesting that the slope is smaller below $R^* \approx 22$ too. This tendency was observed for all other data sets obtained, although the R^* at which the max-

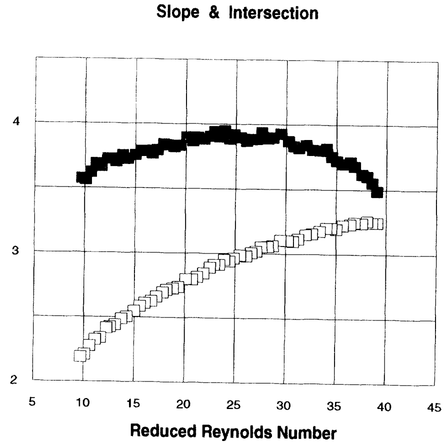

FIG. 3. A variation of slope (A, solid square) and intersect (B, open square) of exponential decay of continuous background of energy spectral density with respect to the reduced Reynolds number.

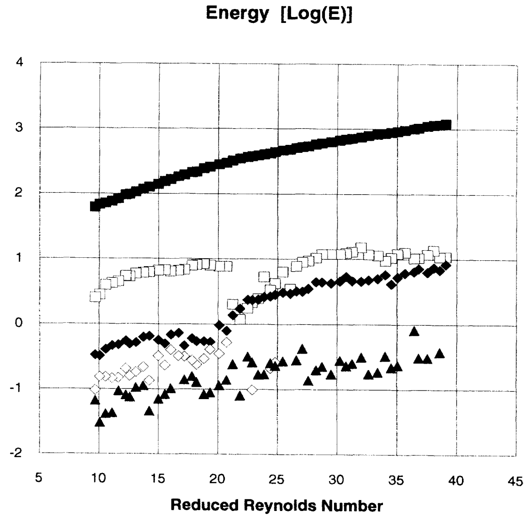

FIG. 4. Variation of peak areas of the first five peaks on the energy spectral density with respect to the reduced Reynolds number. Sequence from the first to fifth is solid square, open square, solid diamond, open diamond, and solid triangle.

imum slope is reached changes from one data set to another. On the other hand, the variation of B which is a measure of the magnitude of continuous background shows a smooth and monotonous increase with R^*. These results suggest that fundamental modes (TVF and WVF) keep developing until $R^* \approx 22$, and concentrate a larger portion of the energy in these modes, but beyond this Reynolds number, participation of other modes starts to develop.

Figure 4 shows the variation of axial energy (peak area) of each axisymmetric mode with respect to Reynolds number. The energy of the k_0 mode increases monotonically. On the other hand, the $2k_0$ harmonic (k_2) shows a strong dip at $R^* \approx 22$ and then slowly recovers. The $3k_0$ (k_3) harmonic starts to increase and the $4k_0$ mode disappears at and above $R^* \approx 22$. The highest harmonic observed, $5k_0$, appears to increase slowly but scattering of the data is large. This is due to a relatively large uncertainty arising from a low energy level compared with the corresponding background. The variation of energy for each harmonic mode shown in this figure also suggests a nonlinear interaction between these modes. For instance, the k_3 mode is excited at $R^* \approx 22$, presumably by obtaining its energy from the k_2 mode. However, since the participation of the fundamental mode is (93–95)% of the total energy in comparison to its harmonics at less than 5%, and in addition, since we had not been able to resolve WVF from TVF in the experiment, the nonlinear relationship cannot be singled out quantitatively.

In conclusion, our experiments support results from numerical simulations that the energy spectral density is generally exponential (linear in log-linear plot). The components which follow SR symmetry show a slight increase at lower wave numbers, which was observed in the simulation too. The decay rate (slope) changes smoothly with respect to Reynolds number, but it shows a peak value at $R^* \approx 22$. Above and below this value, it decreases slowly, whereas the background level increases monotonically over the entire range of the Reynolds number studied here. The variation of energy of each of the components with SR symmetry shows the same behavior as that of the decay rate, and a magnitude of background. The fundamental mode increases monotonically but the higher modes show sudden changes such as dip or ramp at $R^* \approx 22$. These results suggest that some new mode sets in at this R^*.

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