Comment on "Gauge-Independent Analysis of Chern-Simons Theory with Matter Coupling"

There have been numerous attempts over the past several years to extend the anyon concept to the domain of relativistic quantum field theory. Probably the most well known of these is that of Semenoff [1] who claimed that upon combining a spin zero field operator with an appropriate exponential of the current operator two desired goals were achieved—namely, the statistics of the field become anomalous and the interaction was eliminated from the equations of motion. Such a result (if correct) would have been an appropriate basis for claiming a relativistic field theory of anyons. In fact the necessity of this set of *two* criteria for an anyon theory must seem rather evident inasmuch as the anyon view of interacting nonrelativistic flux tubes succeeded only because it allowed interactions to be replaced by alterations of the underlying particle statistics. In the case considered by Semenoff, however, it was subsequently shown [2,3] that the claimed elimination of the interaction term required an incorrect identity, thus leaving unfulfilled the desired goal of an anyon field theory.

Recently an approach fairly similar in spirit to that of Semenoff has been presented [4] which claims to circumvent the criticisms of Refs. [2] and [3] by avoiding the use of gauge fixing. It is the purpose of this Comment to point out that (a) there is a serious error in Ref. [4] which invalidates one of its principal claims and (b) independent of that error Ref. [4] does not offer a relativistic field theory of anyons.

The error (a) occurs in the last equation of Ref. [4] which essentially claims that $D_i = \partial_i + iA_i$ $(i=1,2)$ can be replaced by ∂_i if one includes a line integral of A_i in the exponential which defines "the gauge-invariant $\hat{\phi}$." However, this is true only if

$$\partial_i \int^x dx \cdot A = A_i(x) , \qquad (1)$$

an equation which is well known to be generally valid only for the case of one spatial dimension. This incorrect result of Ref. [4] then leads to the subsequent claim that "explicit dependence on the potential has been eliminated by the use of careted variables." This itself is a rather surprising statement since it has been known ever since this theory was first formulated [5] that it is a "photonless gauge theory," i.e., a model which allows the gauge fields to be written explicitly in terms of the current operator. Thus the elimination of explicit reference to the gauge fields certainly does not require a new (gauge-independent or otherwise) reformulation of the model.

It is worth noting that a definition of the derivative of a path dependent quantity by means of

$$\partial_i \chi(x,P) = \lim_{dx_i \to 0} \frac{\chi(x_i + dx_i, P') - \chi(x_i, P)}{dx_i} , \qquad (2)$$

with P' an extension of P by dx_i, does not remedy the situation. This is most easily demonstrated by the counterexample $A=(y,0)$ which yields for the right-hand side of (1) the result $\frac{1}{2}(x-x_0)$ for $i=2$ and the path choice [6] given by Eq. (10) of Ref. [4]. One would obtain the correct answer for $A_i(x)$ in this case if the dx_i term were absent in the numerator of (2) so that the entire contribution arose from the difference between the paths P' and P. However, this further modification of the derivative would then not reduce to the conventional derivative when acting on path independent quantities, thereby creating additional difficulties in the calculations described in Ref. [4].

Criticism (b) consists of the observation that even upon ignoring point (a) Ref. [4] is not a relativistic field theory of anyons. This is actually an immediate consequence of the first paragraph of this Comment which has pointed out that it is not enough to define operators which have peculiar statistics. That can, of course, always be done in any field theory. In fact it is essential that one also *eliminate* (not merely rewrite) the effect of the gauge field coupling, leaving only the anomalous statistics as a residue. In Ref. [4] a rewriting of the equations without explicit reference to the gauge field [actually an invalid claim in view of the criticism (a)] has been substituted for the indispensable criterion of the reduction of the scalar field equation to a Klein-Gordon equation. Since the latter has not been accomplished in Ref. [4], it clearly does not provide a field theory of anyons.

This work was supported by the Department of Energy Grant No. DE-FG02-91ER40685.

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Received 22 July 1992
PACS numbers: 11.15.Tk, 11.10.Ef

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[6] The path P' used in this calculation is a straight line from (x_0,y_0) to x_i+dx_i. If instead one were to use $P'=P+dx_i$ the result is $(x-x_0)$ in the example rather than $\frac{1}{2}(x-x_0)$. The effect in this case arises from the dx_i term in the numerator of Eq. (2).

