Quantum Freezing of the Fractional Quantum Hall Liquid

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We compare the free energy computed from the ground state energy and low-lying excitations of the 2D Wigner solid and the fractional quantum Hall liquid, at magnetic filling factors of $\nu = 1/3$ and $1/5$, to show that there is an experimentally accessible region of density and magnetic field where the fractional quantum Hall liquid freezes as the temperature is *raised* and then melts again at higher temperatures.

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The transition from the fractional quantum Hall state, the so-called Laughlin liquid (LL), to the electron crystal state, the so-called Wigner solid (WS), has been under active investigation for some time now [1–6]. The phase boundary between these two fascinating quantum states of matter is in principle extremely interesting and complex. Moreover, it now appears that much of the interesting parameter region can be reached in experimentally accessible samples.

For a 2D electron system consisting of carriers of mass m^* immersed in a dielectric medium ϵ with a density $n = \pi a^2$, the phase boundary between the LL and the WS, in the absence of impurities, is characterized by three dimensionless parameters. The first is the filling factor $\nu = 2\ell^2/a^2 \equiv 2E_F/\hbar\omega_c$ where $\ell = (\hbar c/eB)^{1/2}$, $E_F = \hbar^2/m^*a^2$, and $\omega_c = eB/m^*c$. The second is the dimensionless density $r_s = a/a_B$ where $a_B = \hbar^2\epsilon/m^*e^2$. In the regime where r_s is small (high density) it is convenient to think of r_s as equivalent to a Landau level mixing parameter $\lambda \equiv (e^2/\epsilon a)/\hbar\omega_c = r_s\nu/2$. Finally we need a dimensionless temperature $\Gamma \equiv (e^2/\epsilon a)/k_BT$.

Most microscopic theoretical investigations of the phase boundary between LL and WS have focused on the $\Gamma = 0$, $r_s = 0$ limits [7,8]. Very recently Price, Platzman, and He [9] have included density into a theory of the LL. They chose a variational wave function which was of the form of a Laughlin wave function multiplied by a Jastrow factor with one variational parameter and computed the energy of the liquid as a function of density. The Jastrow factor mixes in higher Landau levels, improving the liquid energy by making the LL look more like a solid.

The energy of this new correlated state was computed and compared directly with the energy of a WS at the same density. The energy of the solid was obtained from the recent variational calculation of Zhu and Louie [10]. The results of the calculation gave a good estimate of the critical density r_s^c at which the system went from LL to WS. At $\nu = 1/5$, for example, the computed crossover was at $r_s^c \approx 15$ in the absence of impurities, and at $\nu = 1/3$, the crossover was found to be at $r_s^c \approx 22$. A rough estimate of the effects of impurities, since impurities favor the solid, suggested that the solid was shifted down in energy by an amount proportional to $1/r_s$ and that the new value of r_s^c for realistic samples at, for example, $\nu = 1/5$ was $r_s^c \approx 12$. Since there is very little difference in energy between the liquid and solid at $\nu = 1/5$, impurities may shift the crossover point significantly. At $\nu = 1/3$, however, the difference is larger and impurities were seen to shift r_s^c by a smaller amount. Since good samples are readily available in this density range, experiments near such density-driven transitions are possible.

In this Letter we will quantitatively extend the analysis of the LL-WS phase boundary to finite temperatures in the neighborhood of the critical density r_s^c. The analysis will show that it should be possible to observe a new and rather interesting phase transition which occurs on the high density side ($r_s < r_s^c$) of the critical density. In particular the LL will freeze as the temperature is raised and then melt at a somewhat higher temperature. We will show that the freezing of the LL with an increase in temperature is physically similar to the behavior of liquid ^{4}He near the minimum pressure required to solidify it [11].

In the absence of impurities, the WS for $\nu \leq 1/3$ can to a very good approximation be thought of as a harmonic solid with slight corrections due to anharmonic and exchange effects [12]. The picture becomes more accurate as ν decreases. For such a harmonic solid the free energy per particle is given by

$$F^{\mathrm{WS}} = E^{\mathrm{WS}}(r_s) + T\sum_{k,\sigma} \ln(1 - e^{-\omega_{k,\sigma}/T}), \qquad (1)$$

where $\omega_{k,\sigma}$ ($\sigma = \pm$) are the phonon frequencies. Here $E^{\mathrm{WS}}(r_s)$ is the energy per particle computed, for example, by Zhu and Louie [10]. We evaluate the second term above by averaging over the Brillioun zone by the method of Cunningham [13], using phonon spectra calculated in the same way as Bonsall and Maradudin [14]. At very

low temperatures the excitation spectrum is dominated by the low-lying shear mode

$$\omega_{k,-} \approx 0.526 \left(\frac{\nu}{r_s}\frac{e^2/\epsilon}{2a_B}\right)(ka)^{3/2}. \tag{2}$$

In this case

$$F^{\mathrm{WS}} \approx E^{\mathrm{WS}} + 0.701 \left(\frac{r_s}{\nu}\frac{2a_B}{e^2/\epsilon}\right)^{4/3} T^{7/3}. \tag{3}$$

By now it is generally agreed that the excitation spectrum of the LL at a fixed density is very accurately described by the so-called single-mode approximation of Ref. [15]. More specifically, at infinite densities, the low-lying excitation is a magnetoroton branch which has a gap Δ_R at some wave vector $k_R \sim n^{1/2}$. This means that the free energy of the liquid will deviate exponentially from its zero temperature value. Thus it is clear that there is some regime where the more energetically favored liquid freezes as the temperature increases, due to the decrease in free energy (higher entropy) of the solid. This freezing happens close enough to r_s^c that the energy difference between liquid and solid is small compared to the gap in the liquid excitation spectrum. This is always possible assuming only that there is a real first-order phase transition between LL and WS.

The excitations in the neighborhood of k_R are well approximated by

$$\Delta(k) = \frac{(k-k_R)^2}{2m_R} + \Delta_R. \tag{4}$$

At low temperatures ($T \ll \Delta_R$) we need only consider the lowest-lying excitations.

Both the roton gap and the effective mass in the single-mode approximation depend only on the structure factor in the ground state, and both these quantities have been computed in the $r_s = 0$ (lowest Landau level) limit. Of course this low-lying branch will be density dependent; however, we will argue that the density dependence is small.

The single-mode approximation can be extended to arbitrary density by assuming that the magnetoroton wave function is

$$\psi_k = \frac{1}{\sqrt{N}}\bar{\bar{\rho}}_k|0\rangle, \tag{5}$$

where

$$\bar{\bar{\rho}}_k = \sum_i A_i^D(k)B_i(k). \tag{6}$$

Here $k = k_x + ik_y$,

$$A_i^D(k) = (e^{-k^* a_i^\dagger/\sqrt{2}} e^{k a_i/\sqrt{2}})_{\mathrm{diagonal}}, \tag{7}$$

and

$$B_i(k) = e^{-ik^* b_i/\sqrt{2}} e^{ikb_i^\dagger/\sqrt{2}}. \tag{8}$$

In Eqs. (6) and (7) we have separated the density operator into factors involving inter-Landau-level transitions $A_i(k)$ and taken the diagonal part so that $A_i^D(k)$ does not change the Landau-level number, and a part $B_i(k)$ which explicitly involves intra-Landau-level terms. Here a_i and b_i are the standard magnetic field harmonic oscillator ladder operators [16]. The a_i's are the inter-Landau-level operators and the b_i's are the intra-Landau-level operators.

To compute the energy of the states defined by Eq. (5) is difficult. However, when the quantity $\delta = \Delta E_{\mathrm{K.E.}}/\hbar\omega_c \ll 1$, i.e., the number of particles in higher Landau levels is small, then it is possible to show to lowest order in δ that the density dependent roton spectrum has the same analytic form as given previously,

$$\Delta(k) = \frac{\bar{f}(k)}{\bar{s}(k)}, \tag{9}$$

where

$$\begin{aligned}\bar{f}(k) &= \int \frac{d^2q}{(2\pi)^2}\frac{2\pi}{q} e^{-(|k|^2+|q|^2)/2}(e^{(q^*k-k^*q)/2}-1) \\ &\quad \times [\tilde{s}(q) - \tilde{s}(k+q)], \\ \bar{s}(k) &= s(k) - (1 - e^{-|q|^2/2}), \\ \tilde{s}(k) &= e^{|k|^2/2}\bar{s}(k),\end{aligned} \tag{10}$$

and $s(k) = \frac{1}{N}\langle 0|\rho_k^\dagger \rho_k|0\rangle$ is the new density dependent structure factor. For the variational wave function of [9], Fig. 1 shows that $\delta \ll 1$ is a good approximation. We find, for example, that near $\nu = 1/3$ at $r_s = 20$ that Δ_R is softened about 20%. For the purpose of our semiquantitative argument such changes are unimportant.

Assuming only small changes in the roton spectrum as a function of density and that only noninteracting rotons in the neighborhood of the minimum contribute ($T \ll \Delta_R$), the free energy per particle of the liquid is

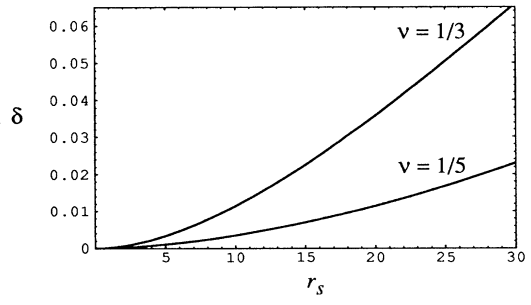

FIG. 1. The excess kinetic energy $\delta = E_{\mathrm{K.E.}}/\hbar\omega_c$ for the liquid phase. Near the melting point at $\nu = 1/5$, $\delta \approx 0.01$, and at $\nu = 1/3$, $\delta \approx 0.05$.

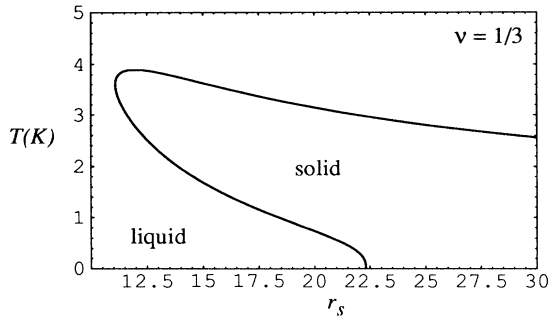

FIG. 2. The liquid-solid phase boundary at $\nu = 1/3$, where $m^* = 0.3m_e$, the mass of the bare electron. Impurity effects are small on the scale of this plot.

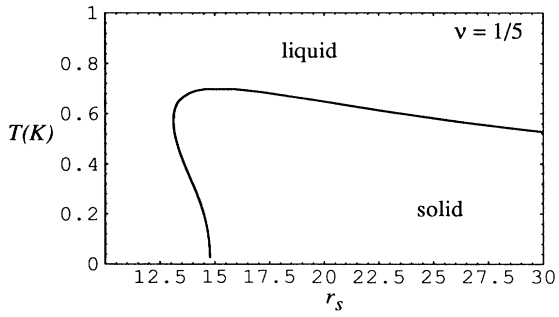

FIG. 3. The liquid-solid phase boundary at $\nu = 1/5$, where $m^* = 0.3m_e$. Impurity effects are not shown in this plot.

$$F^{\mathrm{LL}} = E^{\mathrm{LL}}(r_s) + (2\pi m_R)^{1/2}\frac{k_R\ell^2}{\nu}T^{3/2}e^{-\Delta_R/T}. \tag{11}$$

Setting the two free energies F^{LL} and F^{WS} equal gives us the phase boundary.

In Fig. 2 we have plotted this phase boundary for a hole sample with effective mass $m^* = 0.3m_e$ at a filling factor of $\nu = 1/3$. The phase boundary at $\nu = 1/3$ behaves as expected: there is a temperature region for $r_s \lesssim r_s^c$ where the liquid is stable at $T = 0$. The liquid freezes, in this case at a temperature of ~ 1 K, followed by a magnetoroton entropy triggered melting at ~ 3 K. The effects of impurities will shift the boundary to the left by a small amount in r_s, and may also push the top of the phase boundary down somewhat. At $\nu = 1/5$, shown in Fig. 3, impurities play a much larger role, since the energy difference between liquid and solid is much smaller, but we would expect the general features of the phase diagram to remain; that is, below r_s^c, the system remains a liquid at all temperatures, well above r_s^c the system is a solid at low temperatures, melting at some point probably lower than shown in Fig. 3, and just above r_s^c a narrow reentrant regime exists.

This calculation, which is based on very simple physical properties of the LL and WS phases, shows how quantum freezing comes about in this system. The actual temperatures and densities are within an experimentally accessible range. However, the effects of impurities, which are significant, can only be estimated.

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