Spin-Orbit Berry Phase in Conducting Rings

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The spin Berry phase can be observed in conductance experiments on rings of noncentrosymmetric materials. It results in destructive interference and in the shift of the Aharonov-Bohm oscillations. The spin-orbit interaction in vacuumlike Aharonov-Bohm experiments leads to the opposite sign of the magnetoresistance in comparison with the weak localization regime in disordered conductors. A time-dependent uniaxial strain results in current in a ring.

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The concept of Berry's phase [1] is applied in different areas of modern physics [2]. This topological phase arises as a result of the adiabatic variation of the external parameters. Its most simple example is the phase obtained by the spin wave function in the presence of a magnetic field. When the value of the field is constant and its direction follows adiabatically a closed trajectory the spin wave function acquires an additional phase factor besides the standard phase in the static magnetic field. The Berry phase is proportional to the solid angle subtended in a space by the magnetic field. Berry [1] also demonstrated that the Aharonov-Bohm (AB) effect [3] can be considered as an example of the adiabatic topological phase. Since this effect manifests itself in several remarkable quantum phenomena [4, 5] it is interesting to find out whether the spin topological phase also results in coherent effects.

Persistent currents in mesoscopic rings induced by the Berry phase were studied by Loss and co-workers [6]. Stern [7] demonstrated that the spin phase effect on the conductivity of the rings is similar to the influence of the Aharonov-Bohm flux effect and discussed motive forces connected with this phase. However, the manner in which the magnetic field is varied in [6, 7] leads to rather difficult experiments. Meir, Gefen, and Entin-Wohlman [8] have considered the spin-orbit scattering effect in mesoscopic systems. They found that spin-orbit interaction modifies the magnetic flux Φ dependence of the spectrum into a $\Phi \pm \delta$ dependence for two relevant spin directions (δ depends on spin-orbit scattering). Therefore, they could find the dependence of any property on spin-orbit scattering.

In this paper we demonstrate that the spin-orbit interaction in low dimensional or lowered symmetry conductors leads to topological spin phase effects in conducting rings. Even in the absence of an external magnetic field the electron spin in low dimensional structures is influenced by the momentum-dependent effective magnetic field because the electron Hamiltonian $\mathcal{H}$ includes a term linear in momentum $\mathbf{p}$,

$$\mathcal{H} = \frac{p^2}{2m} + \frac{\hbar}{2}\sum_{i,j} \sigma_i \beta_{ij} p_j, \tag{1}$$

which describes the spin-orbit splitting of the electron states at $p \neq 0$. Here m is the effective mass; σ_i are the Pauli matrices. If the electron momentum $\mathbf{p}$ subtends a closed trajectory in the momentum space during the electron motion, the effective magnetic field in Eq. (1) leads to the Berry phase effect. We demonstrate that the conductance of a quasi-one-dimensional ring is an oscillating function of the topological flux induced by the momentum-dependent magnetic field. We show that time-dependent uniaxial strain results in a motive force in the ring, similar to one discussed in [7].

We consider a quasi-one-dimensional ring of radius $\mathbf{r}$, which could be defined in the two-dimensional electron gas (2DEG) of a semiconductor heterostucture. All the symmetry aspects concerning the effective field will be discussed below. First we take the normal to the heterostructure interface $z \parallel (001)$. For a rectangular quantum well in a A_3B_5 crystal the Hamiltonian in the external magnetic field $\mathbf{B} \parallel z$ is

$$\mathcal{H} = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})_x^2 + (\mathbf{p} - \frac{e}{c}\mathbf{A})_y^2}{2m} + \frac{\hbar}{2}\beta\left[\sigma_x\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)_x - \sigma_y\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)_y\right] + \hbar\omega_s\sigma_z, \tag{2}$$

where $\mathbf{A}$ is the vector potential and ω_s is the Larmor frequency, $x \parallel (100), y \parallel (010)$. Adopting a cylindrical coordinate system and the tangential component of the vector potential $A_\varphi = \Phi/2\pi r$ we have the following Hamiltonian for the electrons in a closed ring:

$$\mathcal{H} = \hbar\omega\left(-i\frac{\partial}{\partial\varphi} - \frac{\Phi}{\Phi_0}\right)^2 + \hbar\omega_s\sigma_z + \hbar\omega_0(\sigma_x\cos\varphi - \sigma_y\sin\varphi)\left(-i\frac{\partial}{\partial\varphi} - \frac{\Phi}{\Phi_0}\right). \tag{3}$$

Here $\omega = \hbar/2mr^2$, $\omega_0 = \hbar\beta/2r$, Φ is the magnetic field flux, $\Phi_0 = \frac{2\pi\hbar c}{e}$ is the flux quantum. The eigenvalues of the Hamiltonian (3) are

$$\varepsilon_\pm = \hbar\omega(\ell^2 + \tfrac{1}{4}) \pm \hbar\sqrt{\omega_0^2(\ell^2 - \tfrac{1}{4}) + (\omega_s + \omega\ell)^2}, \tag{4}$$

where $\ell = n - \frac{1}{2} + \frac{\Phi}{\Phi_0}$, and n is an integer number. At $\ell >> 1$ the wave functions are given by

$$\Psi_+ = \exp i(n - \tfrac{1}{2})\varphi \begin{pmatrix} -\cos\frac{\theta}{2}\exp i\frac{\varphi}{2} \\ \sin\frac{\theta}{2}\exp -i\frac{\varphi}{2} \end{pmatrix}, \tag{5}$$

$$\Psi_- = \exp i(n - \tfrac{1}{2})\varphi \begin{pmatrix} \sin\frac{\theta}{2}\exp i\frac{\varphi}{2} \\ \cos\frac{\theta}{2}\exp -i\frac{\varphi}{2} \end{pmatrix}, \tag{6}$$

where $\tan\theta = -\frac{\omega_0\ell}{\omega_s+\omega\ell}$. First we consider for simplicity the case when $\Phi = 0$ whereas Zeemann splitting is present. Then Eqs. (4)–(6) show that the two waves propagating in opposite directions in a ring obtain different phases due to the effective field. The situation is similar to the usual AB effect. It is clearer from the quasiclassical description of the electron in a ring with the Hamiltonian (3). In this case the electron in the zeroth order is a rotator with the frequency $\Omega = \frac{p}{mr}$ (p is the momentum along the ring). If the electron wavelength λ is much less than the size of the circle ($\lambda \ll \pi r$) the electron motion is quasiclassical and the wave function is $\Psi(\varphi,t) = \Psi_r(\varphi,t)\chi(\varphi,t)$, where $\Psi_r(\varphi,t)$ is the quasiclassical rotator wave function and χ is a spinor. Inserting $\Psi(\varphi,t)$ into the time-dependent Schrödinger equation with the Hamiltonian (3) and taking into account the first order to $\hbar$ we have

$$i\hbar\frac{\partial\chi}{\partial t} = \boldsymbol{\sigma}\cdot\boldsymbol{\omega}_e\chi. \tag{7}$$

Here $\boldsymbol{\omega}_e = (+\hbar\beta mr\Omega\cos\Omega t, -\hbar\beta mr\Omega\sin\Omega t, \omega_s)$ is the effective Zeemann frequency for the anticounterclockwise rotator; for the counterclockwise motion Ω is to be changed by $-\Omega$. Equation (7) is the equation of motion for a spin $\frac{1}{2}$ in a magnetic field which follows a cone-shaped path, and its exact solution can easily be written. In the adiabatic situation $\omega_0, \omega_s \gg \Omega$ we see that during the period $T = 2\pi/\Omega$ the wave function, besides the dynamical phase

$$\Phi_D = \Big(\frac{mr^2\Omega^2}{2} \pm \sqrt{\omega_0^2 + \omega_s^2}\Big)\frac{2\pi}{\Omega}, \tag{8}$$

acquires also the topological Berry phase for the spin $\frac{1}{2}$,

$$\Phi_B = \pm\pi\Big(1 - \frac{\omega_s}{\sqrt{\omega_0^2 + \omega_s^2}}\Big) = \pm\pi(1 - \cos\theta), \tag{9}$$

which is exactly half of the solid angle subtended by effective magnetic field. The angle between the z direction and the cone, θ, satisfies $\cot\theta = \frac{\omega_s}{\omega_0}$.

Now consider a ring coupled to the current leads (Fig. 1). We assume that the electrons propagating in the leads are described by the Hamiltonian (2) with $p_y = 0$; they traverse the ring in the counter- or anticounterclockwise direction and the resulting transmission probability is influenced by an interference. We study the ballistic motion of electrons in the absence of scattering and thus neglect also the spin-flip processes. In this case spin-up and spin-down electrons traverse the ring independently. We assume that the electron spin is not changed while the electron passes a junction. In this situation the transmission amplitude of the ring is derived using the same procedure as was used by Buttiker, Imry, and Azbel [9].

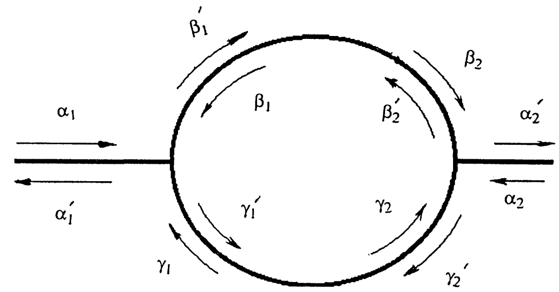

FIG. 1. Ring connected to current leads.

For simplicity, we study the perfect symmetric ring with equivalent branches. A wave of unit amplitude with "spin up" or "spin down" coming from the current lead is transmitted into the two branches with equal amplitude $\eta^{\frac{1}{2}}$ and reflected back with amplitude $\sqrt{1-2\eta}$. For $\eta = 0$, one junction completely reflects electrons back to the lead (limit of the weak coupling). For $\eta = \frac{1}{2}$, the junction is completely transparent for electrons (the strong coupling limit).

In the absence of the electron scattering the amplitudes in the upper branch, for example, are transferred according to

$$\begin{pmatrix} \beta_2 \\ \beta_2' \end{pmatrix} = \begin{pmatrix} \exp i\varphi_1 & 0 \\ 0 & \exp i\varphi_2 \end{pmatrix}\begin{pmatrix} \beta_1' \\ \beta_1 \end{pmatrix}, \tag{10}$$

where $\varphi_1 \neq \varphi_2$ are the phases to be calculated for the given energy of the transmitting particle and for the certain electron spectrum in the ring. The phase φ_1 is the phase acquired by an electron traversing the branch in the counterclockwise direction; the phase $(-1)\varphi_2$ is acquired in the course of the anticounterclockwise motion. In the absence of a magnetic field and the spin-orbit splitting of the electron states $\varphi_1 = -\varphi_2 = \pi k_0 r$. In the presence of a magnetic field, or due to the spin-orbit splitting, the phases are different in the absolute value.

Using Eq. (10) and following [9] we obtain the electron wave function of unit amplitude $\alpha_1 = 1$ [the spin-up or spin-down solution of the Hamiltonian (3) with $p_y = 0$] is transmitted to another lead with amplitude

$$\alpha_2' = \frac{\eta(e^{i\varphi_1} + e^{-i\varphi_2})(1 - e^{i(\varphi_1 - \varphi_2)})}{(\sqrt{1-2\eta}e^{i(\varphi_1-\varphi_2)} + 1)^2 - (e^{i\varphi_1} + e^{-i\varphi_2})^2 b^2}, \tag{11}$$

where $b = \frac{1}{2}(\sqrt{1-2\eta}+1)$. Equation (11) coincides with Eq. (4.24) in [9] apart from the nonessential phase factor due to the different choice of the transfer matrix in Eq. (10). The physical situation can be explained as follows. The waves which have traversed the ring in the counter- and anticounterclockwise direction acquire the phases φ_1 and $-\varphi_2$. They interfere at the second junction and the transmitted amplitude in the lead is $\eta(\exp i\varphi_1 + \exp -i\varphi_2)$. This interference is analogous to the vacuum AB effect and it vanishes in the limit $\eta \Rightarrow 0$. The multiple passages of the waves in a ring give rise to resonant states with width proportional to η and result in the conventional resonant tunneling. If the energy of the tunneling electron is equal to the eigenvalue of the closed ring the transmission reaches its maximum. There are also oscillations due to the multiple passages within one of the branches of the ring. In the strong coupling case ($\eta \simeq \frac{1}{2}$) these oscillations, as well as vacuumlike AB effect, are of order of unity.

In general, the energy of the tunneling electron E does not coincide with the eigenvalues of energy in a closed ring. Therefore the solution of the equation $E = \varepsilon_\pm$ determines noninteger numbers n [Eq. (4)]. It has four solutions which determine the phase $\kappa = \pi n$ in a semicircle for the counter- and anticounterclockwise motion of electrons with spin up and spin down. We see from Eqs. (5) and (6) that the two components of spinors acquire different phases. This can be taken into account and it is nonessential for the final results. If the Zeemann and the spin-orbit splittings are absent, we obtain two phases $\varphi_1 = \pi(k_0 r + \frac{1}{2}\Phi/\Phi_0)$ and $\varphi_2 = -\pi(k_0 r - \frac{1}{2}\Phi/\Phi_0)$, where k_0 is the wave vector of the incident wave. The interference of the waves with such phases results in the Aharonov-Bohm oscillations of the transmission probability.

Writing the equation $E = \varepsilon_\pm$ in the form

$$\begin{aligned} E &= \frac{(\hbar k_0^\pm)^2}{2m} + \hbar\sqrt{(\hbar\beta k_0^\pm)^2 + \omega_s^2} \\ &= \hbar\omega\left(\ell^2 + \tfrac{1}{4}\right) \pm \hbar\sqrt{\omega_0^2(\ell^2 - \tfrac{1}{4}) + (\omega_s + \omega\ell)^2}, \end{aligned} \tag{12}$$

and using adiabatical and quasiclassical conditions we have for the spin-up electrons

$$\begin{aligned} \varphi_s &= \pi n_s \\ &= (-1)^s \pi k_0^+ r + \frac{\pi}{2}\left(1 - \frac{\omega_s}{\sqrt{(\hbar\beta k_0^+)^2 + \omega_s^2}}\right) - \pi\frac{\Phi}{\Phi_0}. \end{aligned} \tag{13}$$

Here $k_0^\pm$ are the wave vectors of the incident electrons with spin up and spin down, which are different for the given energy; $s = 1, 2$. For the spin-down electrons, k_0^+ is to be replaced by k_0^-, and the sign of $\frac{\pi}{2}$ is to be changed to the opposite.

The phase obtained by the electron wave function in the course of traversing the hole ring, $2\varphi_1$ or $2\varphi_2$, contains the Berry phase Φ_B, described by Eq. (9), besides the standard phase $\Phi_s^\pm = 2\pi k_0^\pm r$. The phase due to the Aharonov-Bohm effect is the only magnetic field orbital effect in our system because the current leads are one dimensional.

As we mentioned above, the most pronounced effect caused by the interference exists in the strong coupling limit. Then the transmission probability of the ring for different spins $T_\pm$ reads

$$T_\pm = \frac{4\sin^2\frac{\Phi_s^\pm}{2}}{\tan^2\frac{\Phi_t^\pm}{2} + 4\sin^2\frac{\Phi_s^\pm}{2}}, \tag{14}$$

where the topological phase $\Phi_t^\pm = \Phi_B^\pm - 2\pi\frac{\Phi}{\Phi_0}$ includes the spin Berry phase $\Phi_B^\pm$ and the Aharonov-Bohm phase $2\pi\frac{\Phi}{\Phi_0}$. If the spin-orbit splitting is absent and the Berry phase is $\Phi_B = 0$, we have [9]

$$T_\pm = 4\sin^2\frac{\Phi_s^\pm}{2} \Bigg/ \left(\tan^2\frac{2\pi\Phi}{\Phi_0} + 4\sin^2\frac{\Phi_s^\pm}{2}\right). \tag{15}$$

We see that the transparency is decreased by the magnetic field at small magnetic flux. This sign of the effect corresponds to positive magnetoresistance.

If the spin-orbit interaction is essential then $\Phi_B \simeq \pi - \delta\Phi(B)$, and at small fields $\delta\Phi(B) \ll 1$ we get $T_\pm \propto B^2$. Thus, the transparency of a ring is increased by the magnetic field (negative magnetoresistance).

We see that the signs of the effect are opposite to the signs of magnetoresistance in the weak localization effect. The reason for this difference is the following. If the spin-orbit splitting is absent, the interference of the waves in a ring has a constructive character, and at $B = 0$ the transmission probability is unity. The magnetic field suppresses the interference and the magnetoresistance is positive. In the weak localization regime this suppression means that the probability to find a particle in the initial point is lowered, i.e., the diffusion coefficient increases, and the effect is opposite (negative magnetoresistance).

In the presence of the spin-orbit interaction the interference is destructive; at $B = 0$, $\Phi_B = \pi$ and the transparency of the ring is equal to zero. In this case the magnetic field increases the conductivity and the magnetoresistance of the ring is negative. In weak localization the suppression of the destructive interference by the magnetic field leads to positive magnetoresistance [10].

It can be seen that while the spin Berry phase takes values within the interval $[0, \pi]$, the phase Φ_t is not limited. If $\Phi_t = \pi(2m+1), m$ is an integer number, and the conductance of the ring is zero. The correspondent magnetic fields depend on the strength of the spin-orbit interaction, governed by the Fermi energy E_F. When $\Phi_t = 2\pi m$, the conductance of a ring reaches its maximum. The spin Berry phase leads to the shifts of the minima and maxima of the conductance. These shifts can be tuned by tuning the spin-orbit splitting.

We would like to mention that magnetic fields in extrema are different for the spin-up and spin-down tunneling electrons. The conductance is determined by both the spin-up and spin-down contributions and consequently the most pronounced effect takes place when electrons are polarized and only one kind of them gives rise to the conductance.

Another type of oscillation of the conductance, which is clearly seen from Eq. (14), is determined by the phase Φ_s. We attribute this effect to the location of tunneling electrons in one of the branches of a ring. The phase obtained by the electron wave function during a cycle of motion in a branch between two contacts is equal to Φ_s. If $\cos\Phi_t \neq 1$ and $\Phi_s = \pi(2m+1)$ the conductance reaches maximum; if $\Phi_s = 2\pi m$ the conductance is zero.

Consider the experimental situation. The term linear in the electron momentum in the electron Hamiltonian is allowed by symmetry [11] in asymmetric quantum wells, in rectangular quantum wells in noncentrosymmetric materials, and in tellurium type and wurzite type bulk crystals. It can also be induced by uniaxial strain in A_3B_5 crystals, where an effective magnetic field cubic in the electron momentum is present in the bulk material. The latter effective field can also contribute to the spin Berry phase.

We have found a rather promising geometry for experiment. If the normal to the 2DEG plane in a A_3B_5 crystal structure is directed along $z' \parallel (111)$ the Hamiltonian is

$$\mathcal{H} = \frac{\mathbf{p}_\perp^2}{2m} + \hbar(b_1 + b_2\epsilon)(\boldsymbol{\sigma} \times \mathbf{p})_{z'} + \hbar\omega_s\sigma_{z'} \tag{16}$$

where $\mathbf{p}_\perp \perp z'$, ϵ is the deformation perpendicular to the plane. The coefficient b_1 is determined by the Rashba term [12] as well as by the size quantization in noncentrosymmetric crystals; the term proportional to b_2 is caused by the uniaxial strain. The cubic-in-momentum effective magnetic field is absent. The effective field can be varied by the external strain, and could even be made equal to zero. If a ring is confined in the 2DEG plane, the total magnetic field follows a cone-shaped path in the course of the electron motion, and all the effects we discussed for the geometry with $z \parallel (001)$ are present.

As was shown by Stern [7], the time-dependent spin phase induces a motive force and creates a current in a ring according to Ohm's law. In the geometry of experiment which we propose the motive force can be induced by the time-dependent external strain. The usual acoustoelectric effect does not contribute to the current and the entire effect is caused by the time-dependent flux.

It should be noted that the coefficient b_1 in Eq. (16) can also be tuned by a variation of the external electric field applied perpendicular to the plane of the ring. The reason is the variation of the quantum well profile.

The estimations show that the most promising material for the observation of the spin Berry phase is InAs due to the large value of the electron g factor ($g = 15$), and a sizable value of the spin-orbit splitting of the electron states.

In summary, we have shown that the spin Berry phase can be observed in the conductance experiments on rings in noncentrosymmetric materials. The spin Berry phase results in destructive interference. We found that the Berry effect produces a phase shift of the Aharonov-Bohm oscillations. Spin-orbit effects in weak localization can also be interpreted as a result of the spin Berry phase, but the sign of magnetoresistance in rings in the ballistic case is opposite to the sign of magnetoresistance structures in the weak localization regime.

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