Destabilization of the Internal Kink by Energetic Circulating Ions

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A stability analysis is carried out for the $m=1$, $n=1$ internal kink mode, in the presence of energetic circulating particles. It is found that, including the effect of finite radial particle-orbit excursion, the $m=1$ internal kink mode is strongly destabilized by the resonance interaction with the energetic passing particles. Such an instability could explain the experimental observations of the "fishbone" oscillations during tangential neutral beam injection [W. W. Heidbrink *et al.*, Phys. Rev. Lett. **57**, 835 (1986)].

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The study of the fishbone instability is of particular importance to magnetic confinement fusion. Recent tokamak experiments [1-4] have shown that a mode with dominant poloidal and toroidal wave numbers $m=1$ and $n=1$ is strongly destabilized by the injection of fast neutrals. Such an instability (denoted as "fishbone oscillation") is correlated to particle bursts corresponding to the loss of energetic beam ions. The current view [5-7] is that a fishbone oscillation is an internal kink destabilized by the resonant interaction with the energetic ions trapped inside the $q=1$ surface ($q \simeq rB_\phi/RB_\theta$ is the safety factor). Such an interpretation fails to explain the observation of fishbone oscillations during tangential injection [2], where neutral beams are injected parallel to the magnetic axis and trapped particles are virtually absent. In this Letter, we investigate the stability of the internal kink in the presence of an energetic circulating ion population. We show that by retaining the effect of finite orbit excursion and the bulk ion diamagnetic drift frequency, the circulating beam particles can indeed destabilize the $m=1$ internal kink mode. The growth rate of the instability is comparable to the one of the trapped-particle-induced fishbone mode and in general agreement with the experimental data.

The stability analysis essentially follows the approach adopted in previous articles [7]. We consider a plasma consisting of bulk ions and electrons and energetic ion beams injected parallel to the magnetic field. All species are treated kinetically and, for simplicity, we assume that the beam particles are purely circulating ($v_\perp \ll v_\parallel$), the bulk ions and electrons are isotropic, and the ions are electrostatically confined. According to these assumptions, we choose the following form of the equilibrium distribution functions: $f_{i0} = F_i(\varepsilon)$, $f_{e0} = F_e(\varepsilon, P_\phi)$, and $f_{b0} = F_b(\varepsilon, P_\phi, \mu)$, where $\varepsilon = mv^2/2 + Ze\Phi_0$, $P_\phi = mRv_\phi + Ze\Psi/c$, and $\mu \simeq mv_\perp^2/2B$ (with $\mu B \ll \varepsilon$ for the beam particles). Furthermore, the structure of the instability is mainly that of an ideal magnetohydrodynamic (MHD) mode ($E_\parallel = 0$). In order to carry out the analysis, we introduce the inverse aspect ratio $\epsilon = r_s/R \ll 1$ (r_s is the radius of the $q=1$ surface) and the Ohmic tokamak expansion $\beta \sim \epsilon^2$, $q \sim 1$, $\beta_\theta \sim 1$. We consider a small population of energetic particles ($n_b \ll n_i$ and $T_b \gg T_i$). For such energetic particles, we retain the effect of finite radial orbit excursion by writing the particle trajectory as $r(t) = \bar{r} - \Delta_b \cos\theta(t)$, with $\bar{r}$ the constant of motion and $\Delta_b = q(\bar{r}) v_\parallel/\Omega_{cb}$ (Ω_{cb} is the beam cyclotron frequency). To capture the essential physics and to model the experiment of Ref. [2], we order $\beta_b/\beta_i \sim \epsilon$, $T_i/T_b \sim (\Delta_b/r_s)^2 \sim \epsilon^{3/2}$. We introduce the $E \times B$ displacement $\boldsymbol{\xi}_\perp \equiv ic \times (\mathbf{E}_1 \times \mathbf{B}_0)/\omega B^2$, and we consider a perturbation dominated by a large $m=1$, $n=1$ component and a small $m=2$ sideband,

$$\xi = \xi_{r1} \exp[-i\theta - i\phi - i\omega t] + \epsilon \xi_{r2} \exp[-2i\theta - i\phi - i\omega t] + O(\epsilon^2) .$$

Retaining the bulk ion finite Larmor radius effects, the eigenvalue equation describing the linear evolution of the radial displacement for the $m=1$ internal kink mode in toroidal geometry can be written in the following form:

$$\frac{1}{r}\frac{d}{dr} r^3 [-4\pi\rho_i \omega(\omega + \omega_i^*) + F^2] \frac{d\xi_r}{dr} = [g(r) + h(r)]\xi_r , \tag{1}$$

where $F \equiv -B_\theta(1-q)/r$, $\omega_i^* = cT_i/Z_i e B_0 r_{pi} r_s$ is the bulk ion-diamagnetic-drift frequency, $g(r) \sim \epsilon^2 F^2$ represents the fluid potential energy given in Ref. [8], and $h(r)$ represents the kinetic potential energy

$$h(r) = \frac{2ir}{\xi_r} \int_{-\pi}^{\pi} e^{i\theta + i\phi + i\omega t} \mathbf{b} \times \boldsymbol{\kappa} \cdot \nabla \sum_j (p_{j\perp 1}^K + p_{j\parallel 1}^K) d\theta , \tag{2}$$

$$p_{j(\perp,\parallel)1}^K = -im_j^2 \int d\mathbf{v} \left(\frac{v_\perp^2}{2}, v_\parallel^2 \right) \left(\omega \frac{\partial F_j}{\partial \varepsilon} - \frac{\partial F_j}{\partial P_\phi} \right) \hat{s}_j , \tag{3}$$

$$\hat{s}_j \equiv -\int_{-\infty}^{t} (v_\parallel^2 + v_\perp^2/2) \boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp dt' . \tag{4}$$

The quantity $\mathbf{b}$ is the unit vector in the direction of the magnetic field and $\boldsymbol{\kappa} = \mathbf{b} \cdot \nabla \mathbf{b}$ is the magnetic field line curvature. The structure of the ideal internal $m=1$ mode is characterized by the presence of a transition layer of thickness $\delta \sim \epsilon^2 r_s$ ("inner" region), centered around the $q=1$ surface, where the inertial effects are important. Away from this layer ("outer" region), the plasma inertia can be neglected. A dispersion relation can be derived by matching the solution in the transition layer to the outer solution. We first solve the eigenvalue equation in the outer region, and, using standard techniques, we derive the asymptotic behavior of the eigenfunction in proximity to the $q=1$ surface,

$$\xi_r(r \simeq r_s) = \xi_0 \left[H(r_s - r) + \frac{\delta W_F + \delta W_K}{\pi x} \right], \tag{5}$$

where

$$\delta W_F = -\pi \int_0^{r_2} r g(r) dr / (B_\theta s)^2_{r_s},$$

$$\delta W_K = -\pi \int_0^{r_s} r h(r) dr / (B_\theta s)^2_{r_s},$$

$x \equiv (r - r_s)/r_s$, and $H(x)$ is the Heaviside unit step function. The quantity δW_F is related to the negative of the ideal MHD energy functional δW_{MHD}. The effect of the resonant interaction between the wave and the beam particles is included in the total kinetic potential energy δW_K through the orbit integral $\hat{s}_j$ [Eq. (4)]. The resonance condition for circulating particles is simply $\omega - k_{\parallel m'}(\bar{r}) v_\parallel = 0$, where $k_{\parallel m'}(\bar{r}) = [m'/q(\bar{r}) - 1]/R_0$ $(m'=0,1,2)$. For the $m=1$ internal kink mode, $v_\parallel/R_0 \gg \omega_r \sim \omega_i^*$ for each particle species, and strong resonance occurs only in the neighborhood of the $q=1$ surface $[k_\parallel(\bar{r} \simeq r_s) \simeq 0]$. It is easy to show that the resonant condition is only satisfied by the beam particles localized within the region $r_s - \Delta_b < r < r_s + \Delta_b$. Notice that the main resonant contribution is provided by the interaction with the poloidal electric field inside the $q=1$ surface and not by the large radial electric field in the inertial layer. The resonant interaction within the layer is virtually absent because the radial component of the particle magnetic drift velocity changes sign over a complete period of the particle orbits. It follows that there is no net interaction with the radial electric field. Furthermore, the orbit excursion Δ_b for an energetic beam particle is much larger than the inner layer width; therefore very little time is allowed for such an interaction. In contrast, the particles transiting in the region $r_s - \Delta_b < r < r_s + \Delta_b$ experience the effect of the electric field when they penetrate inside the $q=1$ region ($\xi_r \simeq 0$ for $q > 1$). Their poloidal drift velocity inside the $q=1$ surface is in phase with the poloidal electric field and the resonance condition is satisfied for $\bar{r} = r_s + \omega/(k_\parallel)'_{r_s} v_\parallel$. For a quantitative estimate of the resonant interaction, we write the beam particle orbits in the following form:

$$\theta(t') = \chi(t') + \alpha \sin\chi(t') + O(\epsilon \sin\chi), \tag{6}$$

$$\phi(t') - \phi(t) = \frac{v_\parallel}{R_0}(t' - t) + O(\epsilon \sin\chi), \tag{7}$$

$$r(t') = \bar{r} - \Delta_b \cos\theta(t') + O(\epsilon^{3/2}), \tag{8}$$

where

$$\chi(t') - \chi(t) = [-v_\parallel / q(\bar{r}) R_0 + \omega_E](t' - t),$$

$\alpha = (s+1)\Delta_b/\bar{r}$, and $\bar{r} = r + \Delta_b \cos\theta$. The quantity ω_E represents the $\mathbf{E}_0 \times \mathbf{B}_0$ drift frequency $[\omega_E \equiv c\mathbf{E}_0 \times \mathbf{B}_0 \cdot \nabla\theta / B^2]$ and $s = -rq'/q$ is the magnetic shear. The lowest order resonant component of the orbit integral can be written in the following form:

$$\hat{s}_b = \frac{v_\parallel^2}{R_0} \frac{\xi_0 (1/2\pi) \int_{-\pi}^{\pi} \cos\chi' H(r_s - \bar{r} + \Delta_b \cos\chi') d\chi'}{-i[\omega + \omega_E - k_\parallel(\bar{r}) v_\parallel]} e^{-i\chi - i\phi - i\omega t}. \tag{9}$$

Equation (15) can be used to evaluate the kinetic potential energy δW_K. For a purely circulating beam particle population, the equilibrium can be approximated by a slowing down distribution function,

$$F_b = \frac{\sqrt{2} m_b^{3/2}}{\pi \varepsilon_I} p_b(r) \frac{H(\varepsilon_I - \varepsilon)\delta(\mu B_0/\varepsilon)}{\varepsilon_0^{3/2} + \varepsilon^{3/2}} \left[\frac{1}{2}(1 - \sigma) + \sigma H(\pm v_\parallel) \right], \tag{10}$$

where $p_b(r)$ is the beam particle energy density $[p_b = \int d\mathbf{v}\, \varepsilon F_b]$. The parameter σ can be adjusted according to the kind of injection; $\sigma = 0$ for a balanced beam and $\sigma = 1$ for a beam injected in the $\pm B_0$ directions. The paramenter ε_0 is the low-energy transition to the bulk plasma and ε_I is the injection energy. Typically, $\varepsilon_I \gg \varepsilon_0$.

Combining Eqs. (2), (3), and (9) with the assumption that $\omega \sim \omega_i^* \ll (\partial F_b/\partial P_\phi)/(\partial F_b/\partial \varepsilon)$, a short calculation shows that the function $h(r)$ of Eq. (2) has the form

$$h(r) = -\frac{8}{\pi} \frac{1}{R_0^2} \int d\mathbf{v}\, \varepsilon^2 \frac{\partial F_b}{\partial P_\phi} \frac{\Delta_b}{|\Delta_b|} \int_{-\pi}^{\pi} \cos\theta \frac{H(1 - |z|)\sqrt{1 - z^2}}{\omega + \omega_E - k_\parallel(r_s + z\Delta_b) v_\parallel} d\theta, \tag{11}$$

where $z = [\bar{r}(r,\theta) - r_s]/\Delta_b$. The kinetic potential energy δW_K can be easily derived by inverting the order of integration $\int dr \int d\theta \int d\mathbf{v} = \int d\mathbf{v} \int d\theta \int dr$ and using $dr \simeq \Delta_b dz$. For $\omega_i \ll (\omega_r + \omega_E)$ and neglecting the real part of $\delta W_K [\mathrm{Re}(\delta W_K) \ll \delta W_F]$, the result is

$$\delta W_K \simeq -i \frac{2}{3} \frac{r_s}{R_0} \left[\frac{\Delta_b^{\mathrm{inj}}}{r_{pb}} \frac{\beta_{b\theta}}{s^3} \right]_{r_s} [1 + O(\epsilon^{3/4})], \tag{12}$$

where $\beta_{b\theta}=8\pi p_b/B_\theta^2$ is the beam poloidal beta and $\Delta_b^{\rm inj}=qc(2m_b\varepsilon_I)^{1/2}/Z_beB$ is the particle-orbit excursion at the injection energy. The quantity $r_{pb}=-[dp_b/p_bdr]^{-1}$ is the beam pressure gradient scale length.

In the inner layer of thickness $\delta\sim\epsilon^2 r_s$, the eigenvalue equation [Eq. (1)] can be simplified, yielding

$$\frac{d}{dr}\left\{r^3[-4\pi\rho_i\omega(\omega+\omega_i^*)+F^2]\frac{d\xi_r}{dr}\right\}=0\,. \qquad (13)$$

The solution of Eq. (13) leads to the well-known expression for the eigenfunction,

$$\xi_r(r\simeq r_s)=\xi_0\left[\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\left(\frac{x}{[-\hat\omega(\hat\omega+\hat\omega_i^*)]^{1/2}}\right)\right], \qquad (14)$$

where $\hat\omega=\omega R/sv_A$ and $\hat\omega_i^*=\omega_i^*R/sv_A$. The dispersion relation can be derived by matching the inner and outer solution for $r\rightarrow r_s$, yielding

$$[-\hat\omega(\hat\omega+\hat\omega_i^*)]^{1/2}=(\delta W_F+\delta W_K)/\mathrm{sgn}\{\mathrm{Re}[-\hat\omega(\hat\omega+\hat\omega_i^*)]^{1/2}\}\,. \qquad (15)$$

The roots of this equation exist only for δW_F positive. In the limit of $\omega_i\rightarrow 0$ and for $0<\delta W_F<\hat\omega_i^*/2$, the dispersion relation gives two finite frequency roots,

$$\hat\omega_r=-\tfrac{1}{2}\{\hat\omega_i^*\pm[(\hat\omega_i^*)^2-4\delta W_F^2]^{1/2}\}\,. \qquad (16)$$

The two frequencies characterize the well-known low and high frequency branch of the internal kink mode [9] that are stable in the absence of resonant particles. The imaginary part of the dispersion relation [Eq. (15)], shows that the fishbone branch, with the "$-$" sign, is driven unstable by the resonant interaction with the beam particles. The growth rate of this instability can be approximated by the following expression,

$$\gamma=\frac{2v_A/3R}{[-1+(\hat\omega_i^*)^2/4\delta W_F^2]^{1/2}}\frac{r_s}{R_0}\left[\frac{\Delta_b^{\rm inj}}{r_{pb}}\frac{\beta_{b\theta}}{R^2}\right]_{r_s}. \qquad (17)$$

Observe that the instability starts growing when the plasma has approached the condition for ideal marginal stability of the internal kink mode ($\delta W_F\simeq 0$). In proximity to the ideal marginal stability, a convenient expression for the growth rate can be written in the following form,

$$\frac{\gamma}{\Omega_{ci}}=\frac{8}{3}\frac{r_s}{R_0}\left[\frac{r_{pi}}{r_{pb}}\frac{\beta_b}{\beta_i}\frac{\Delta_b^{\rm inj}}{sr_s}\right]_{r_s}\delta W_F\,, \qquad (18)$$

where β_i and β_b are the toroidal bulk ion and beam betas, and r_{pi} is the bulk ion-pressure gradient scale length. Since $\delta W_F\sim\beta_i$, $r_{pi}\sim r_{pb}$, and $\Omega_{ci}\sim\Omega_{cb}$, Eq. (18) shows that the scaling of the growth rate is $\gamma\sim\beta_b v_{\parallel b}/R_0$. For the following equilibrium parameters (typical of a tokamak such as PBX), $r_s/R_0\simeq 1/9$, $\beta_b\simeq 0.1\beta_i$, $s\simeq 0.4$, $\varepsilon_I\simeq 44$ keV, $\Delta_b^{\rm inj}/r_s\simeq 0.4$, $r_{pi}/r_{pb}\simeq 1.5$, $m_i=2m_p$, $B_\phi\simeq 0.84$ T, and $\delta W_F=0.005$, Eq. (18) yields $\gamma\approx 10^4$ sec^{-1}, which corresponds to a growth time of approximately 100 μsec. This result is in good agreement with the experimental observations of Ref. [2]. The expression of the growth rate for the circulating-particle-driven fishbone mode [Eq. (18)] has been compared with the one of the trapped-particle-driven fishbone mode given in Ref. [7]. For hydrogen isotope beam particles, the result is

$$\frac{\gamma^{\rm circ}}{\gamma^{\rm trap}}=\frac{4}{3\pi^2}\left[\frac{r_{pi}}{r_{pb}}\frac{\Delta_b^{\rm inj}}{sR_0}\frac{\varepsilon_I}{T_i}\frac{\beta_b^{\rm circ}}{\bar\beta_b^{\rm trap}}\right]_{r_s}, \qquad (19)$$

where $\bar\beta_b^{\rm trap}$ is the trapped particle beta, averaged inside the $q=1$ surface. For PBX parameters and for $\beta_b^{\rm circ}\simeq\beta_b^{\rm trap}$, it is readily found that $\gamma^{\rm circ}\approx\gamma^{\rm trap}$, in agreement with the experimental observations [2].

We have shown that the $m=1$ internal kink mode can be strongly destabilized by energetic circulating ions and that the growth rate of the instability is about equal to the one of the trapped-particle-induced fishbone mode. In a burning plasma, the circulating alpha particles with large radial orbit excursion could also destabilize the internal kink causing a substantial loss of energetic particles. The resonant interaction of circulating alphas with the internal kink needs further investigation to determine how detrimental its effect can be on alpha particle confinement.

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