Spatiotemporal Intermittency in the Faraday Experiment

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The spectral fluctuations of random capillary wave fields appear to be intermittent. This intermittency is related to local creation and annihilation of coherent structures: ordered regions that have definite symmetry. Both the occurrence of non-Gaussian fluctuations and the existence of (quasi) long-range order question the applicability of a thermodynamic description of spatiotemporal chaos in two dimensions.

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The Faraday experiment is the parametric excitation of ripples on the surface of a vertically oscillated fluid [1]. We discuss the regime where the wavelength λ of the ripples is much smaller than the linear size L of the container and where the state of the surface becomes independent of the shape of the container boundary. Theory and experiment agree that for increasing amplitudes A a flat surface yields a stationary square wave field at a critical amplitude A_c [2]. At larger amplitudes the square pattern becomes time dependent and loses its spatial coherence. The onset of spatiotemporal disorder is at a critical reduced amplitude $\epsilon=\epsilon_c$, $\epsilon=(A-A_c)/A_c$. The resulting pattern, being disordered in both space and time, bears a striking resemblance to hydrodynamic turbulence and the question is whether this resemblance could be made quantitative. In this Letter we show for the first time that spectral fluctuations of the pattern are, in fact, intermittent, much as are the fluctuations of the velocity field in turbulence.

Spatiotemporal chaos has been studied in one-dimensional convection experiments that have suggested the applicability of a thermodynamic description [3]. The idea is that the local dynamics provides enough randomness so that larger scales (i.e., scales larger than the coherence length ξ but much smaller than the size of the system L) that depend on data from many coherence lengths exhibit simple statistical properties [4]. Another key observation both in simulations of amplitude equations in one dimension [5] and in 1D convection experiments [6] has been that the transition to spatiotemporal chaos follows the scenario of directed percolation. However, the value of the critical exponents found was not universal.

Disordered Faraday crispations were reported by Ezerskii, Korotin, and Rabinovich [7] and analyzed quantitatively by Tuffilaro and Gollub [8]. The measured temporal and spatial correlation functions showed a critical slowing down of the surface characteristic frequency at $\epsilon=\epsilon_c$ that was accompanied by a sudden loss of translational and orientational order. A further study by them has explored the use of spectral fluctuations to characterize the random surface [9].

An attractive property of the Faraday experiment is its short time scale. At the transition point it is seconds rather than hours in convection experiments. A profound problem, however, is the nonuniformity of the amplitude ϵ over the fluid surface. Experiments at large values of L/λ have to be done at frequencies of the order of 200 Hz. At these frequencies it is a true challenge to suppress resonances of the mechanical structure that supports the fluid layer and transduces the vibrations of the exciter. Our experiment has a $L=130$ mm diam container that is filled to a level of 10 mm with silicon oil [10]. The container is mounted on a conical structure that transduces the force from a Bruel and Kjaer 4808 vibration exciter. Both the bottom and the top are 5 mm thick glass plates. The structure was designed to suppress parasitic vibrations. The amplitude is measured interferometrically with a relative accuracy of 2×10^{-4}. By arranging the interferometer in a differential fashion, amplitude differences between two opposing points on the perimeter of the container could be measured. At our excitation frequency of $\Omega=160$ Hz, the distortion of the container is less than 1% [11]. This number, therefore, imposes a limit on the homogeneity of the control parameter in our experiment. Only in a few other cases in the literature has a number for the amplitude inhomogeneity been mentioned; the smallest number quoted there is 2% [12].

The temperature of the working fluid is stabilized at 294.00 ± 0.03 K; the frequency is constant to 1 part in 10^6. The surface waves, with $\lambda=2.83$ mm, are visualized by shining a parallel beam of light through the transparent bottom plate and observing the wave image on a diffusing screen attached to the top glass plate of the container. The image is registered by a home-made charge-coupled-device (CCD) line camera that is tightly synchronized with the driving frequency so that images are taken at exactly equal wave phase. The 512 pixels of the camera are illuminated for one quarter of the wave period; the intensity of the signal is digitized with 12 bits.

The scenario that leads to a disordered surface state starts with a stationary square pattern. This pattern becomes time dependent at $\epsilon=0.09$ through nucleation of defects. At a slightly larger value of the control parameter ($\epsilon=0.13$) a triangular state is born that consists of three running waves and is necessarily time dependent. The power spectrum develops a series of peaks at multiples of 0.9 Hz on a sloping background, but there is no low-dimensional chaos. Finally, at $\epsilon=0.25$ this triangular state gives way to a disordered surface. The observed scenario is in accord with that found by Christiansen, Al-

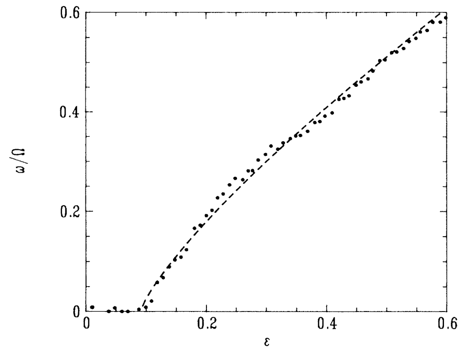

FIG. 1. Dots: characteristic frequency of Faraday crispations as a function of reduced driving amplitude ϵ. A regular triangular pattern exists in the interval [0.13, 0.25]. Dashed line: fit $\omega \simeq (\epsilon-\epsilon_c)^\beta$ with $\epsilon_c=0.09$ and $\beta=0.8$.

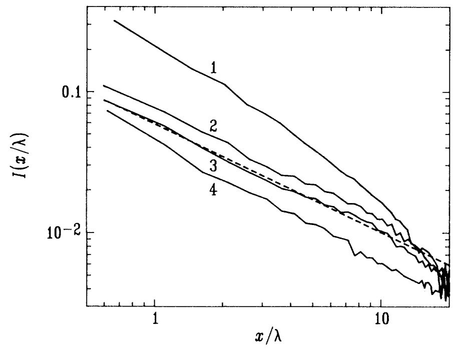

FIG. 2. Full lines: mutual information I as a function of distance x/λ, where λ is the crispation wavelength. The reduced amplitude ϵ is 0.25, 0.30, 0.35, and 0.50 for curves 1 through 4, respectively. $I(x/\lambda)$ oscillates with period $\frac{1}{2}$; the line connects its local maxima. Not shown is the rapid decay of I (over a factor 25 for curve 3) in the first half wavelength. At $x/\lambda>10$ the noise in I gives rise to spurious maxima. Dashed line: a fit of $I(x)$ by $x^{-0.8}$.

strom, and Levinsen [13]. It differs from that observed in a square container where the intervening triangular state appears to be absent and the square pattern becomes chaotic directly [8]. As the control parameter ϵ is increased and the fluid surface moves through the above sketched scenario, the characteristic fluctuation frequency ω increases. The frequency ω shown in Fig. 1 was determined from the decay of information functions [14] computed from time series of 64000 point samples. At the critical amplitude $\epsilon=\epsilon_c$ $(=0.09)$ the fluctuation frequency rises as $\omega \simeq (\epsilon-\epsilon_c)^\beta$, where the exponent β $(=0.8)$ is slightly but significantly smaller than 1. Because Fig. 1 is the result of several scans up and down the ϵ scale, it demonstrates the absence of hysteresis. Quite recently it has been suggested that the order-disorder transition is a first-order phase transition [15], this is, however, clearly contradicted by Fig. 1.

Correlation functions are not independent of the distribution functions of the quantities whose correlation is measured. In our case the intensity $u(x)$ in the wave image is a strongly nonlinear function of the actual wave heights; it is determined by the focusing and defocusing of incident light by the surface ripples. This causes highly skewed probability distribution functions of the light intensity measured in a point. An estimate of the surface coherence that is not dependent on the shape of the distribution function of $u(x)$ is the mutual information I [14]. It is directly expressed in terms of the joint probability $P(u_1,u_2)$ that two points at position x_1,x_2 have their intensities equal to u_1 and u_2, respectively, as $I(x_1,x_2) = \int du_1 du_2 P(u_1,u_2)\log_2[P(u_1,u_2)/P(u_1)P(u_2)]$. Figure 2 shows the line-averaged spatial information function $I(x)=[1/(L-x)]\int^{L-x}dx' I(x',x'+x)$. It is based on 32000 line samples of the surface image. At the lowest value of ϵ, where the surface is slowest, they were taken with a sampling frequency of 2 Hz. Figure 2 shows the height of the wave maxima in $I(x)$, $x=i\lambda/2$, $i=1,2,\ldots$. There is a rapid (exponential) decay over the first $\lambda/2$ where $I(x)$ drops by approximately a factor of 20. The information functions, which we view as a better measure of the surface coherence, clearly illustrate the existence of long-range correlations, even at the largest ϵ value $(\epsilon=0.50)$ [16]. Except at the lowest value of ϵ, the information function has a conspicuous algebraic decay: $I(x)\simeq x^{-\delta}$, with an exponent δ $(=0.8)$. Given the limited dynamical range of the decay in Fig. 2, it is the existence of long-ranged correlations rather than the precise shape of $I(x)$ that we want to emphasize.

As the basic surface structure is wavelike, it is most appropriate to study its stochastic properties in the spectral domain. Figure 3 shows the average wave spectrum at $\epsilon=0.45$. It was obtained from an average over $n=32000$, 512-pixel line samples $u_j(x)$, $j=1,n$ whose spectra, $u_j(k)=\int e^{-ikx}u_j(x)H(x)dx$, were determined using a standard Hanning window $H(x)$ for each line sample. The average spectrum is $\langle|u(k)|^2\rangle=(1/n)\sum_{j=1}^{n}|u_j(k)|^2$. For both the spectrum and the correlation function the individual line images $u_i(x)$ were normalized by subtract-

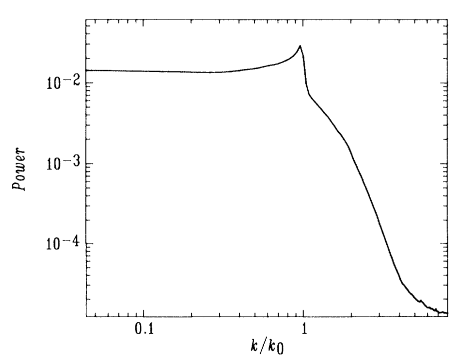

FIG. 3. Power spectrum $\langle|u(k)|^2\rangle$ of surface fluctuations measured on a line at $\epsilon=0.45$.

ing the long-time averaged mean $\langle u(x)\rangle$ (which was featureless) and dividing by the rms fluctuation $[\langle u(x)^2\rangle - \langle u(x)\rangle^2]^{1/2}$. The spectrum bears a strong resemblance to one computed from a numerical simulation of spatiotemporal chaos in the Kuramoto-Shivasinsky equation [17]. It is flat at small wave numbers and drops sharply at $k=k_0$. The flat part of the spectrum might be taken as a sign for equipartition of energy.

Of much more interest than the average spectrum is the statistical nature of the fluctuations of the instantaneous spatial spectra. Small wave vectors are an average over many coherence lengths and therefore provide a coarse graining. The spectral fluctuations at these wave numbers can reveal whether the coarse-grained system moves close to equilibrium. In that case, the spectral fluctuations would be Gaussian. In order to show the deviation of Gaussian statistics we measure the normalized fourth moment (the kurtosis) of the spectrum. It is defined as $G_4(k)=\langle[\mathrm{Re}\,u(k)]^4\rangle/\langle[\mathrm{Re}\,u(k)]^2\rangle^2$, and is identically equal to 3 for Gaussian wave signals $\mathrm{Re}\,u(k)$.

Figure 4 shows $G_4(k)$ for a range of ϵ values. At the lowest value of ϵ ($\epsilon=0.25$), there are large deviations from Gaussianity at the $1/\sqrt{3}$ wave number of the triangular pattern and its higher harmonics. We emphatically notice that the pattern at this value of ϵ is chaotic and has a decaying correlation function (see Fig. 2). At larger values of the amplitude the sharp peaks in the flatness merge into a broad feature centered at $k/k_0=1/\sqrt{2}$. These k values are related to macroscopic patches of a definite (triangular and square) symmetry but random orientation that are seen to briefly exist in the disordered state. Incidentally, the measured third-order moment G_3 is indistinguishable from zero.

Those coherent structures strongly affect the statistics of the spectral fluctuations. Figure 5 shows the probability distribution function (PDF) of $\mathrm{Re}\,u_i(k)$ at $k/k_0=0.31$ and 0.70, respectively. While the PDF is Gaussian at $k/k_0=0.31$, it has wide intermittent tails at $k/k_0=0.70$. We also believe that those structures are responsible for the long tails of the coherence functions (Fig. 2). Of course, the deviations from Gaussian behavior are consistent with the long-range correlations of the surface fluctuations. Our results should be viewed in light of a recent discussion whether chaos in extended systems provides only *local* disorder [15]. Both the observed deviation from Gaussianity and the existence of long-range order suggest that chaos in the system is in fact *not* local.

It is well known that in hydrodynamic turbulence intermittency shows in non-Gaussian PDF's of velocity differences. For the first time we have shown that spatiotemporal intermittency in two dimensions is also associated with intermittent probability distribution functions. Earlier work has reported compatibility with Gaussian statistics of spectral fluctuations in the spa-

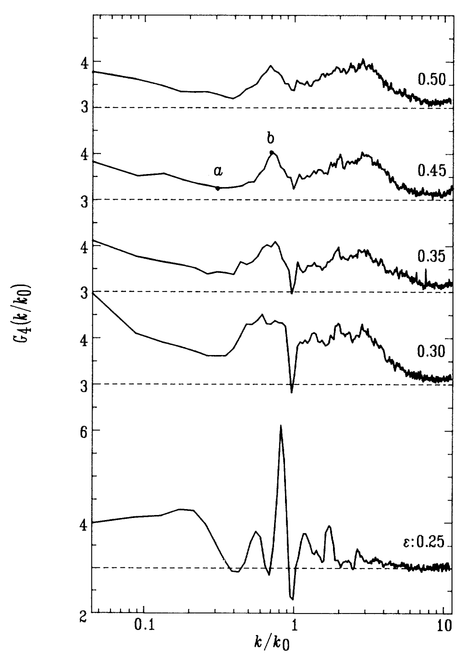

FIG. 4. Full lines: the kurtosis $G_4(k)$ of spectral fluctuations as a function of wave number k/k_0, where k_0 is the crispation wave number. The value of G_4 in the case of Gaussian fluctuations ($G_4=3$) for each curve is indicated by a dashed line. The dots at a and b point to the corresponding probability distribution functions shown in Fig. 5.

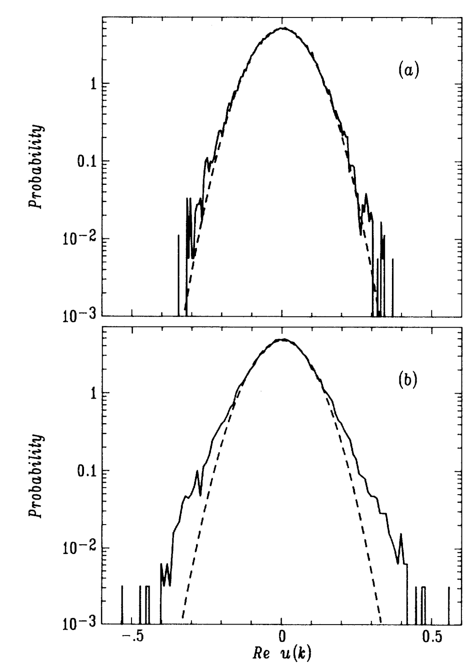

FIG. 5. Full lines: probability distribution functions of spectral fluctuations at $k/k_0=0.31$ and 0.70 for curves (a) and (b), respectively. The distribution functions were measured at $\epsilon=0.45$ (see Fig. 4). Dashed lines: Gaussian distribution functions.

tiotemporal chaotic regime [9]. However, the number of samples we have collected is 2 orders of magnitude larger, which enabled us to see the *deviations* of Gaussian statistics in this regime.

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[11] We have performed experiments using two different mechanical designs, one with an amplitude inhomogeneity of 3% and one with an inhomogeneity of 1%. The results differed in that the critical amplitude A_c was 25.1 μm and 27.7 μm in the constructions with 3% and 1% inhomogeneity, respectively. The latter value is closer to the value predicted theoretically for the instability of the stationary square pattern ($A_c = 29.0\ \mu$m). Qualitatively, both fluctuation and coherence spectra were the same. We believe in the necessity of a significant improvement of the amplitude inhomogeneity by a few orders of magnitude. It can only be achieved by a radically different mechanical design of the experiment in which we are currently involved.

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[16] Also the spatial correlation function shows long-range order. At $\epsilon > \epsilon_c$ it exhibits a rapid initial decay over a single wavelength, followed by a much more gradual decay over macroscopic distances. We have found no simple exponential behavior of time-averaged spatial correlation functions as has been reported in Ref. [8] where the correlation function of 2D snapshots of the surface was measured. We found that at any given instant these correlation functions are highly anisotropic and it is not clear how the outcome of this procedure can be compared to the result of a long-time average.

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