Noise Suppression in Semiconductor p-i-n Junctions: Transition from Macroscopic Squeezing to Mesoscopic Coulomb Blockade of Electron Emission Processes

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(Received 24 February 1992)

We analyze noise suppression properties of a constant-current driven p-i-n heterojunction. It is shown that the junction capacitance and temperature determine the minimum measurement time required for obtaining sub-Poissonian electron injection. As the capacitance of the junction is reduced, the noise spectrum develops a "regulated single electron emission peak" at the single electron charging frequency, indicating regulation of the individual thermionic emission events and therefore a transition from macroscopic squeezing to Coulomb blockade regime.

PACS numbers: 73.40.Kp, 42.50.Dv, 79.40.+z

Recently, there has been considerable interest in the suppression of shot noise in a semiconductor p-n junction driven by a high-impedance constant current source. If these junctions are driven by a large source resistance, the junction voltage drop due to an electron thermionic emission or tunneling event persists and *may* influence the following event. If there is negligible dissipation to the pump source (reservoir) from the junction (system), the fluctuation from the pump source, namely the Johnson-Nyquist thermal noise, is also negligible. The continuous charging of the junction at a constant rate guarantees that for long observation times, the number of electrons injected across the junction is also constant. This is truly a classical and macroscopic effect. However, a semiconductor p-n junction laser converts such a regulated electron stream into quantum mechanical squeezing of the generated radiation: The sub-Poissonian injection of electrons makes it possible to generate a number-phase squeezed state of light [1,2]. The photon number noise was suppressed to 14 dB below the standard quantum limit (SQL), at a sacrifice of the enhanced phase noise under the constraint of the Heisenberg uncertainty principle [3]. This is a macroscopic quantum effect. The value of the second order correlation function for the light generated by a constant-current driven macroscopic p-n junction is below but very close to the Poisson limit, indicating that anticorrelation between successive photon emission events is very small and that practically no information on the photon emission times exists: The anticorrelations only become important for a large number of photons.

In mesoscopic metal-insulator-metal (M-I-M) [4] and semiconductor p-n junctions [5,6], on the other hand, it was predicted that even a single electron tunneling or thermionic emission process is regulated with a fixed time interval $\tau = e/I$, as the voltage drop due to a single event is much larger than the thermal fluctuations (i.e., Coulomb blockade). If such a regulated single electron stream is converted into a photon stream with a relatively short time delay (i.e., $\tau_{\rm rad} << \tau$), a stream of single-photon states with a well-defined clock (i.e., a highly nonclassical state) can be generated [5,6].

The suppression of the driving current shot noise in a macroscopic conductor connected to a large resistor is an important factor in the observation of the described effects [7]. However, a constant-current source alone does not dictate the correlations between successive injection events. The transition from a "macroscopic regulation" of many electrons in the "squeezing" regime to the strict regulation of individual electron injection events in the "Coulomb blockade" regime has not been explored from the viewpoint of mutual coupling of the junction and the pump source.

In this theoretical work, we show that the junction capacitance ($C_{\rm dep}$) and the operating temperature (T) determine the transition from the macroscopic to mesoscopic regime through the ratio $r = e^2/kT\,C_{\rm dep}$ of the single electron charging energy to the characteristic energy of the thermal fluctuations. We find the following: (1) In the macroscopic and high temperature limit ($r << 1$), the electron injection process is sub-Poissonian with variance given by $1/r$. (2) For $r > 1$ (mesoscopic and low temperature limit), the individual injection process is regulated, so that a nonstochastic spike appears at the single-electron charging frequency I/e with a squeezed background noise. We also find that for measurement times ($T_{\rm meas}$) short compared to the thermionic emission time $\tau_{\rm te}$, the injection process is Poissonian, even with an ideal constant-current source. The relative magnitude of the three fundamental time scales τ, $\tau_{\rm te}$, and $T_{\rm meas}$ completely determine the noise characteristics of the carrier injection. If the radiative recombination process is fast compared to these time scales, the same noise properties will be transcribed to the generated light field. The squeezed-state generation in macroscopic p-n junctions [2,8] and the recently proposed Coulomb-blockade regulated single-electron injection (and single-photon stream generation) from low-capacitance mesoscopic junctions [5,6] appear as special cases of our theory.

We consider a p-i-n AlGaAs–GaAs heterojunction driven by an ideal constant-current source. The carrier transport in such a junction occurs by thermionic emission of electrons from the n-type AlGaAs layer into the p-type GaAs layer, across an undoped (i) AlGaAs section. The rate of thermionic emission of electrons is given by

$$\kappa_{\rm te}(t) = \frac{A_{\rm eff}\,T^2\,A^*}{e} \times \exp\left[\frac{e}{kT}\left(V_j(t) - V_{bi} - \frac{e}{2C_{\rm dep}}\right)\right]. \qquad (1)$$

In our model, the junction depletion layer capacitance $C_{\rm dep} = \epsilon\, A_{\rm eff}\,/\,L_i$, where $A_{\rm eff}$ is the effective area of the junction, ϵ is the dielectric constant of the medium, and L_i is the length of the depletion layer. A^* in Eq. (1) is the Richardson's constant [5]. As shown in Ref. [5], under ideal constant-current operation ($dI/dt = 0$), the time dependence of the thermionic emission is exponential,

$$\kappa_{\rm te}(t) = \kappa_{\rm te}(0)\exp\left(\frac{t}{\tau_{\rm te}} - r\,n_e(t)\right) , \qquad (2a)$$

where

$$\tau_{\rm te} = \frac{k\,T\,C_{\rm dep}}{e^2}\,\frac{e}{I} = \frac{1}{r}\,\frac{e}{I} . \qquad (2b)$$

The time constant $\tau_{\rm te}$ as defined in Eq. (2b) gives the time scale over which the thermionic emission rate changes appreciably and will be termed "thermionic emission time." In Eq. (2a), $n_e(t)$ denotes the number of thermionically emitted electrons in time interval $(0,t)$. The $\exp[-\,r\,n_e(t)\,]$ term can be regarded as providing a "feedback mechanism": Emission of an electron results in a decrease in the thermionic emission rate and makes a second emission event less likely. This feedback is at the heart of Coulomb-blockade regulation where the decrease in the emission rate is strong enough to strictly block a second emission event for another single-electron charging time e/I. It is also the very physical origin of macroscopic squeezing where the decrease in the emission rate is small so that only the regulation of many electrons is possible.

We simulate the junction dynamics using the Monte Carlo method detailed in Ref. [5]: Since our goal is to identify the fundamental limitations imposed by the junction dynamics itself, we assume here that the driving constant-current source is ideal and thereby remove the effects of the finite source resistance and the thermal noise generated in it. In Figs. 1–4, we fix the operating temperature at $T = 3\,\mathrm{K}$ and the current at $I = 0.3$ nA. The quantities that are varied are the counting time $T_{\rm meas}$ and the junction capacitance (i.e., $\tau_{\rm te}$) through its area.

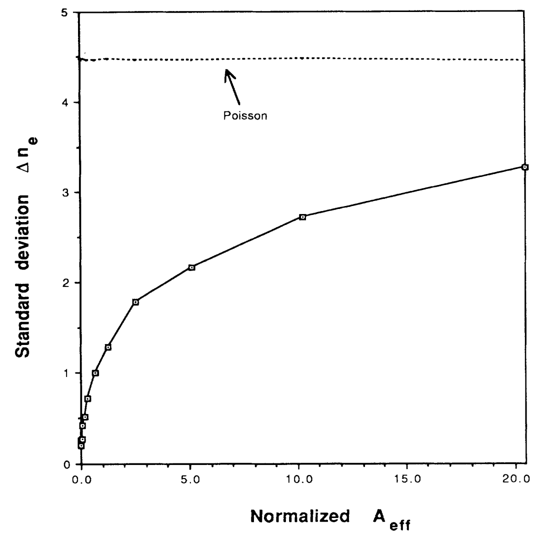

FIG. 1. Standard deviation of the emitted electron number in $T_{\rm meas} = 20\,e/I$ as a function of the normalized junction area. Normalization is such that $A_{\rm eff} = 1$ corresponds to a junction capacitance that satisfies $kTC_{\rm dep}/e^2 = 1.0$. The standard deviation of a random point process with the same expectation value is also given for reference (dotted curve).

We first consider the standard deviation ($\Delta\,n_e$) of the number of electrons injected into the p-type GaAs layer and evaluate its dependence on the junction area (Fig. 1). For the chosen (fixed) measurement time $T_{\rm meas} = 20\,e/I$, the expectation value $\bar{n}_e$ of the thermionically emitted electrons is 20. We observe that for very large junction areas ($\tau_{\rm te} > T_{\rm meas}$, or $A_{\rm eff} > 20$), the value of $\Delta\,n_e$ is very close to the Poisson value of 4.47: In this limit the electron injection events have practically no effect on the thermionic emission rate. As a result, for the chosen observation time, we have *a random point process* with a constant rate, even though the junction is driven by a *perfect constant-current source.* For junction areas that give $e/I < \tau_{\rm te} < T_{\rm meas}$ ($1 < A_{\rm eff} < 20$, in Fig. 1), $\Delta\,n_e$ is approximately proportional to the square root of the area (or $C_{\rm dep}$) and is clearly below the Poisson limit. Finally, for $e/I > \tau_{\rm te}$ (i.e., $A_{\rm eff} < 1$), $\Delta\,n_e$ decreases very sharply: This is the Coulomb-blockade regime where the individual electron injection events become deterministic as the single-electron charging energy $e^2/C_{\rm dep}$ exceeds kT. It is important to note that the junction-voltage fluctuations are complementary to those of the injected electrons: For $e/I \simeq \tau_{\rm te}$, $\Delta V_j \simeq kT/e$ whereas for $e/I << \tau_{\rm te}$, $\Delta V_j << kT/e$.

Next, we consider the dependence of the standard deviation on the measurement time. Figure 2 shows this dependence for a fixed junction area that gives

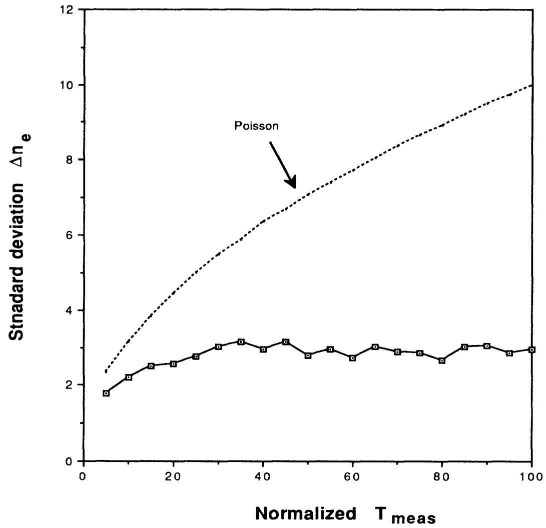

FIG. 2. Standard deviation of the emitted electron number as a function of the measurement time $T_{\rm meas}$, for $kTC_{\rm dep}/e^2 = 13.0$. $T_{\rm meas}$ is normalized to e/I. The dotted curve gives the Poisson limit.

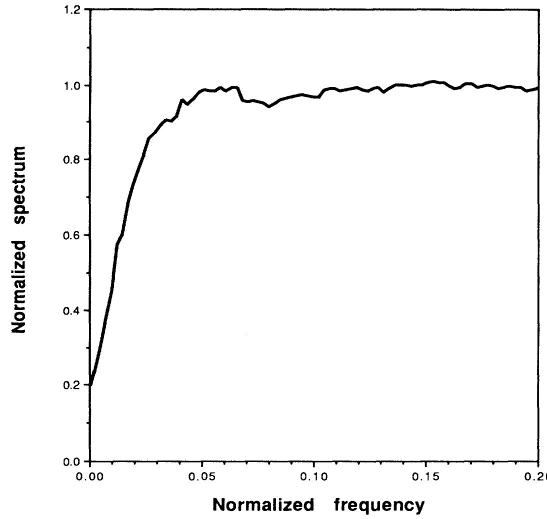

FIG. 3. The spectrum $S_e(f)$ of the electron stream for a "macroscopic" junction ($kTC_{\rm dep}/e^2 = 13.0$). $S_e(f)$ is normalized to the spectral density of a random point process with the same expectation value for n_e. Frequency scale is normalized to I/e.

$kTC_{\rm dep}/e^2 = 13$: The principal result here is that the standard deviation is independent of the observation time $T_{\rm meas}$, provided $T_{\rm meas} > \tau_{\rm te}$. This independence is at the heart of squeezed-state generation by macroscopic *p*-*n* junctions: As the measurement time becomes arbitrarily large so does the squeezing which is generally measured as improvement over the Poisson limit. Finally, in agreement with the results of Fig. 1, for $T_{\rm meas} < \tau_{\rm te}$ (i.e., $T_{\rm meas} < 13\,e/I$), the emission process approaches Poissonian.

It can be shown analytically that in the limit of $r << 1$, the probability for observing n_e electron injection events in an observation time $T_{\rm meas}$ is

$$P(n_e, T_{\rm meas}) \simeq \frac{1}{N(r,\bar{n}_e)}\,\frac{1}{n_e!}\,\bar{n}_e^{n_e}\exp[-\bar{n}_e]\exp\left(-\frac{r}{2}(n_e-\bar{n}_e)^2\right) , \tag{3}$$

where $\bar{n}_e = T_{\rm meas}I/e$ and $N(r,\bar{n}_e)$ is the normalization factor. If $\bar{n}_e\; r << 1$ (i.e., $\tau_{\rm te} >> T_{\rm meas}$), then the last term in Eq. (3) can be neglected and one obtains the usual Poisson distribution with $\Delta n_e = \sqrt{\bar{n}_e}$. If, on the other hand, $\bar{n}_e r >> 1$ (i.e., $\tau_{\rm te} << T_{\rm meas}$), then the n_e dependence is predominantly determined by the last term (i.e., n_e is Gaussian distributed). The standard deviation in this limit is

$$\Delta n_e = \frac{1}{\sqrt{r}} = \sqrt{\frac{k\,T\,C_{\rm dep}}{e^2}} . \tag{4}$$

Using our Monte Carlo procedure and changing other junction parameters such as the temperature T and the width of the depletion layer L_i, we have confirmed this result numerically. Δn_e given in Eq. (4) can be considered as a *fundamental noise limit for macroscopic squeezing* in constant-current driven *p*-*i*-*n* heterojunctions. Even in the limit $r << 1$ where emission or tunneling of a single electron creates a very small voltage drop, the combined effect of many electrons is sufficient to control and regulate the electron emission to within several Δn_e. Finally, note that in accordance with Fig. 2, Δn_e obtained from Eq. (4) does not depend on $T_{\rm meas}$, even though $\bar{n}_e$ does.

Figure 3 shows the normalized spectrum $S_e(f)$ of the injected electron stream for the same junction parameters as Fig. 2. The noise for frequencies $f < 1/(2\pi\tau_{\rm te})$ is well below the shot-noise limit ($S_e = 1$). As mentioned before, in the limit of radiative recombination rates fast compared to $\tau_{\rm te}^{-1}$ and I/e, the spectrum of the generated photon stream will be identical to that of $S_e(f)$. The *squeezing bandwidth* of such a light source will in general be determined by $\min[1/(2\pi\tau_{\rm te}), 1/(2\pi\tau_{\rm rad})]$, where $\tau_{\rm rad}$ is the radiative recombination time in the *p*-type GaAs layer. Optical squeezing in semiconductor *p*-*n* junctions has been observed in this limit of large junction capacitances (i.e., $r << 1$): For the numerical parameters reported in Ref. [3], $r \simeq 10^{-7}$ and $e/I \simeq 1.6\times10^{-17}$ s, so that the (calculated) squeezing bandwidth is $1/(2\pi\tau_{\rm te}) \simeq 1\,{\rm GHz}$. This is in very good agreement with the measured squeezing bandwidth of 1.1 GHz [3].

As the junction area is decreased, the squeezing band-

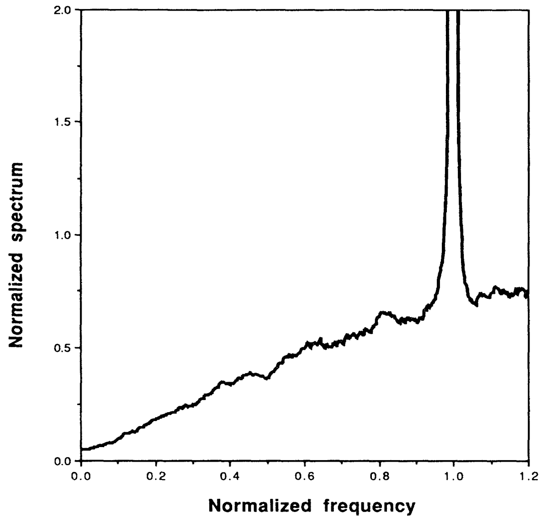

FIG. 4. The spectrum $S_e(f)$ of the electron stream for a "mesoscopic" junction ($kTC_{\rm dep}/e^2 = 0.13$). $S_e(f)$ is normalized to the spectral density of a random point process with the same expectation value for n_e. Frequency scale is normalized to I/e.

width increases along with decreasing $\tau_{\rm te}$, keeping the same functional relation $f_{3\,\rm dB} = 1/(2\pi\tau_{\rm te})$. The qualitative changes in the spectrum occur for $kTC_{\rm dep}/e^2 < 1$. Figure 4 shows $S_e(f)$ for $kTC_{\rm dep}/e^2 = 0.13$: We observe that the spectrum develops a nonstochastic peak at $f = I/e$, indicating that the electron thermionic emission events are now regulated and occur (roughly) at constant time intervals. As the junction area is further reduced, *regulated single-electron emission peaks* at the harmonics of I/e also appear, together with diminishing noise in between. The background noise spectrum is still determined by the squeezing bandwidth $1/(2\pi\tau_{\rm te})$ ($\simeq 1.3$ for Fig. 4).

A detailed analysis of the regulation of the electron thermionic emission or resonant tunneling events by Coulomb blockade has been given in Refs. [5] and [6], together with the implications for the light field generated by subsequent radiative recombination. In this case, for a given measurement time interval, we have complete information not only about the total number of injected electrons (emitted photons), but also the injection (emission) time of each individual electron (photon).

The ratio r of the single-electron charging energy to the characteristic energy of the thermal fluctuations determines the extent of the correlations between the injected electrons: In the mesoscopic limit ($kTC_{\rm dep}/e^2 << 1$), each electron is aware of the previous one due to the significant change in the junction voltage that the last thermionic emission event created: This is the Coulomb-blockade regime discussed above. In the macroscopic limit ($kTC_{\rm dep}/e^2 >> 1$), individual thermionic emission events practically have no effect on the expected injection time of the next electron. A large number of emission events, however, do have a combined effect on $\kappa_{\rm te}$ and it is this feedback that keeps the standard deviation below the Poisson limit.

In summary, we have analyzed the noise in the electron thermionic emission events in constant-current driven *p-i-n* heterojunctions. We have shown that the ratio of the single-electron charging energy $e^2/C_{\rm dep}$ to kT (i.e., r) is an important parameter in determining the noise characteristics. By deriving an expression for the probability distribution of the number of injected electrons, we have obtained a fundamental noise limit for a macroscopic junction. In the macroscopic limit with $r << 1$, we get squeezed electron injection and photon generation processes, with squeezing extending from $f = 0$ to $f = 1/(2\pi\tau_{\rm te}) = rI/(2\pi e)$. For $r >> 1$, we obtain nonstochastic peaks at $f = I/e$ in the spectrum, indicating regulated emission processes.

The authors would like to thank W. Richardson for helpful discussions. The work of A. Imamoḡlu was supported by the National Science Foundation through a grant for the Institute for Theoretical Atomic and Molecular Physics at Harvard University and Smithsonian Astrophysical Observatory.

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