Quantum Noise Reduction in Optical Amplification

Z. Y. Ou, S. F. Pereira, and H. J. Kimble

Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125

(Received 9 December 1992)

Quantum fluctuations in optical amplification are investigated with a nondegenerate optical parametric amplifier whose internal idler mode is coupled to a squeezed vacuum. Reductions of the inherent quantum noise of the amplifier are observed with a minimum noise level 0.7 dB below the usual noise level of the amplifier with its internal idler mode in a vacuum state. With a small coherent field as the signal input, the amplified output exhibits an improvement in signal-to-noise ratio of 0.5 dB for the case of a squeezed vacuum as compared to a vacuum state for the amplifier's internal mode.

PACS numbers: 42.50.Lc, 42.50.Dv

Quite apart from its significance in technological terms, the question of the fundamental noise performance of an amplifier has a long history in physics because of its intimate relationship to quantum measurement [1-5]. Indeed, if "noiseless" amplification were possible, then the microscopic quantum world could be magnified to a macroscopic scale for our casual inspection. However, the principles of quantum mechanics require an amplifier to add noise to any input that it processes, as has been codified by Caves in a fundamental theorem for phase-insensitive amplifiers and in an uncertainty relation for phase-sensitive amplifiers [5]. Whatever the particular origin (e.g., spontaneous emission in a laser amplifier), the fundamental excess noise associated with the amplification process can be viewed as arising from the coupling of the signal input to the internal modes of the amplifier and hence depends upon the state of these modes, which in the best case until now has been a vacuum state. For phase-insensitive amplification, the quantum noise added by internal vacuum-state modes is phase insensitive and gives rise to noise at the output which is equivalent to half of a noise photon at the input in the limit of large gain. A coherent field as the signal input will thus suffer a 3 dB degradation in signal-to-noise ratio in the large gain limit; furthermore, nonclassical features of the signal input will be lost for amplification with power gain greater than 3 dB for this kind of amplifier [6].

If instead the internal modes of the amplifier are coupled to a squeezed vacuum rather than to the usual vacuum state, then the added quantum noise at the signal output will be phase dependent reflecting the reduced and enhanced fluctuations of a squeezed state relative to the vacuum [7]. Yurke and Denker [8] and others [9-11] have in this way suggested that the excess noise for phase-insensitive amplifiers can be effectively eliminated by arranging for signal information to be encoded on the quadrature-phase amplitude of the input corresponding to that with reduced noise at the amplifier's output. Of course in this case the added quantum noise demanded by Caves's amplifier uncertainty principle goes mostly into the (unused) conjugate quadrature; the originally phase-insensitive amplifier is thus converted to a phase-sensitive amplifier by squeezing its internal modes.

In this Letter we present results from an experiment which implements these ideas related to the quantum limits of amplification. In particular, by coupling the squeezed vacuum to the idler mode (internal mode) of a nondegenerate optical parametric amplifier (NOPA), we observe phase-sensitive amplification for a vacuum-state input to the signal mode and record a minimum amplified noise level 0.7 dB below the usual phase-insensitive noise level for the amplifier with its idler mode in a vacuum state. Note that the best possible noise reduction for the amplified signal for the case of a perfectly squeezed idler and large gain is 3 dB, in which case there would be no degradation in signal-to-noise ratio. For our system with a small coherent field as the signal input, the amplifier exhibits an improvement in signal-to-noise ratio of 0.5 dB for a squeezed input to the idler as compared to the usual vacuum input.

As a base line for our discussion of quantum amplification, we consider the following general model for a phase-preserving linear amplifier [4,5]:

$$\hat{a}^{\text{out}} = \sqrt{G}\,\hat{a}^{\text{in}} + \hat{F} , \tag{1}$$

where $\hat{a}^{(\text{in,out})}$ represent the signal modes for input and output and G is the amplifier's power gain for the signal channel. The operator $\hat{F}$ is associated with the internal modes of the amplifier and is responsible for added quantum noise in amplification. For an ideal NOPA, $\hat{F} = \sqrt{G-1}\,\hat{b}^\dagger$ with $\hat{b}$ representing the input idler mode. If signal information is encoded on the quadrature-phase amplitude $\hat{X}_a(\theta) \equiv \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}$, then the noise of the signal field at the amplifier's output as specified by the variance of $\hat{X}_a^{\text{out}}$ is given by

$$N_a^{\text{out}}(\theta) = G N_a^{\text{in}}(\theta) + (G-1) N_b^{\text{in}}(\theta) , \tag{2}$$

where $N_i(\theta) \equiv \langle (\hat{X}_i - \langle \hat{X}_i \rangle)^2 \rangle$ $(i=a,b)$ and $\hat{X}_b(\theta) \equiv \hat{b}e^{-i\theta} + \hat{b}^\dagger e^{i\theta}$. This relationship is illustrated by the simple diagram in Fig. 1(a) where the noise of the signal output is seen to arise from two contributions, namely, direct amplification of input signal noise and noise added from

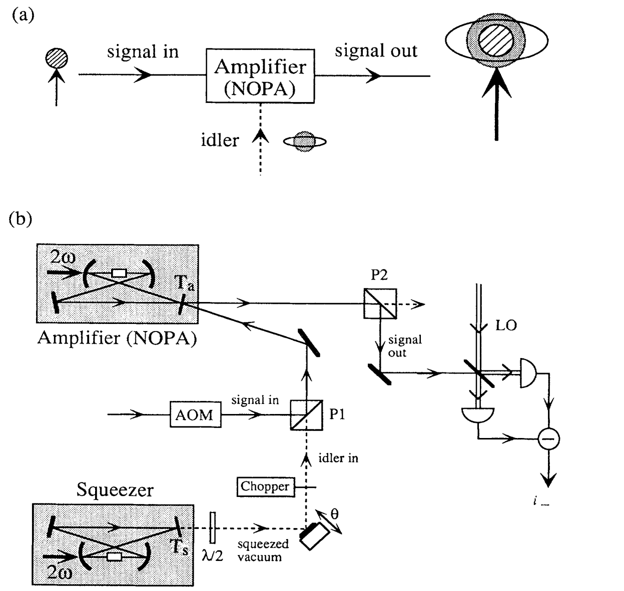

FIG. 1. (a) Simple diagram to illustrate amplification by a nondegenerate parametric amplifier. The signal input is in a coherent state, while the idler is in either a vacuum state (represented by a circle) or a squeezed state (ellipse). (b) Principal components of the actual experiment.

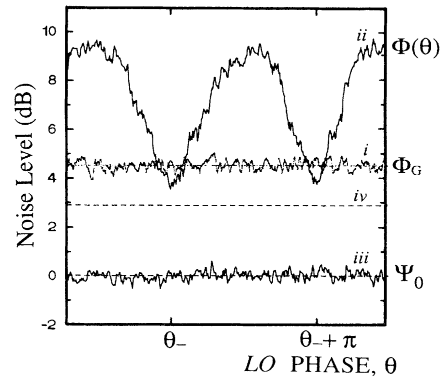

FIG. 2. Spectral density of photocurrent fluctuations for $i_-(\Omega_0)$ generated by the amplifier's signal output $\hat{a}_{\rm out}$ as a function of local oscillator phase θ. Trace i is the amplified noise level Φ_G from the NOPA when the idler mode is in a vacuum state, while trace ii is the noise level $\Phi(\theta)$ when the idler is in a squeezed-vacuum state. In each case, the signal input to the amplifier is a vacuum state. Trace iii gives the vacuum-noise level Ψ_0 for the balanced detector; Ψ_0 lies 20 dB above the electronic noise floor. The dashed line iv is the noise level expected for a lossless system with a perfectly squeezed idler. Spectrum analyzer acquisition parameters are resolution bandwidth of 100 kHz, video bandwidth of 100 Hz, analyzer frequency center frequency $\Omega_0/2\pi = 1.1$ MHz, and sweep time of 0.28 s.

coupling to internal modes of the amplifier (in this case, the idler mode). With a coherent state as the signal input and with the idler mode in a vacuum state, we have $N_a^{\rm in}(\theta) = 1 = N_b^{\rm in}(\theta)$ (circular areas). Hence the quantum noise gain $G_q \equiv N_a^{\rm out}/N_a^{\rm in} = 2G-1$, so that the signal-to-noise ratio R of the output relative to the input becomes $R = (2-1/G)^{-1} \to \frac{1}{2}$ for $G \gg 1$ (3 dB degradation in signal-to-noise ratio). On the other hand, with the idler mode coupled to a squeezed vacuum (elliptical shape), the noise in the quadrature specified by θ_- is reduced while that in the orthogonal quadrature θ_+ is enhanced, so that $N_{a\pm}^{\rm out} = G + (G-1)N_\pm$ and $R_\pm [1 + (1 - 1/G)N_\pm]^{-1}$, where $N_\pm \equiv N_b^{\rm in}(\theta_\pm)$. Hence $N_{a-}^{\rm out} \to G$ and $R_- \to 1$ for $N_- \to 0$, which approximates noiseless amplification for the quadrature phase of the signal beam specified by θ_-.

Turning from this discussion of an ideal NOPA, we present in Fig. 1(b) an overview of our actual experimental arrangement, which consists of a nondegenerate parametric amplifier whose idler mode is itself coupled to a squeezed vacuum (generated by a second parametric amplifier). In fact both the "amplifier" and the "squeezer" are NOPAs of the type described in detail in Ref. [12]. Briefly stated, each is operated as a frequency degenerate but polarization nondegenerate subthreshold optical parametric oscillator by employing noncritically phase-matched type-II down-conversion in KTP at 1.08 μm within a traveling-wave cavity. The inputs for the amplifier shown in Fig. 1(b) are the outputs of the polarizer $P1$ and are orthogonally polarized signal (p-polarized) and idler (s-polarized) beams which are mode matched through the input coupler $T_A = 0.07$ into the amplifier cavity. A squeezed vacuum for the idler input to the amplifier is generated by projecting the signal and idler modes from the squeezer along a direction 45° relative to these polarizations [12] and is accomplished with the half-wave plate ($\lambda/2$) and polarizer ($P1$) shown in Fig. 1(b). Also injected into $P1$ is a coherent signal beam which has been frequency shifted (single sideband) by $\Omega_0/2\pi = 1.1$ MHz relative to the frequency ω of degeneracy for the amplifier and squeezer. By using a mechanical chopper to block and unblock the field from the squeezer, we can couple either vacuum or squeezed vacuum to the idler mode of the NOPA. The amplified signal from the output of the NOPA is separated from the idler output by polarizer $P2$ and is sent to a balanced homodyne detector where information about the quadrature-phase amplitudes of the signal beam is obtained from measurements of the spectral density of photocurrent fluctuations of i_-.

The noise behavior of our amplifier in the absence of a coherent signal is investigated by blocking the injected signal at $P1$ thus leaving the input to the signal mode of the amplifier in a vacuum state. As illustrated in Fig. 2, when the idler mode of the amplifier is also coupled to a vacuum state (squeezed input to $P1$ blocked), we observe a phase-insensitive increase in the noise level of the signal output, denoted under these conditions by Φ_G (trace i). By contrast, with a squeezed vacuum for the idler input, a

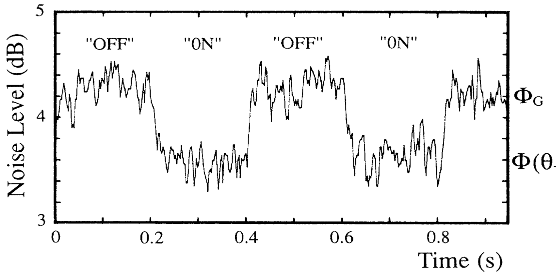

FIG. 3. Amplified noise level of the signal output with a vacuum state for the signal input. "OFF" represents the noise level Φ_G for a vacuum-state input to the idler mode. "ON" gives the noise level $\Phi(\theta_-)$ for a squeezed-state input to the idler mode with phase offset adjusted for minimum noise. The noise levels $[\Phi_G, \Phi(\theta_-)]$ are referred to the vacuum-noise level Ψ_0 (0 dB) for the balanced detector. Acquisition parameters are as in Fig. 2 except that the video bandwidth is 30 Hz and sweep time is 0.96 s.

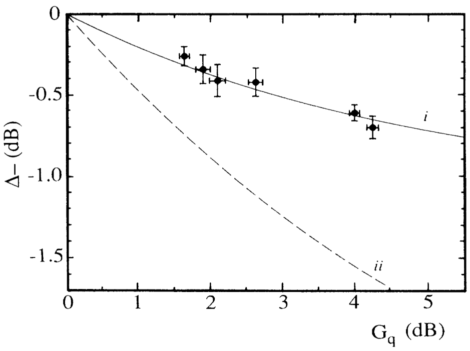

FIG. 4. Quantum noise reduction Δ_- for the amplified signal as a function of the detected quantum noise gain G_q for a squeezed idler input with $N_-=0.52$. The solid curve i is the theoretical prediction for our system as discussed in the text. The dashed curve ii is calculated from Eq. (2) for lossless amplification and detection with perfect squeezing for the idler input.

phase-sensitive noise level $\Phi(\theta)$ for the signal output is observed as the phase (θ) of the squeezed vacuum is scanned (trace ii). The point of reference for these measurements is the vacuum-state level Ψ_0 for the balanced detector (trace iii). Notice that for $\theta \simeq \theta_- + p\pi$ (p integer), the phase-sensitive noise level $\Phi(\theta)$ of trace ii drops below the phase-insensitive noise level Φ_G of trace i. In qualitative terms, trace i represents phase-insensitive amplification of the vacuum-state input illustrated by the error circles in Fig. 1(a), while trace ii corresponds to phase-sensitive amplification with a squeezed idler as indicated by the error ellipses. The dashed line in Fig. 2 (trace iv) is the calculated noise level (see below) for the case of perfect injected squeezing ($N_- \to 0$) and corresponds to the maximum noise reduction possible for the gain G employed in Fig. 2.

To measure the quantum noise reduction more accurately than in Fig. 2, we manually tune the phase of the injected squeezed vacuum to reach the operating point $\theta = \theta_-$ corresponding to minimum noise for the signal output and then chop the output from the squeezer "on" and "off." Such a measurement at the highest available pump power is recorded in Fig. 3, from which we find a quantum noise reduction $\Delta_- \equiv \Phi(\theta_-)/\Phi_G = 0.85$ (-0.7 dB) and quantum noise gain $G_q \equiv \Phi_G/\Psi_0 = 2.6$ (4.2 dB), where we stress that Δ_- and G_q refer to the observed photocurrent fluctuations. By repeating these measurements at several operating points, we obtain the noise reduction Δ_- versus the noise gain G_q as plotted in Fig. 4, with the error bars reflecting trace-to-trace fluctuations in Δ_- and G_q. Also shown in Fig. 4 is the result of a model calculation for our NOPA which generalizes the ideal case of Eq. (2) to include the total internal loss of the amplifiers determined to be $l=0.32\%$ from measurements of the passive reflection dip of the cavity and the external losses $1-\xi = 0.30 \pm 0.04$ due to propagation from the amplifier to the detector and to detection with finite quantum and heterodyne efficiencies [12]. In addition to measuring the various efficiency factors, we also examine the squeezing generated by both the amplifier and squeezer cavities. For example, with the amplifier cavity tuned off resonance to act as a high reflector, we observe that the photocurrent noise reduction for the field from the squeezer is -1.8 dB. With independent knowledge of ξ, we then deduce that $N_- = 0.52$ for the idler field $X_b^{\text{in}}(\theta_-)$ injected into the amplifier for the data in Figs. 3 and 4. By analogous techniques, we operationally determine all the experimental parameters required for the theoretical curve i in Fig. 4. For comparison, the dashed curve ii in Fig. 4 is derived by assuming no loss $\xi=1$, $l=0$ as in Eq. (2) and perfect squeezing ($N_-=0$). This curve corresponds to the best possible noise reduction $\Delta_- = R_-$ that can be achieved for a coherent input signal and asymptotically approaches -3 dB for $G \to \infty$.

Given this characterization of the noise performance of our system, we next investigate its behavior with a small coherent signal injected into the signal mode of the amplifier and concentrate in particular on the *detected* signal-to-noise ratio (SNR) for the signal output relative to the *inferred* signal-to-noise ratio for the amplifier's input field, that is, on the ratio $R \equiv (\text{SNR})_{\text{out}}/(\text{SNR})_{\text{in}}$. Note that even without the amplifier (cavity detuned from resonance), the ratio $R_\xi = \xi < 1$ due to linear losses $(1-\xi)$ in propagation and detection in passing from the amplifier's output to the measured photocurrent. Further note that even with $G_q = 0$ (pump turned off and cavity tuned on resonance), the passive internal loss l of the amplifier itself will further degrade performance to a value $R_{\xi l} < R_\xi$. Both R_ξ and $R_{\xi l}$ are indicated in Fig. 5 for our system. When the amplifier is in fact turned on $(G_q > 1)$ with the idler coupled to vacuum, the SNR of the output is degraded yet further to a value $R(G_q)$ due to the fundamental noise added in amplification as indi-

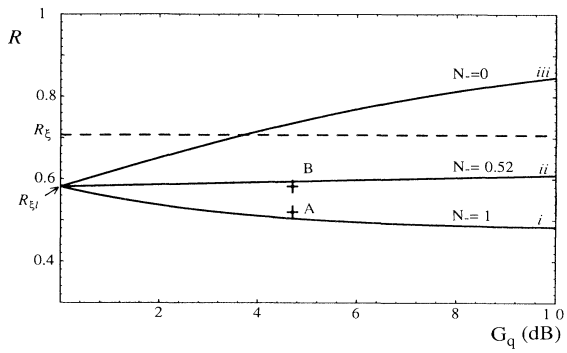

FIG. 5. Signal-to-noise ratio R of the amplified output relative to that of the input as a function of quantum noise gain G_q. The following values are assumed for the variance of the injected idler mode: Curve i, $N_- = 1.0$ (vacuum state); curve ii, $N_- = 0.52$; and curve iii, $N_- = 0.0$ (perfect squeezing). The dashed line at $R_\xi = 0.71$ corresponds to the *detected* SNR due to passive losses in propagation and detection. The level $R_{\xi l} = 0.58$ represents a reduction of R_ξ due to passive internal losses of the amplifier for $G_q = 0$. Points A and B are measured with a vacuum-state idler and a squeezed-state idler, respectively. Points A and B as well as curves i–iii are for $(\mathrm{SNR})_i = 4.8$.

cated in Fig. 5 by curve i drawn for $N_- = 1$. By contrast if squeezed vacuum is injected into the idler mode of the amplifier ($N_- < 1$), then $R(G_q)$ increases since there is in this case less contamination of the amplified signal field by the idler mode. In Fig. 5 we draw two curves to illustrate this point; curve ii with $N_- = 0.52$ corresponds to our experiment, while curve iii with $N_- = 0$ represents the case of perfect squeezing injected into the amplifier [still, however, with finite internal amplifier losses l and with finite propagation and detection losses $(1-\xi)$]. Note that the family of curves presented in Fig. 5 depends upon the actual SNR of the input [here $(\mathrm{SNR})_i = 4.8$] and that these curves are drawn for parameters appropriate to our experiment. Indeed, the two points indicated in Fig. 5 are obtained with the idler mode coupled to vacuum ($N_- = 1$) yielding $R(G_q = 3) = 0.52$ (point A) and with the idler mode coupled to squeezed vacuum ($N_- = 0.52$) yielding $R_-(G_q = 3) = 0.58$ (point B). We thus demonstrate an improvement in SNR by squeezing the internal idler mode of the amplifier.

An apparently peculiar aspect of Fig. 5 is that the SNR after the amplification process can exceed the SNR for the directly detected signal field without amplification [that is, $R_-(G_q) > R_\xi$ for $N_- \to 0$]. This somewhat surprising improvement can be understood by recalling that passive losses are responsible for reducing R_ξ and $R_{\xi l}$ below unity. Although the internal amplifier losses expressed by l cannot be overcome, amplification with $N_- \to 0$ can result in the amplified fluctuations of the signal field being dominant over the added vacuum noise introduced by the losses $(1-\xi)$. In this way we can achieve $R_-(G_q) > R_\xi$ and indeed for $l \to 0$ we can recover the true SNR of the input $[R_0(G_q) \to 1]$ even in the presence of propagation and detection losses [13].

In summary, by coupling squeezed vacuum to the internal idler mode of a nondegenerate parametric amplifier, we have observed quantum-noise reduction in the amplification process and have demonstrated an improvement of signal-to-noise ratio for the amplified output relative to the usual case of a vacuum-state idler mode. Although we have considered only the situation for which the signal beam is limited by vacuum fluctuations and for which the amplifier is operated in a linear (undepleted) regime, it is of considerable interest to investigate as well situations involving the amplification of nonclassical field states with noise below the vacuum-state limit [6] and the operation of the amplifier in a nonlinear regime with depletion [14]. In fact with squeezed vacuum coupled to the idler mode, it should be possible to demonstrate that manifestly quantum states can be amplified in the signal channel with $G > 2$ (the so-called photon cloning limit) while still maintaining nonclassical characteristics [9–11]. Beyond its relevance to quantum amplification, our experiment is significant in that it provides an example of the alteration of the fundamental quantum fluctuations of a system interacting with a squeezed reservoir [15].

This work was supported by the Office of Naval Research and by the National Science Foundation. We gratefully acknowledge the contributions of Professor J. Yarrison-Rice made during a summer research visit.

[1] K. Shimoda, H. Takahasi, and C. H. Townes, J. Phys. Soc. Jpn. **12**, 686 (1957).
[2] R. Serber and C. H. Townes, in *Quantum Electronics*, edited by C. H. Townes (Columbia Univ. Press, New York), pp. 233–255.
[3] H. Heffner, Proc. IRE **50**, 1604 (1962).
[4] H. A. Haus and J. A. Mullen, Phys. Rev. **128**, A2407 (1962).
[5] C. M. Caves, Phys. Rev. D **26**, 1817 (1982).
[6] C. K. Hong, S. Friberg, and L. Mandel, J. Opt. Soc. Am. B **2**, 494 (1985).
[7] See special issues on squeezed states of light: J. Opt. Soc. Am. B **4**, 1450 (1987); J. Mod. Opt. **34**, 709 (1987).
[8] B. Yurke and J. S. Denker, Phys. Rev. A **29**, 1419 (1984).
[9] G. J. Milburn, M. L. Steyn-Ross, and D. F. Walls, Phys. Rev. A **35**, 4443 (1987).
[10] C. R. Gilson, S. M. Barnett, and S. Stenholm, J. Mod. Opt. **34**, 949 (1987).
[11] J. A. Vaccaro and D. T. Pegg, J. Mod. Opt. **34**, 855 (1987).
[12] Z. Y. Ou, S. F. Pereira, and H. J. Kimble, Appl. Phys. B **55**, 265 (1992).
[13] C. M. Caves, Phys. Rev. D **23**, 1963 (1981).
[14] M. J. Collett and D. F. Walls, Phys. Rev. Lett. **61**, 2442 (1988).
[15] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).