Structure of the Z^0 Resonance and the Physical Properties of the Z^0 Boson

Robin G. Stuart [(a)]

Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 México, Distrito Federal, Mexico

(Received 20 August 1992)

The proximity of multiparticle production thresholds means that the Z^0 resonance is formed, not by a single particle, but by a "multiplet" of distinct Z^0 bosons differing slightly in mass. The effects that these thresholds can have on the gauge-invariant pole expansion near the Z^0 resonance are studied. It is shown how this expansion naturally yields physical quantities from which the mass and partial widths of the unstable Z^0 bosons can be defined. The recent discussion on the definition of the Z^0 boson mass is reviewed and commented upon, stressing the distinction between the renormalized and physical masses.

PACS numbers: 14.80.Er, 11.10.Gh, 11.20.Fm

(1) *Introduction.*— In a previous publication [1], henceforth referred to as **I**, it was pointed out that most calculations of the electroweak radiative corrections to the process $e^+e^- \rightarrow f\bar{f}$ near the Z^0 resonance contain spurious higher-order gauge-dependent contributions. An expansion of the full scattering amplitude about its pole was proposed to correct the problem. In the same publication it was also noted that the value of the Z^0 mass in the on-shell renormalization scheme [2], the one currently being extracted by LEP experiments, differs significantly from the traditional one based on the position of the pole of the scattering amplitude. This last point was also discussed independently by Willenbrock and Valencia [3]. It was subsequently shown [4,5] that the usual definition of the renormalized Z^0 boson in the on-shell renormalization scheme is gauge dependent. The above points have generated much discussion [6–10] and a certain amount of confusion.

The gauge dependence of the scattering amplitude near the resonance and the gauge dependence of the Z^0 mass in the on-shell scheme are distinct and unrelated matters. The former arises as a consequence of the nonperturbative character of the resonance. Resummations of higher-order corrections that are necessary to describe the resonant line shape are often done in a gauge-dependent manner and are the source of the problem which can be manifest in any renormalization scheme. Adopting a gauge-invariant definition for the renormalized Z^0 mass does not automatically cure the problem of gauge dependence of the scattering amplitude near resonance.

Much of the discussion of the definition of the Z^0 boson mass is based on the mistaken notion that the on-shell definition is, in some way, the fundamental or natural one. The renormalized mass of any particle in any renormalization scheme is just a bookkeeping device that arises as an artifact of perturbation theory. This can be seen by comparing the on-shell and $\overline{MS}$ (modified minimal subtraction) renormalization schemes. The latter is a perfectly respectable scheme that is in many ways superior to the on-shell scheme [11,12]. Assume that the on-shell renormalized Z^0 mass in 't Hooft–Feynman gauge is $M_Z^{\rm OS}=91.17$ GeV. Then for a top quark mass $m_t=120$ GeV and a Higgs mass $M_H=100$ GeV, the renormalized Z^0 mass in the $\overline{MS}$ scheme is $M_Z^{\overline{MS}}=91.71$ GeV [13] for a 't Hooft mass of $\mu=91.17$ GeV. From an experimental point of view, the numerical difference between these two is huge. Yet both represent equally valid and workable choices for the renormalized Z^0 mass. The exact value of $M_Z^{\overline{MS}}$ manifestly depends on the value taken for μ, thereby emphasizing the unphysical character of the renormalized mass. As the renormalized mass is not a physical quantity, it is not *a priori* required to be gauge invariant. The renormalization conditions that one imposes in order to fix the renormalized parameters are arbitrary to within constraints imposed by the Ward identities. When not tied to S-matrix elements, they can be, and often are, gauge dependent. The renormalized parameters obtained from them can therefore also be gauge dependent but this gauge dependence is just a manifestation of the well-known renormalization scheme ambiguity [14] that reflects the freedom available in the choice of perturbative approximation to the exact S-matrix element. The gauge dependence coming from the renormalized parameters must cancel in exact S-matrix elements but does so across different orders of the perturbation expansion. Difficulties only arise when one attempts to identify the renormalized mass as the physical mass. In that case one is forced to introduce *ad hoc* definitions. That one can work with gauge-dependent parameters is noted by Veltman [10].

While gauge invariance is not necessarily required for renormalized parameters, this is an essential feature for the parameters that one would like to ascribe as physical properties to elementary particles and for S-matrix elements that supposedly represent observable physical processes. The pole expansion introduced in **I** both produces exactly gauge-invariant S-matrix elements near the Z^0 resonance and, as will be shown here, provides a means of unambiguously identifying gauge-invariant physical parameters from which the mass and partial widths of the Z^0 boson may be defined.

Borrelli *et al.* [15] have discussed the value of expressing cross sections in terms of physical observables,

© 1993 The American Physical Society

such as masses and partial widths, rather than model-dependent parameters, such as $\sin^2\theta_W$, as a basis for model-independent analyses of LEP data. The catch is that the definitions of the physical mass and partial decay widths of the Z^0 boson are not clear cut especially at high precision. The parameters adopted by Borrelli *et al.* are themselves rather arbitrary although they can be related to the physical parameters of the Z^0 boson considered here.

In this paper we review the gauge-invariant pole expansion of **I** taking care to address certain technical points that were not originally considered. In particular, the presence of a multitude of multiparticle production thresholds in the Z^0 resonance region means that the resonance is comprised of a "multiplet" of Z^0 bosons each having its corresponding pole on an unphysical Riemann sheet. The exact factorization of the residues at these poles is explicitly demonstrated, which leads to a generalization of scattering amplitudes to the case of external unstable particles and allows partial widths to be defined for these unstable Z^0 bosons. These quantities appear naturally and explicitly in a pole expansion of the scattering amplitude.

(2) Gauge-invariant pole expansion near the resonance.— We will be concerned with the process $e^+e^- \to f\bar{f}$ near the Z^0 resonance. Far from the resonance, the pole expansion given in **I** does not need to be invoked because ordinary perturbation theory can and should be used without Dyson summation. We shall limit ourselves to considering two-particle final states. Hence QED bremsstrahlung corrections are excluded along with their associated IR-divergent virtual photon corrections. Such corrections together form a gauge-invariant and unitary set. All external fermions will be considered massless and s and t will denote the usual Mandelstam variables, $s=(p_{e^+}+p_{e^-})^2$ and $t=(p_{e^-}+p_f)^2$. We shall be cavalier in ignoring renormalization and thus the mass that appears here, M_0, is the bare mass of the Z^0. The one-loop renormalization of the gauge-invariant pole expansion was considered in Ref. [16].

The vector-boson self-energies and mixings all take the form

$$\Pi_{\mu\nu}(q^2)=\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\Pi_T(q^2)+\left(\frac{q_\mu q_\nu}{q^2}\right)\Pi_L(q^2)\,, \tag{2.1}$$

where μ,ν are Lorentz indices, q^2 is the momentum squared, and Π_T,Π_L are the transverse and longitudinal parts. For massless initial-state fermions, as assumed here, only the transverse parts of the vector-boson self-energies contribute.

The complete one-particle-irreducible (1PI) transverse parts of the self-energies for the photon and Z^0 are denoted by $\Pi_{\gamma\gamma}(s)$, $\Pi_{ZZ}(s)$ and the 1PI photon-Z^0 mixing by $\Pi_{\gamma Z}(s)$ or $\Pi_{Z\gamma}(s)$. The complete 1PI corrections to the photon-e^+e^- and to the Z-e^+e^- vertices are written $V_{i_{h^e}\gamma}(s)[\bar{v}(p_{e^+})\gamma_\mu\gamma_{h^e}u(p_{e^-})]_e$ and $V_{i_{h^e}Z}(s)[\bar{v}(p_{e^+})\times\gamma_\mu\gamma_{h^e}u(p_{e^-})]_e$, respectively. Here u and v are spinor wave functions and $h^e=L,R$ is the helicity of the electron current. Similarly the complete 1PI photon-$f\bar{f}$ and Z-$f\bar{f}$ vertex corrections with final-state fermion helicity h^f are $V_{\gamma f_{h^f}}(s)[\bar{u}(p_f)\gamma_\mu\gamma_{h^f}v(p_{\bar{f}})]_f$ and $V_{Zf_{h^f}}(s)[\bar{u}(p_f)\gamma_\mu\gamma_{h^f}\times v(p_{\bar{f}})]_f$. The full 1PI multiparticle exchange corrections between initial and final states takes the form

$$B_{i_{h^e}f_{h^f}}(s,t)[\bar{v}(p_{e^+})\gamma_\mu\gamma_{h^e}u(p_{e^-})]_e\cdot[\bar{u}(p_f)\gamma_\mu\gamma_{h^f}v(p_{\bar{f}})]_f\,.$$

In what follows, the helicities and fermion currents will not be written out explicitly. The given expressions may be considered as form factors in definite helicity amplitudes. These helicity amplitudes are generally sufficient to construct any required S-matrix element including those in which bremsstrahlung is taken into account.

Baulieu and Coquereaux [17] have given the general form for the transverse parts of the dressed photon and Z^0 propagators and of the photon-Z^0 mixing. Using their results the complete scattering amplitude for the process $e^+e^-\to f\bar{f}$ is given by

$$\begin{aligned}A(s,t)=&V_{i\gamma}(s)\frac{s-M_0^2-\Pi_{ZZ}(s)}{[s-\Pi_{\gamma\gamma}(s)][s-M_0^2-\Pi_{ZZ}(s)]-\Pi_{\gamma Z}^2(s)}V_{\gamma f}(s)\\&+V_{i\gamma}(s)\frac{\Pi_{\gamma Z}(s)}{[s-\Pi_{\gamma\gamma}(s)][s-M_0^2-\Pi_{ZZ}(s)]-\Pi_{\gamma Z}^2(s)}V_{Zf}(s)\\&+V_{iZ}(s)\frac{\Pi_{Z\gamma}(s)}{[s-\Pi_{\gamma\gamma}(s)][s-M_0^2-\Pi_{ZZ}(s)]-\Pi_{\gamma Z}^2(s)}V_{\gamma f}(s)\\&+V_{iZ}(s)\frac{s-\Pi_{\gamma\gamma}(s)}{[s-\Pi_{\gamma\gamma}(s)][s-M_0^2-\Pi_{ZZ}(s)]-\Pi_{\gamma Z}^2(s)}V_{Zf}(s)+B(s,t)\,.\end{aligned} \tag{2.2}$$

The first term can be split into a resonant and nonresonant part and the resonating pieces collected with the result that

$$\begin{aligned}A(s,t)=&\left(\left[V_{i\gamma}(s)\frac{\Pi_{\gamma Z}(s)}{s-\Pi_{\gamma\gamma}(s)}V_{iZ}(s)\right]\cdot\left[V_{Zf}(s)+\frac{\Pi_{Z\gamma}(s)}{s-\Pi_{\gamma\gamma}(s)}V_{\gamma f}(s)\right]\right)\Big/\left(s-M_0^2-\Pi_{ZZ}(s)-\frac{\Pi_{\gamma Z}^2(s)}{s-\Pi_{\gamma\gamma}(s)}\right)\\&+\frac{V_{i\gamma}(s)\cdot V_{\gamma f}(s)}{s-\Pi_{\gamma\gamma}(s)}+B(s,t)\,.\end{aligned} \tag{2.3}$$

Note that in order to obtain the exact factorization of the numerator of the first term in Eq. (2.3), it is essential that the resonant piece of the photon propagator be included. It is convenient to localize the dominant behavior of the single term with a simple structure. Hence we write

$$A(s,t)=\frac{R_{iZ}(s_p)\cdot R_{Zf}(s_p)}{s-s_p}+\frac{R_{iZ}(s)\cdot R_{Zf}(s)-R_{iZ}(s_p)\cdot R_{Zf}(s_p)}{s-s_p}+\frac{V_{i\gamma}(s)\cdot V_{\gamma f}(s)}{s-\Pi_{\gamma\gamma}(s)}+B(s,t)\,, \tag{2.4}$$

in which

$$R_{iZ}(s)=\left[V_{i\gamma}(s)\frac{\Pi_{\gamma Z}(s)}{s-\Pi_{\gamma\gamma}(s)}+V_{iZ}(s)\right]F_{ZZ}^{1/2}(s)\,, \tag{2.5a}$$

$$R_{Zf}(s)=F_{ZZ}^{1/2}(s)\left[V_{Zf}(s)+\frac{\Pi_{Z\gamma}(s)}{s-\Pi_{\gamma\gamma}(s)}V_{\gamma f}(s)\right]. \tag{2.5b}$$

$F_{ZZ}(s)$ is defined from

$$s-M_0^2-\Pi_{ZZ}(s)-\frac{\Pi_{Z\gamma}^2(s)}{s-\Pi_{\gamma\gamma}(s)}=\frac{1}{F_{ZZ}(s)}(s-s_p)$$

and s_p is a solution to the equation

$$s_p-M_0^2-\Pi_{ZZ}(s_p)-\frac{\Pi_{Z\gamma}^2(s_p)}{s_p-\Pi_{\gamma\gamma}(s_p)}=0 \tag{2.6}$$

lying near the physical sheet in the resonance region. It should be emphasized that Eqs. (2.2)-(2.4) are exact expressions for the scattering amplitude. They are equivalent to Eq. (2.11) of **I** but here the factorization of the residue is displayed explicitly. The structure of Eq. (2.4) is remarkably simple being just a leading Breit-Wigner resonant term plus a background that is regular at $s=s_p$. The pole position s_p, the residue $R_{iZ}(s_p)\cdot R_{Zf}(s_p)$, and the background, comprised of the last three terms in (2.4), are all separately and exactly gauge invariant. They can be independently expanded as well-behaved series to any finite order in the coupling constant without introducing gauge dependence into the scattering amplitude. Provided s is not too close to a production threshold, the background can be expanded as a Taylor series in s and then truncated at some order.

A large number of multiparticle thresholds lie in the Z^0 resonance region. For example the thresholds for $Z^0\rightarrow W^+b\bar{b}s\bar{c}$ and $Z\rightarrow 10(b\bar{b})$, along with a host of others, sit densely under the umbrella of the resonance. These thresholds are admittedly very weak but all are associated with a branch point that opens up a new unphysical sheet. The solution obtained for Eq. (2.6) depends on exactly where (i.e., between which pair of thresholds) one crosses the real s axis from the physical region to an unphysical second sheet.

The question arises as to which of the poles, corresponding to the many possibile solutions of Eq. (2.6), represents the "true" Z^0 boson. In the resonance region, all of the poles that can be reached by crossing the real s axis from the physical region onto an unphysical second sheet lie roughly the same distance from the axis and exert a roughly equal influence on the physics. None can be attributed any special status. A manifestation of this is that, given sufficient experimental resolution, the resonance is not being adequately described by a Breit-Wigner function plus background corrections and one needs to determine as many pole positions as the experimental precision requires and allows. The Z^0 resonance can therefore not be considered as a single spectral line but rather as a closely spaced multiplet. In effect the resonance possesses a fine structure.

An upper limit for the maximum separation between components of the Z^0 multiplet can be straightforwardly estimated. Had the top quark had a mass $m_t\sim M_Z/2$, the $t\bar{t}$ threshold would have been the leading one in the Z^0 resonance region. The difference between the solutions to Eq. (2.6) on the Riemann sheet reached by crossing the real s axis immediately below the $t\bar{t}$ threshold and that reached by crossing immediately above is approximately

$$\Delta s_p\approx \mathrm{disc}_{t\bar{t}}\Pi_{ZZ}(s_p) \tag{2.7a}$$

$$\approx 2\pi i\frac{s_p}{16\pi^2}\left(1-\frac{4m_t^2}{s_p}\right)^{1/2}\left\{\frac{2}{3}(\beta_L^2+\beta_R^2)\left(1+\frac{2m_t^2}{s_p}\right)-(\beta_L-\beta_R)^2\left(\frac{2m_t^2}{s_p}\right)\right\} \tag{2.7b}$$

that yields a separation in $\sqrt{s}$ of $|\Delta\sqrt{s_p}|\sim\mathcal{O}(100\ \mathrm{MeV})$. Here $\mathrm{disc}_{t\bar{t}}\Pi_{ZZ}(s_p)$ is the discontinuity in $\Pi_{ZZ}(s)$ at s_p upon encircling the $t\bar{t}$ threshold in an anticlockwise direction [18]. The quantities β_L,β_R represent the left- and right-handed couplings of the top quark to the Z^0. Note that there is a suppression both by powers of α and by the velocity factor $(1-4m_t^2/s_p)^{1/2}$ but that the separation would produce easily resolvable effects at LEP.

The effect of thresholds on resonance peaks has been studied by Bhattacharya and Willenbrock [19]. They noted that near a threshold there will be two poles of physical relevance and that two sets of mass data should be quoted.

Given the known experimental fact that m_t is considerably larger than $M_Z/2$ [20], the dominant splitting, within the standard model, could come from a process like $Z^0\rightarrow Hf\bar{f}$ [21] for a suitable Higgs mass M_H. Suppression by powers of α and three-body phase-space factors put the splitting generated by this threshold, should it exist, beyond the resolving power of LEP. From these considerations it follows that, because the leading thresholds lie far below the resonance region, the position of the pole that is reached by crossing the real s axis in the resonance region is unique for present theoretical and experimental purposes.

(3) The physical mass and partial decay widths of the Z^0 boson.—The quantity s_p appearing in Eq. (2.4) has traditionally been used to define the mass and total width of an unstable particle. It should be stressed that the complex number s_p as a whole constitutes a physical property of the Z^0 boson. It is often split up into real and imaginary parts in an attempt to define a real mass M and total width Γ. Such definitions are purely matters of convention with the two most common being

$$s_p = M_1^2 - iM_1\Gamma_1 \text{ and } s_p = (M_2 - i\Gamma_2/2)^2 . \qquad (3.1)$$

There is no good physical reason to prefer one over the other and the two definitions yield significantly different numerical values. Thus the physical meaning of these quantities is dubious especially when the width is large or precision is high. In the case of the Z^0, $M_1 = M_2 - 8$ MeV, $\Gamma_1 = \Gamma_2 + 0.2$ MeV, and M_1 lies roughly 34 MeV below the on-shell scheme renormalized Z^0 mass in 't Hooft-Feynman gauge.

The factorization of the residue at the pole (2.4) is a manifestation of a general property of the scattering amplitude (see Ref. [22], p. 245). For stable particles it is known that the factors are themselves scattering amplitudes. For unstable particles the residue factors provide a means of defining generalized scattering amplitudes with external unstable particles that satisfy crossing and unitarity relations [23,24]. The residue factors of (2.5) are therefore gauge invariant by themselves and (2.5b) represents the amplitude for the process $Z^0 \to f\bar{f}$. Its magnitude constitutes a physical observable from which one might choose to define the partial decay width of the Z^0 to massless fermions as

$$\Gamma_{Z\to f\bar{f}} = \frac{1}{48\pi |s_p|^{1/2}} [|R_{Zf_L}(s_p)|^2 + |R_{Zf_R}(s_p)|^2] . \qquad (3.2)$$

Note that the above definition is not precisely the same as the partial width that is standardly calculated. The amplitude considered here is evaluated at the complex momentum, $s = s_p$, that corresponds to the unstable Z^0's being on shell. Most calculations of the partial width are evaluated at some real s, normally the renormalized mass. As was seen above, the renormalized mass is essentially arbitrary and there is no fundamental real momentum value associated with an unstable particle. The partial width calculated in this way will depend on which arbitrary momentum value one chooses and therefore cannot be a physical property of the Z^0 boson itself. Far from thresholds the definition (3.2) is greater by a factor of roughly $1 + \Gamma_Z^2/(8M_Z^2)$ as compared to the standard one evaluated at $s = M_Z^2$ and thus the difference is normally extremely small. Close to thresholds, however, the standard perturbative calculation blows up due to the Z^0 wave function renormalization factor, $F_{ZZ}^{1/2}(M_Z^2)$, being singular there [19,25,26]. This problem is not present in Eq. (3.2) because F_{ZZ} is evaluated at complex s_p.

It would be very satisfying, but is not at all necessary, if an exact simple relationship could be found between the sum of the partial widths as given above and the total width obtained by some decomposition of the position of the pole, s_p. Such a relation remains elusive. One can conclude, however, that the total width defined in either of Eqs. (3.1) is equal to the sum of the partial widths defined from the residue factors, Eq. (3.2), up to terms of relative weight $\mathcal{O}(10^{-4})$.

The author wishes to thank P. Landshoff for stressing the effects that thresholds can have. Useful discussions with R. Akhoury, A. Martin, and A. Mondragon are also gratefully acknowledged.

[a]Present address: Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109-1120.

[1] R. G. Stuart, Phys. Lett. B **262**, 113 (1991).
[2] A. Sirlin, Phys. Rev. D **22**, 971 (1980).
[3] S. Willenbrock and G. Valencia, Phys. Lett. B **259**, 373 (1991).
[4] A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991).
[5] A. Sirlin, Phys. Lett. B **267**, 240 (1991).
[6] T. F. Treml and G. Kunstatter, Winnipeg Report No. PRINT-91-0480 (to be published).
[7] B. F. L. Ward, Phys. Lett. B **296**, 209 (1992).
[8] J. Ellis, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics,* edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992), Vol. 2, p. 29.
[9] G. Passarino, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics* (Ref. [8]), Vol. 1, p. 56.
[10] H. Veltman, DESY Report No. DESY 92-076 (to be published).
[11] R. G. Stuart, Z. Phys. C **34**, 445 (1987).
[12] G. Passarino and M. Veltman, Phys. Lett. B **237**, 537 (1990).
[13] B. A. Kniehl and R. G. Stuart, Comput. Phys. Commun. **72**, 175 (1992).
[14] P. M. Stevenson, Nucl. Phys. **B203**, 472 (1982).
[15] A. Borrelli, M. Consoli, L. Maiani, and R. Sisto, Nucl. Phys. **B333**, 357 (1990).
[16] R. G. Stuart, Phys. Lett. B **272**, 353 (1991).
[17] L. Baulieu and R. Coquereaux, Ann. Phys. (N.Y.) **140**, 163 (1982).
[18] J. C. Polkinghorne, Nuovo Cimento **25**, 901 (1962).
[19] T. Bhattacharya and S. Willenbrock, Brookhaven Report No. BNL-56481 (to be published).
[20] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **68**, 447 (1992).
[21] F. Diakonos and W. Wetzel, Heidelberg Report No. HD-THEP-88-21 (to be published).
[22] R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge Univ. Press, Cambridge, 1966).
[23] H. P. Stapp, Nuovo Cimento **32**, 103 (1964).
[24] J. Gunson, J. Math. Phys. **6**, 827 (1965); **6**, 845 (1965); **6**, 852 (1965).
[25] J. Fleischer and F. Jegerlehner, Phys. Rev. D **23**, 2001 (1981).
[26] B. A. Kniehl, Nucl. Phys. **B357**, 439 (1991).