Comment on "Finite-Temperature Phase Transition in Metallic Spin Glass Alloys"

In a recent Letter, Matsubara and Iguchi [1] have argued from a finite-size scaling analysis of Monte Carlo data that the Ruderman-Kittel-Kasuya-Yosida (RKKY) model of metallic spin glass exhibits a finite-temperature phase transition in three dimensions. A few years ago, we have concluded from a similar study [2] that this model exhibits a $T_c=0$ transition with critical exponents $\eta=-1$ and $\nu\approx 0.9$. We show below that the data of Ref. [1] are, in fact, consistent with our conclusion about the nature of this transition if a sample-size dependence of the data arising from a noncritical effect inherent to the RKKY model is taken into account.

In Ref. [1], a finite-size scaling analysis is carried out for the data for the spin-glass susceptibility,

$$\chi_{SG}=\frac{1}{N}\sum_{i,j}\langle\langle \mathbf{S}_i\cdot\mathbf{S}_j\rangle_T^2\rangle_c\,, \tag{1}$$

where $\langle\cdots\rangle_T$ and $\langle\cdots\rangle_c$ represent thermal and configurational averages, respectively. In Ref. [2], we studied a closely related quantity $q^{(2)}$ defined as

$$q^{(2)}=\frac{1}{N(N-1)}\sum_{i\neq j}\langle\langle \mathbf{S}_i\cdot\mathbf{S}_j\rangle_T^2\rangle_c=(\chi_{SG}-1)/(N-1)\,. \tag{2}$$

For a $T_c=0$ transition with $\eta=-1$, these two quantities are expected to exhibit the following finite-size scaling behavior:

$$\chi_{SG}(N,T)\sim N\tilde{\chi}(TN^{1/3\nu}),\quad q^{(2)}(N,T)\sim\tilde{q}(TN^{1/3\nu})\,. \tag{3}$$

We pointed out in Ref. [2] that the measured values of $q^{(2)}$ (and χ_{SG}) exhibit a noncritical sample-size dependence arising from the very nature of the RKKY interaction. Because of the tendency of spin pairs separated by short distances to line up parallel or antiparallel to each other, the spins in typical low-temperature configurations are not distributed uniformly over the unit sphere. This causes the values of $q^{(2)}$ and χ_{SG} to be larger for smaller samples at low temperatures. Since this size dependence has nothing to do with that arising from critical effects, it is necessary to separate it from the raw data before carrying out a scaling analysis. In Ref. [2], this was (approximately) done by considering "corrected" values of $q^{(2)}$ defined by

$$q^{(2)}_{\rm corr}=q^{(2)}/3q^{(2)}(0)\,,$$
$$q^{(2)}(0)=\frac{1}{N(N-1)}\sum_{i\neq j}\langle\langle(\mathbf{S}_i\cdot\mathbf{S}_j)^2\rangle_T\rangle_c\,. \tag{4}$$

A different correction procedure which reduces to Eq. (4) for low T and large N has been suggested by Henley [3]. We believe that the apparent failure of the scaling form (3), shown in Fig. 4(a) of Ref. [1], results from the neglect of this effect. Our raw data for $q^{(2)}$ also show deviations from scaling very similar to that found in Fig. 4(a) of Ref. [1]. In order to determine whether the corrected values of $q^{(2)}$ obtained from the data of Ref. [1] would scale according to Eq. (3), we have estimated the values of $q^{(2)}(0)$ for the appropriate sample parameters by first obtaining simple empirical expressions for the N and T dependence of $q^{(2)}(0)$ from fits to our data and then using the temperature concentration scaling appropriate for RKKY systems [4]. The resulting scaling plot for $q^{(2)}_{\rm corr}$ is shown in Fig. 1 where we have combined our data ($N=20$, 44, 81, 161, and 312) with those extracted from Ref. [1] ($N=102$, 200, 346, 819, and 1600) [5]. The two sets of data points are found to collapse nicely to the same scaling curve, thereby validating the conclusion reached by us in Ref. [2] about the nature of the transition. It is not clear to us whether the "corrected" data of Ref. [1] would also be consistent with the scaling behavior appropriate for a finite-T_c transition. Even if they are, it would not be convincing evidence for a finite-T_c transition because such a scaling fit would involve three adjustable parameters, whereas the form (3) involves only one.

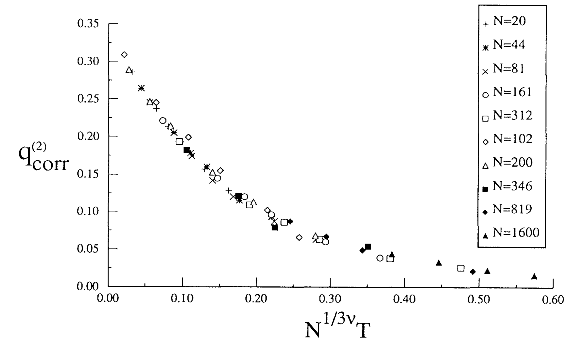

FIG. 1. $T_c=0$ scaling plot [Eq. (3)] of the "corrected" data for $q^{(2)}$ with $\nu=0.85$. The temperature T is measured in units of the reduced temperature defined in Ref. [2].

Amitabha Chakrabarti
Department of Physics, Kansas State University
Manhattan, Kansas 66506

Chandan Dasgupta
Department of Physics, Indian Institute of Science
Bangalore 560012, India

Received 23 October 1992
PACS numbers: 75.10.Hk, 75.10.Nr, 75.40.Mg

[1] F. Matsubara and M. Iguchi, Phys. Rev. Lett. **68**, 3781 (1992).
[2] A. Chakrabarti and C. Dasgupta, Phys. Rev. Lett. **56**, 1404 (1986); J. Phys. C **21**, 1613 (1988).
[3] C. L. Henley, Ann. Phys. (N.Y.) **156**, 324 (1984).
[4] J. Souletie and R. Tournier, J. Phys. (Paris), Colloq. **32**, C1-172 (1971).
[5] The $N=43$ data of Ref. [1] are not included in this plot because the corresponding $q^{(2)}$ values are substantially larger than our $N=44$ data at appropriately scaled temperatures. This discrepancy arises from a breakdown of temperature-concentration scaling for very small systems. The small deviations from scaling observed for the $N=102$ data arise due to the same reason. In contrast, our $N=312$ data agree well with the $N=346$ data of Ref. [1].