Meissner Effect in Quantum Hall State Josephson Junction

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The Meissner effect is analyzed in the Josephson junction predicted recently for a certain double-layer quantum Hall system. As a result of the Josephson current the magnetic field parallel to the layers is squeezed into a sine-Gordon vortex with the flux quantized in the unit of $2\pi/e$. A distinctive feature of the vortex is the appearance of an electric potential well perpendicular to the magnetic field within the layers. We also estimate the maximum strength of the parallel magnetic field which can be expelled outside the junction.

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The fractional quantum Hall (QH) effect is a remarkable property intrinsic to the planar electron system [1]. Recent experiments have shown [2] that the double-layer electron system possesses even- as well as odd-denominator QH states, which proves that the intrinsic nature of planar electrons is not lost even in the double-layer system. A Chern-Simons (CS) gauge theory of these QH states has been presented [3,4], which successfully accounts for various aspects of the phenomena. This microscopic formulation predicts [4,5] that a certain double-layer QH system acts as a Josephson junction when the filling factor takes an odd denominator; $\nu = 1/m$ with m an odd integer. See also Ref. [6]. This would be the first example where the Josephson effect is predicted in a system of semiconductors. The Josephson effect occurs due to coherent interlayer tunneling of the condensed bosonized electrons just like the Cooper pair tunneling in the superconductor Josephson junction. Since the bosonized electron carries the charge $-e$ as the electron does, the unit charge e appears instead of the $2e$ of the Cooper pair in various formulas of the Josephson effect in the double-layer QH system. Such a condensation is only allowed in a planar system where the statistics transmutation [7] is possible.

The aim of this paper is to analyze whether a kind of Meissner effect occurs in the QH state Josephson junction. We show that the magnetic field parallel to the layers is squeezed into a vortex just as in the superconductor Josephson junction. This is hardly expected because the QH system consists of semiconductors and not of superconductors. We also reveal some new features of the vortex peculiar to the QH state Josephson junction. They are the existence of an electric potential well with the size of the vortex and of an electric current parallel to the magnetic field B_P within the layers; both are absent in the superconductor Josephson junction.

Let us recapitulate the CS theory of a double-layer electron system, whose details are found in Refs. [4,8]. In this approach we represent electrons in terms of boson field ψ_α with the aid of the CS gauge field a_k^β together with the coupling constants $K^{\alpha\beta}$ called statistics parameters, where α and β are the layer indices: $\alpha, \beta = 1, 2$. The CS gauge fields have no independent dynamics and they are determined by the constraint equations

$$\varepsilon_{ij}\partial_i a_j^\alpha = 2\pi \sum_\beta K^{\alpha\beta}\psi_\beta^\dagger\psi_\beta, \quad (1)$$

where $K^{\alpha\beta} = K^{\beta\alpha} =$ integer. They ensure that $2\pi K^{\alpha\beta}$ flux quanta of the statistical gauge field a_j^α are attached to each bosonized electron ψ_β in the layer β, transmuting it back into the original electron together with their relative statistics [3,4,8,9].

The QH state is characterized by the uniform condensation of all ψ_α, $\langle\psi_\alpha\rangle \neq 0$, which breaks the CS gauge symmetry $\psi_\alpha \to e^{\Lambda_\alpha}\psi_\alpha$. In general, since the resulting two Goldstone modes are absorbed by the two CS gauge fields a_k^α and disappear, the state is incompressible. However, it is compressible when all the statistics parameters $K^{\alpha\beta}$ are the same: $K^{\alpha\beta} = m$ with m being an odd integer. This is because in this case the constraint equations (1) are reduced to only one equation:

$$\varepsilon_{ij}\partial_i a_j = 2\pi m(\psi_1^\dagger\psi_1 + \psi_2^\dagger\psi_2), \quad (2)$$

where $a_k = \frac{1}{2}(a_k^1 + a_k^2)$, and the combination $a_k^1 - a_k^2$ decouples from the system. The system contains only one gauge field a_k, which absorbs only one of the Goldstone modes. Hence, one of the Goldstone modes survives, leading to a gapless mode in the QH state.

This situation is described by our Hamiltonian [4]

$$\mathcal{H} = \sum_\alpha \left(\frac{1}{2M}|(D_1 - iD_2)\psi_\alpha|^2 + \mu_\alpha|\psi_\alpha|^2\right) + \frac{1}{2}\omega_c N - \lambda(\psi_1^\dagger\psi_2 + \psi_2^\dagger\psi_1) + \sum_{\alpha,\beta}\mathcal{V}_{\alpha\beta}[\psi], \quad (3)$$

with $iD_k = i\partial_k + a_k - eA_k$. Here, M is the effective mass of electrons, $-e$ the charge of the electron, N the total number of electrons, μ_α the chemical potential, and λ the strength of the interlayer tunneling. The external magnetic field B is applied perpendicular to the layers with $A_k = -\frac{1}{2}eB\varepsilon_{kj}x^j$, and $\omega_c = eB/M$ is the cyclotron frequency. Later we shall consider an additional magnetic field B_P parallel to the layers ($B_P \ll B$). Terms $\mathcal{V}_{\alpha\beta}[\psi]$ represent the intralayer and interlayer Coulomb interactions that drive the planar electron system into the QH liquid.

In our formalism the lowest Landau level (LLL) projection is implemented by the condition

$$(D_1 - iD_2)\psi_\alpha = 0 \quad (4)$$

in the mean-field approximation.

Switching off the interlayer tunneling ($\lambda = 0$), we find that the condensation of ψ_α occurs at the filling fraction $\nu \equiv 2\pi(\rho_1 + \rho_2)/eB = 1/m$, which is characterized by the mean-field solution

$$a_k = eA_k, \qquad \psi_\alpha = \sqrt{\rho_\alpha}e^{i\theta_\alpha}, \quad (5)$$

with constant density ρ_α and phase θ_α. This state minimizes the Coulomb energy, and hence it gives the ground state. Such a solution is only possible at the filling fraction $\nu = 1/m$, which is found by substituting (5) into the constraint equation (2). Because (2) does not fix the density of the electrons in each layer separately, any change of each density ρ_α does not break the condensation in the QH state as far as $\rho_1 + \rho_2$ is fixed. This suggests the presence of a gapless mode associated with the change of the density $(\rho_1 - \rho_2)$, which can actually be proven within the Gaussian approximation around the mean-field solution [4]. It is the Goldstone mode we mentioned before. We note that its existence has also been studied in other approaches [10]. This gapless mode is the origin of the Josephson effect in the presence of tunneling.

Equation (5) implies that the CS gauge field cancels precisely the external magnetic field perpendicular to the layers. Thus, bosonized electrons do not feel effectively any magnetic field and condense with energy $\frac{1}{2}\omega_c N$, as revealed explicitly in the Hamiltonian (3). It can be shown that the state is described by the Halperin wave function [11]

$$\prod(z_i^1 - z_j^1)^m \prod(z_k^2 - z_\ell^2)^m \prod(z_i^1 - z_k^2)^m \times \exp\left\{-\frac{1}{4}eB\left(\sum|z_i^1|^2 + \sum|z_k^2|^2\right)\right\}, \quad (6)$$

by taking account of the Gaussian fluctuation around the mean-field solution [4]. Note that in this QH state the electrons are strongly correlated between the layers as well as within each layer as indicated by the same power m; hence, to realize such a state the interlayer distance d should be taken to be comparable with the magnetic radius $\ell_B = 1/\sqrt{eB}$.

The small interlayer tunneling term ($\lambda \neq 0$) causes coherent tunneling between the two layers. Using the LLL condition (4), the well-known equations of motion [12] for the coherent tunneling are easily derived [4]:

$$i\partial_0\psi_1 = \mu_1\psi_1 - \lambda\psi_2, \quad i\partial_0\psi_2 = \mu_2\psi_2 - \lambda\psi_1. \quad (7)$$

Substituting (5) into these equations, we find that

$$\partial_t\theta_\alpha = -\mu_\alpha + \lambda\cos\delta - \lambda, \quad (8)$$

where $\delta \equiv \theta_1 - \theta_2$ is the phase difference; here, we have shifted $\mu_\alpha \to \mu_\alpha + \lambda$ so that $\partial_t\theta_\alpha = 0$ when there is no phase difference ($\delta = 0$) and no external voltage ($\mu_\alpha = 0$). It follows [12] from (7) and (8) that the phase difference satisfies

$$\partial_t\delta = eV, \quad (9)$$

and that the Josephson current is given by

$$J_z \equiv \partial_t\rho_1 = J_c\sin\delta, \quad (10)$$

where $J_c \equiv 2\lambda\rho_0$ with ρ_0 being the average electron density on one layer. Here, $eV \equiv \mu_2 - \mu_1$ is the electric potential difference applied across the layers; the index z in the Josephson current indicates that it flows in the direction of the z axis taken perpendicular to the layers.

We now analyze the screening of the external magnetic field parallel to the layers due to the Josephson current. We first show that it is squeezed into a vortex with the flux quantized in the unit of $2\pi/e$. Although the equations governing the parallel magnetic field B_P inside the QH state junction turn out to be essentially the same as those in the superconductor junction [13], their derivation is quite different from the latter case.

Our basic equation is the LLL condition (4), or

$$\begin{aligned} iD_k\psi_\alpha &= (i\partial_k + a_k - eA_k - eA_k^P)\psi_\alpha \\ &= -\sqrt{\rho_0}(\partial_k\theta_\alpha + eA_k^P)e^{i\theta_\alpha} = 0, \end{aligned} \quad (11)$$

where $\psi_\alpha = \sqrt{\rho_0}e^{i\theta_\alpha}$ and A_i^P is the electromagnetic potential describing the parallel magnetic field B_P. Let us take this magnetic field in the direction of the x axis and assume it to be uniform in x; then, θ_α as well as B_P do not depend on x. In the gauge $A_z^P = 0$, the above equations yield

$$\begin{aligned} \partial_y\delta &= -e[A_y^P(z = z_1) - A_y^P(z = z_2)] \\ &= -ed\partial_z A_y^P = eB_P d, \end{aligned} \quad (12)$$

where we have assigned the coordinates z_α to each layer, and $d \equiv z_1 - z_2$ is the distance between the two layers. The above manipulation is justified when B_P is smooth in z between the two layers. The y dependence of the phase difference δ is related to the parallel magnetic field B_P through this equation.

In order to take into account the screening effect due to the induced Josephson current we consider a Maxwell equation, $\partial_y B_P = eJ_z - \varepsilon\partial_t E_z$ with ε being the dielectric constant of the matter between the layers. Using (10) and $E_z = -V/d$ we obtain

$$\partial_y B_P = eJ_c \sin\delta + \varepsilon d^{-1}\partial_t V. \tag{13}$$

A combination of (9), (12), and (13) gives a sine-Gordon equation,

$$(\varepsilon\partial_t^2 - \partial_y^2)\delta + e^2 d J_c \sin\delta = 0\,, \tag{14}$$

inside the junction.

Let us first consider a junction with an infinite size. There exist soliton solutions that are characterized by the boundary condition $\delta(y=+\infty)-\delta(y=-\infty)=2\pi n$, with n being an integer. They represent vortices confined in the y direction within the size $\ell_V \sim 1/\sqrt{e^2 d J_c}$, which is the penetration depth. This topological nature of vortices makes the magnetic flux Φ quantized in the unit of $2\pi/e$:

$$\Phi \equiv d\int_{-\infty}^{\infty} B_P dy = \frac{1}{e}\int_{-\infty}^{\infty}\partial_y\delta dy = \frac{2\pi}{e}n, \tag{15}$$

where use was made of (12).

In the above derivation of the flux quantization we have tacitly assumed that the magnetic flux is confined in the z direction between the two layers. Indeed, the factor d in the second term of formula (15) results from integration over the distance d between the two layers. In order to prove this confinement, we note that because of the current conservation the Josephson current (10) inevitably induces a current $J_y^1 = J_y$ in the y direction on the first layer and a current $J_y^2 = -J_y$ on the second layer. Here, evaluating the accumulated currents due to the Josephson current we find

$$J_y(y) = \int_{-\infty}^{y} J_z(y')dy' = \frac{1}{e^2 d}\partial_y\delta \tag{16}$$

for the static soliton solution satisfying $\partial_y\delta = 0$ at $y=-\infty$. The z dependence of the magnetic flux B_P is controlled by these currents according to a Maxwell equation, $\partial_z B_P = -eJ_y[\delta(z-z_1)-\delta(z-z_2)]$, which holds inside as well as outside the junction. Integrating this Maxwell equation we obtain

$$B_P(y,z) = -e[\theta(z-z_1)-\theta(z-z_2)]J_y(y) + B_0. \tag{17}$$

Here, by requiring the other Maxwell equation (13), the integration constant B_0 is found to be really a constant. It is the magnetic field outside the QH state junction. Inside the junction it reads

$$B_P = eJ_y + B_0 = \frac{1}{ed}\partial_y\delta + B_0 \tag{18}$$

for the static sine-Gordon soliton. Comparing this with (12) it is necessary that $B_0 = 0$. Namely, in the vortex solution the parallel magnetic field is confined between the two layers.

Although our analysis has been carried out in the approximation of layers without thickness, the finite thickness of the layers should obviously not change the above result about the squeezing. The only effect of finite thickness is that the parallel magnetic field confined between the two layers gradually decreases in the thickness of the layer and vanishes outside the junction.

We next consider the screening of an external magnetic field B_0 in the junction with a finite size L. In this case, we obtain

$$B_P = \frac{1}{ed}\left[\partial_y\delta(y) - \partial_y\delta(y=-L)\right] + B_0$$

instead of (18), where $y=-L$ denotes the end point of the junction. Now, comparing this with (12) we find the boundary condition

$$B_0 = (1/ed)\partial_y\delta(y=-L) \tag{19}$$

to be imposed on the phase difference δ for a given external field B_0. According to (19) and (14), when B_0 is sufficiently small it can penetrate into the junction only near the end points with the penetration depth ℓ_V. Namely, $B_P = \partial_y\delta/ed$ vanishes inside the junction. When it is increased it can penetrate inside the junction as the quantized vortices.

Let us make some numerical estimations. The typical size ℓ_V ($\sim 1/\sqrt{e^2 d J_c}$) and the magnetic field B_P ($\sim \pi/ed\ell_V$) of the vortex are of the order of 0.1 mm and 10^{-2} T, respectively, for $\lambda = 1$ K, $d = 100$ Å, and $\rho_0 = 10^{11}/\text{cm}^2$. The maximum strength of the external magnetic field B_0 that can be screened is also of the order of $\pi/ed\ell_V \sim 10^{-2}$ T from (19). Hence, when we adjust the alignment of the magnetic field B (~ 10 T) applied perpendicular to the layers within the accuracy of $\Delta\theta \sim (10^{-2}\text{ T})/(10\text{ T}) \sim 10^{-3}$, the parallel component B_0 of the magnetic field B due to the misalignment may be regarded as small enough to be screened.

We proceed to consider a QH system with DC-voltage feed or DC-current feed, where the applied magnetic field is so strong that the total flux passing between the two layers, $\sim B_0 L d$, becomes much larger than the flux carried by vortices, $\sim (2\pi/e)\times(L/\ell_V)$. Then, the induced Josephson current cannot screen completely the external magnetic field. In this case we may solve (9) and (12) by neglecting the screening effect due to the Josephson current. Then, the phase difference is given by $\delta = eVt + eB_P yd$ in the DC-voltage circuit. The Josephson current oscillates in space and time:

$$J = J_c\sin(eVt + eB_P yd). \tag{20}$$

The electromagnetic radiation induced by this oscillation must be observed in the double-layer system. On the other hand, in the DC-current circuit, we get

$$J = J_c\sin(\delta_0 + eB_P yd). \tag{21}$$

Evaluating the total Josephson current we obtain

$$J_{\text{total}} = L^2 J_c \sin\delta_0 \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0}, \tag{22}$$

with $\Phi_0 = 2\pi/e$ being the unit flux.

We have so far analyzed aspects of the Meissner effect in the QH state Josephson junction which are almost identical to the superconductor case [13]; that is, vortices with flux quantization (15) and penetration depth ℓ_V, Josephson current oscillation (20), and magnetic field dependence of the maximum DC Josephson current (22). The only difference is that the unit charge e appears in place of $2e$ in various formulas familiar in the superconductor case.

We now point out some distinctive features of vortices peculiar to the QH state junction. We only consider the case with the phase difference δ associated with the static sine-Gordon vortex (14). First of all, an electric field E_y in the y direction appears spontaneously due to the static but spatially varying phase difference δ. To see it we analyze (8) on each layer, which leads to $\mu_\alpha = \lambda(1-\cos\delta)$ for $\partial_t\theta_\alpha = 0$. This equation implies that the nontrivial chemical potential (equivalently the electric potential) is induced in association with the vortex soliton. Namely, the electric potential well $(-\mu_\alpha/e)$ appears in both of the layers; note that the electric charge of electron is $-e$. Thus, the electric field E_y is given as

$$E_y = -\frac{1}{e}\partial_y\mu_\alpha = -\frac{\lambda}{e}(\sin\delta)\partial_y\delta, \qquad (23)$$

which is identical in both of the layers. It is easy to check that the electric field E_y and the current J_y do not lead to any dissipative process, i.e., $\int_{-\infty}^{\infty} J_y E_y dy = 0$.

Second, the current J_x parallel to the magnetic field B_P appears in association with this electric field E_y in the QH state: $J_x = \sigma_{xy}E_y$. This current is quite small compared with J_y since $J_x/J_y \approx O(\lambda d) \approx O(10^{-5})$ where $d \approx O(100\ \text{Å})$ and $\lambda \approx O(1\ \text{K})$. These E_y and J_x exist only around the vortex. Their appearance is very peculiar to the QH system; an electric field can never appear in a superconductor.

We conclude this Letter with the following remarks. First, when the current J flows within the layer, there will be a voltage drop $\Delta V \sim JL^2\rho_{xx}$ due to a small but nonvanishing ρ_{xx} in an actual system. It is necessary to require that this drop be sufficiently small ($\Delta V \ll V$) so that the Josephson effect will not be destroyed. Using numerical values given before, this condition turns out to be $\rho_{xx} \ll 10^{-5}\ \Omega$ for $V \sim 10^{-4}$ V and $L \sim 1$ mm. Second, as we have already argued, in order to observe the Meissner effect the alignment of the perpendicular magnetic field needs to be adjusted within the accuracy of $\Delta\theta \sim 10^{-3}$. Finally, the Josephson effect will be observed not only at $\nu = 1/m$ but also in its plateau, where pseudoparticles are excited and trapped. Pseudoparticles do not carry the current in the plateau and hence have no effect on the Josephson current below the critical current J_c [14]. We hope that our predictions about the Josephson effect and the associated Meissner effect are verified in future experiments.

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