Mechanism for the Generation of 10^9 G Magnetic Fields in the Interaction of Ultraintense Short Laser Pulse with an Overdense Plasma Target

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The physical mechanism for the generation of very high "dc" magnetic fields in the interaction of ultraintense short laser pulse with an overdense plasma target originates in the spatial gradients and nonstationary character of the ponderomotive force. A set of model equations to determine the evolution of the "dc" fields is derived and it is shown that the "dc" magnetic field is of the same order of magnitude as the high frequency laser magnetic field.

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Because of the development of compact terrawatt lasers [1] delivering subpicosecond pulses, a new parameter regime has been opened up in the study of laser plasma interaction. At intensities of 10^{18} W/cm^2 or higher a whole host of new phenomena are predicted [2] and some of these predictions have been confirmed by experiments [3]. Recent numerical simulations by Wilks *et al.* [4] of the interaction of an ultraintense laser pulse with an overdense plasma target have revealed extremely high self-generated magnetic fields ~ 250 MG in the overdense plasma. It is the purpose of this Letter to establish the physical mechanism responsible for the generation of such high "dc" magnetic fields. By dc we mean quantities averaged over the fast laser frequency time scale but not over the whole laser pulse.

The first experimental observations of dc magnetic fields generated in laser plasma interaction by Stamper *et al.* [5] were explained in terms of a thermoelectric source [5] that originates in a hot collisional plasma when the gradient of the electron temperature ∇T is not collinear with the electron density gradient ∇n, i.e., $\partial \mathbf{B}^0/\partial t = \nabla \times \mathbf{u} \times \mathbf{B}^0 + \alpha \nabla n \times \nabla T$ where $\mathbf{B}^0$ is the dc magnetic field, $\mathbf{u}$ is the plasma velocity, and α is a coefficient [6]. However, the extremely high magnetic fields observed in the simulations of Wilks *et al.* [4] cannot be explained on the basis of the mechanism that was satisfactory for the earlier experiments because the magnitudes are too high and the physics underlying the above equation for $\mathbf{B}^0$ is not justified for ultraintense laser pulses. The mechanism for magnetic field generation proposed here is a result of dc currents driven by the spatial gradients and temporal variations of the ponderomotive force exerted by the laser on the plasma electrons. We give below a mathematical derivation of this result which predicts that the dc magnetic field to be of the same order of magnitude as the oscillating magnetic field of the laser pulse and this large magnetic field will last only for the duration of the pulse.

We show towards the end of this Letter that for $\omega t < (m_i/m)^{1/2}(\omega/\omega_e)(eA/mc^2)^{-1}$ the ion motion can be neglected during the laser pulse; ω_e is the plasma frequency of the target plasma, m and m_i are the electron and ion mass, respectively. A is the magnitude of the laser vector potential and $f=\omega/2\pi$ is the laser frequency. When the above inequality is violated the role of ion motion [7] in determining the dc field is treated at the end of this Letter. Thus, the system we treat first is described by Maxwell's equations and the relativistic equations that describe the motions of a cold electron fluid. Defining the vector field $\mathbf{P}=m\gamma\mathbf{v}-e\mathbf{A}/c$, where $\mathbf{v}$ is the electron fluid velocity, $\gamma=(1-v^2/c^2)^{-1/2}$, $\mathbf{A}$ is the vector potential of the electromagnetic field, $-e$ is the electron charge, and c is the light velocity, we obtain $\partial\mathbf{P}/\partial t = \nabla\times\mathbf{v}\times\nabla\times\mathbf{P}$ by taking the curl of the electron momentum equation. Thus, if initially at $t=0$, $\nabla\times\mathbf{P}=0$ everywhere, it follows from the above equation that $\nabla\times\mathbf{P}$ vanishes everywhere for all time. If in addition the electron fluid is still everywhere at $t=0$ before the arrival of the laser pulse then $\mathbf{P}=0$ at $t=0$ everywhere, and $\mathbf{P}=\nabla\psi$ for all time. Following the analysis of Chen and Sudan [8], we may then express the system of Maxwell and electron fluid equations as follows:

$$\partial\psi/\partial t + \gamma - 1 - \phi = 0\,, \tag{1}$$

$$\partial n/\partial t + \nabla\cdot n(\mathbf{a}+\nabla\psi)/\gamma = 0\,, \tag{2}$$

$$\epsilon^2\nabla^2\phi = (n-n_i)\,, \tag{3}$$

$$\epsilon^2(\nabla^2\mathbf{a} - \partial^2\mathbf{a}/\partial t^2 - \nabla\partial\phi/\partial t) = n(\mathbf{a}+\nabla\psi)/\gamma\,, \tag{4}$$

with the Coulomb gauge $\nabla\cdot\mathbf{a}=0$, $\gamma=[1+(a+\nabla\psi)^2]^{1/2}$, and $\epsilon^2=\omega^2/\omega_e^2$. The variables in this set are normalized: $\phi\to e\phi/mc^2$, $\mathbf{a}\to e\mathbf{a}/mc^2$, $\psi\to\omega\psi/mc^2$, $n\to n/\bar{n}$, $n_i\to n_i/\bar{n}$, $t\to t\omega$, and $\mathbf{r}\to\mathbf{r}\omega/c$ and the ion density $\bar{n}$ is taken to be uniform within the plasma at $t=0$. Neglecting ion motion is tantamount to setting $n_i=1$. We are also considering singly ionized ions with $Z=1$; generalization to $Z>1$ is trivial. This model has been employed by Chen and Sudan [8] in the limit $\epsilon^{-2}\ll 1$ for treating the propagation and self-focusing of short intense laser pulses in an underdense plasma.

To investigate the generation of a dc magnetic field in an overdense plasma target, we take the opposite limit $\epsilon^2\ll 1$ in Eqs. (1)-(4) and in addition consider the uniform plasma to occupy the region $z>0$. Thus, we take the ion density gradient length at the plasma-vacuum interface to be shorter than the laser wavelength $\lambda = c/f$, which would be the case for solid density plasmas created on a time scale short enough to disallow ion motion, a regime treated earlier by one [7] and two [4] dimensional simulations. The laser wave fronts are incident normally on the plasma-vacuum interface and the wave polarization may be linear or circular. Since we know from linear theory that the skin depth is of order $\lambda_e = c/\omega_e \ll \lambda$ the laser wavelength, we are justified in assuming that the gradients in z are much greater than the transverse gradients in (x,y) determined by the laser spot size. The laser pulse amplitude varies on a time scale slower than the laser period and accordingly we expand the quantities occurring in Eqs. (1)-(4) not in amplitude but in spatial gradients and slow temporal evolution:

$$\mathbf{a}_\perp = \tfrac{1}{2}[\mathbf{A}_\perp(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t)\exp - it + \text{c.c.}] + \epsilon\mathbf{A}_\perp^0(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t), \quad (5a)$$

$$a_z = \tfrac{1}{2}\epsilon^2[\mathbf{A}_z(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t)\exp - it + \text{c.c.}] + \epsilon^3\mathbf{A}_z^0(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t), \quad (5b)$$

$$\phi = \phi_0(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t) + \tfrac{1}{2}\epsilon[\phi_1(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t)\exp - it + \text{c.c.}], \quad (5c)$$

$$\psi = \psi_0(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t) + \tfrac{1}{2}\epsilon[\psi_1(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t)\exp - it + \text{c.c.}], \quad (5d)$$

$$n = 1 + n_0(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t) + \tfrac{1}{2}\epsilon[n_1(z/\epsilon, \epsilon\mathbf{x}_\perp, \epsilon^2 t)\exp - it + \text{c.c.}]. \quad (5e)$$

The quantities with subscript and superscript 0 are dc in the sense that they are time averaged over a laser period and the laser frequency is normalized to unity. On substituting the expansion (5) in Eqs. (1)-(4), to order ϵ^{-1} we obtain $\partial\psi_0/\partial\zeta = 0$ with $\zeta = z/\epsilon$, $\chi = \epsilon\mathbf{x}_\perp$, and $\tau = \epsilon^2 t$; and to order ϵ^0, $\partial\psi_1/\partial\zeta = 0$ and

$$\gamma_0 = (1 + \tfrac{1}{2}|A_\perp|^2)^{1/2} = 1 + \phi_0, \quad (6a)$$

$$\partial^2\phi_0/\partial\zeta^2 = n_0, \quad (6b)$$

$$\partial^2\mathbf{A}_\perp/\partial\zeta^2 = (1 + n_0)\mathbf{A}_\perp/\gamma_0 = (1 + \partial^2\gamma_0/\partial\zeta^2)\mathbf{A}_\perp/\gamma_0. \quad (6c)$$

Equation (6c) has the following solution for linear or circular polarization,

$$\tanh[\tfrac{1}{4}\sinh^{-1}(\mathbf{A}_\perp/\sqrt{2})] = (\exp - \zeta)\tanh[\tfrac{1}{4}\sinh^{-1}(\hat{\mathbf{A}}_\perp/\sqrt{2})], \quad (7)$$

where $\hat{\mathbf{A}}_\perp \equiv \mathbf{A}_\perp(z=0)$. Note that $\hat{\mathbf{A}}_\perp$ is the resultant amplitude of the incoming and reflected laser pulse. The density n_0 and potential ϕ_0 are obtained from the substitution of (7) in Eqs. (6a) and (6b); $\phi_0 = (1 + \frac{1}{2}|\mathbf{A}_\perp|^2)^{1/2} - 1$ and $n_0 = (\cosh\sinh^{-1}|\mathbf{A}_\perp|/\sqrt{2} - 1)^2$. Equation (7) is the *exact* nonlinear solution of the penetration of a uniform electric field in a plasma. In the limit $|\hat{\mathbf{A}}_\perp| \ll 1$ it reverts to the well known solution $\mathbf{A}_\perp = \hat{\mathbf{A}}_\perp \exp - z/\lambda_e$. The ponderomotive force generated by the quiver motion of the electrons and exhibited in $\partial^2\gamma_0/\partial\zeta^2$ piles up the electrons in the sheath thereby increasing the local plasma frequency ω_e and decreasing the local λ_e. This accounts for the much more rapid drop of the field $\mathbf{A}_\perp$ with z as determined by (7) until $|\mathbf{A}_\perp| \lesssim 1$ when the exponential decay becomes valid. A positive surface charge density σ_+ appears at the interface $z=0$ which is equal to $\int_0^\infty d\zeta\, n_0$ in accordance with the conservation of charge.

When the illumination of the laser pulse at $z=0$ is nonuniform spatially then we must carry the calculation to higher order in ϵ. To order ϵ one obtains for $\mathbf{A}_\perp^0$ and the amplitude of the fluctuating density and velocity potential,

$$\partial^2\mathbf{A}_\perp^0/\partial\zeta^2 - (1+n_0)\mathbf{A}_\perp^0/\gamma_0 = \gamma_0^{-1}(1+n_0)\nabla_\perp\psi_0 + \tfrac{1}{2}\{(n_1/\gamma_0 - [(1+n_0)/\gamma_0](\mathbf{A}_\perp\cdot\nabla_\perp\psi_0)/\gamma_0^2)\mathbf{A}_\perp^* + \text{c.c.}\}, \quad (8a)$$

$$n_1 = -i\mathbf{A}\cdot\nabla(1+n_0)/\gamma_0 = \partial^2\phi_1/\partial\zeta^2, \quad (8b)$$

$$\psi_1 = \mathcal{D}^{-2}\mathbf{A}\cdot\nabla(1+n_0)/\gamma_0 - i(\mathbf{A}_\perp\cdot\nabla_\perp\psi_0)/\gamma_0, \quad (8c)$$

where $\mathcal{D} \equiv \partial/\partial\zeta$, and $\mathcal{D}^{-1} = \int^\zeta d\zeta$.

To order ϵ^2 one obtains

$$\partial\psi_0/\partial\tau + [(\mathbf{A}_\perp^0 + \nabla_\perp\psi_0)\cdot\nabla_\perp\psi_0]/\gamma_0 + \tfrac{1}{2}\gamma_0^{-1}(\mathbf{A}_\perp\cdot\nabla_\perp\psi_1^* + \text{c.c.}) = 0, \quad (9a)$$

$$\partial n_0/\partial\tau + \mathbf{A}^0\cdot\nabla(1+n_0)/\gamma_0 + \nabla_\perp[(1+n_0)/\gamma_0]\nabla_\perp\psi_0 + \tfrac{1}{2}\mathbf{A}\cdot\nabla\{n_1^*/\gamma_0 + [(1+n_0)/\gamma_0](\mathbf{A}_\perp^*\cdot\nabla_\perp\psi_0)/\gamma_0^2 + \text{c.c.}\} = 0, \quad (9b)$$

$$\partial^2 A_z/\partial\zeta^2 - (1+n_0)A_z/\gamma_0 = -\mathcal{D}^{-1}\mathbf{A}\cdot\nabla(1+n_0)/\gamma_0. \quad (9c)$$

Finally to order ϵ^3 one obtains

$$\partial^2 A_z^0/\partial\zeta^2 - \gamma_0^{-1}(1+n_0)A_z^0 = \partial\phi_0^2/\partial\zeta\partial\tau - \tfrac{1}{2}\{\gamma_0^{-1}(1+n_0)[(\mathbf{A}_\perp\cdot\nabla_\perp\psi_0)/\gamma_0^2]A_z^* + \text{c.c.}\}. \quad (10)$$

In the limit of $|\nabla_\perp\psi_0| \ll \mathbf{A}_\perp$ and $|\mathbf{A}_\perp\cdot\nabla_\perp\psi_0/\gamma_0^2| \ll 1$, these equations simplify to

$$\partial^2 A_z^0/\partial\zeta^2 - \gamma_0^{-1}(1+n_0)A_z^0 = \partial^2\phi_0/\partial\tau\partial\zeta, \quad (11a)$$

$$\partial^2\mathbf{A}_\perp^0/\partial\zeta^2 - \gamma_0^{-1}(1+n_0)\mathbf{A}_\perp^0 = \gamma_0^{-1}(1+n_0)\nabla_\perp\psi_0, \quad (11b)$$

$$\partial n_0/\partial\tau + \mathbf{A}^0\cdot\nabla(1+n_0)/\gamma_0 + \nabla_\perp\cdot\gamma_0^{-1}(1+n_0)\nabla_\perp\psi_0 = 0\,, \tag{11c}$$

$$\partial\psi_0/\partial\tau + \gamma_0^{-1}(\mathbf{A}_\perp^0 + \nabla_\perp\psi_0)\cdot\nabla_\perp\psi_0 + \tfrac{1}{2}[\mathbf{A}_\perp\cdot\nabla_\perp\mathcal{D}^{-2}\mathbf{A}^*\cdot\nabla\gamma_0^{-1}(1+n_0) + \text{c.c.}] = 0\,. \tag{11d}$$

The case for axisymmetric illumination of the plasma-vacuum interface with intensity maximum at $r=0$ can be solved easily. Taking the $\nabla_\perp\cdot$ operation on Eq. (11b) we obtain

$$\partial^2\nabla_\perp\cdot\mathbf{A}_\perp^0/\partial\zeta^2 - \gamma_0^{-1}(1+n_0)\nabla_\perp\cdot\mathbf{A}_\perp^0 = -\partial n_0/\partial\tau - A_z^0\partial\gamma_0^{-1}(1+n_0)/\partial\zeta\,. \tag{12}$$

Because $\partial/\partial\theta=0$, $\nabla_\perp\cdot\mathbf{A}_\perp^0=\rho^{-1}\partial\rho A_r^0/\partial\rho$ with $\rho=\epsilon r$. Note that Eq. (11a) is a linear inhomogenous second-order equation for A_z^0. The right-hand side is determined by the zero-order solution (7) which in turn is governed by $\hat{\mathbf{A}}_\perp(z=0,\epsilon r,\epsilon^2 t)$, i.e., the spatial and temporal variation of the driving field at $z=0$ which is given. The coefficient $\gamma_0^{-1}(1+n_0)\to|\mathbf{A}_\perp|\sqrt{2}$ in the limit $|\mathbf{A}_\perp|\gg1$ is also determined by (7). The boundary conditions at $\zeta=0$ are determined by charge conservation, i.e., $A_z^0(1+n_0)/\gamma_0|_{\zeta=0}=\partial\sigma_+/\partial\tau$ where $\sigma_+=\int_0^\infty n_0 d\zeta$ is the positive surface charge at $\zeta=0$. Thus, with A_z^0 known and $\partial n_0/\partial\tau$ obtained from (7), we can proceed to solve a similar linear inhomogeneous equation for $\nabla_\perp\cdot\mathbf{A}_\perp^0=F(\rho,\zeta)$. Thus, we have reduced an extremely nonlinear problem to a set of linear equations for the dc or average magnetic field.

The dc magnetic field is given by

$$\begin{aligned}\mathbf{B}^0 &= \nabla\times(\epsilon\mathbf{A}_\perp^0 + \epsilon^3\hat{\mathbf{z}}A_z^0)\\ &= \epsilon\nabla\times\mathbf{A}_\perp^0 + \epsilon^3\nabla_\perp A_z^0\times\hat{\mathbf{z}} \approx \hat{\boldsymbol{\theta}}\epsilon\partial A_r^0/\partial z\\ &= \hat{\boldsymbol{\theta}}\partial A_r^0/\partial\zeta\,.\end{aligned}$$

Thus, the *dc magnetic field is of the same order as the laser oscillating magnetic field and occupies a thickness of the order of the skin depth.* If $a=10$ then the oscillating field $\tilde{B}=10^9$ G and the dc magnetic field is of the same order, which is consistent with the magnitude of the 250 MG field observed by Wilks *et al.* [4] for $a\approx3$. Without explicitly solving these linear equations it is clear that the magnetic field will have a maximum at $z=0$ and falls off rapidly with z at a rate determined by $\mathbf{A}_\perp$. It vanishes on the $r=0$ axis, and has a maximum at some radius that varies with z. Figure 1 gives the schematic nature of the lines of current flow giving rise to B_θ. All of the return current flows as a surface current on the plasma-vacuum interface to satisfy the boundary condition that the dc field vanishes in vacuum.

Ion motion due to the laser ponderomotive force acting as a piston on the ions can be accounted for in the following manner. We neglect the transverse oscillatory motion of the ions and concentrate on the average z motion of the ions which satisfy the continuity and momentum balance equations:

$$\partial n_i^0/\partial t + \partial n_i^0 u^0/\partial z = 0\,, \tag{13a}$$

$$\partial u^0/\partial t + u^0\partial u^0/\partial z = -(m/m_i)\partial\phi_0/\partial z\,, \tag{13b}$$

where n_i^0 and u^0 are the time averaged ion density and z velocity, respectively. We express Eqs. (13a) and (13b) in Lagrangian variables through a coordinate transformation [9] $(z,t)\leftrightarrow(\xi(z,t),T)$ such that $\xi=z$ at $t=0$ and $d\xi/dt=\partial\xi/\partial t+u^0\partial\xi/\partial z=0$, $T=t$. Thus, because of mass conservation, $\partial\xi/\partial z=n_i^0(\xi,T)/N(\xi)$, where $N(\xi)$ is the initial density profile at $T=0$. In the new coordinates Eqs. (13a) and (13b) become

$$\partial(N/n_i^0)/\partial T - \partial u^0/\partial\xi = 0\,, \tag{14a}$$

$$\partial u^0/\partial T = -(m/m_i)(n_i^0/N)\partial\phi_0/\partial\xi\,. \tag{14b}$$

Combining these two equations and recognizing that $\phi_0\equiv\phi_0(z/\epsilon)=\phi_0(\xi/\epsilon)$ we obtain

$$\partial^2(N/n_i^0)/\partial T^2 = -(m/m_i\epsilon^2)(\partial/\partial\xi')(n_i^0/N)\partial\phi_0/\partial\xi'\,, \tag{15}$$

where $\xi'=\xi/\epsilon$ and $\phi_0=\phi_0(\xi',\chi,\tau)$. From (15) if $T<T_0\equiv(m_i/m)^{1/2}(\omega/\omega_e)\phi_0^{-1}$ with $\phi_0\sim\hat{A}_\perp$ for $\hat{A}_\perp\gg1$, we may neglect ion motion in the solution of Eq. (6c) as stated earlier but compute ion motion from (15) employing the previously obtained solution for ϕ_0. However, for $T>T_0$ Eq. (6c) must be solved in conjunction with Eq. (15). The displacement of the plasma-vacuum interface $\xi(0,T)$ depends on the laser intensity profile with $\mathbf{x}_\perp$

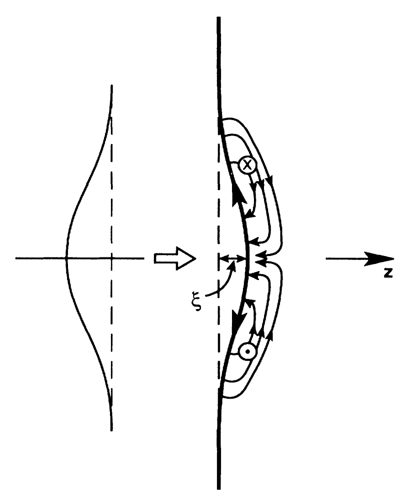

FIG. 1. Schematic of normal current contours driven by ponderomotive force; laser intensity profile is on left, $\xi(0,T)$ is displacement of the plasma-vacuum interface; the z extent of the dc magnetic field is of the order of the skin depth.

through the dependence of ϕ_0 on $A_\perp$ and the interface will therefore acquire the concave appearance observed in the simulations of Wilks *et al.* [4]. This solution will be valid until $n_i^0(\xi,T)$ becomes singular and multistreaming takes place. At this point a kinetic treatment for ions becomes necessary. Ions are reflected and ion viscosity will develop between incoming and reflected streams [7] due to two-stream collective instabilities. The principal dissipative mechanisms for the laser are (i) the ion-ion instability and (ii) collective electron-ion instabilities driven by the quiver motion for which a kinetic treatment is required even for the electrons.

It is important to note that the magnitude of the magnetic field $\mathbf{B}^0$ is determined by $\hat{A}_\perp$ the laser vector potential at the plasma-vacuum interface. This in turn is determined by the incident and reflected laser wave amplitudes. Initially the laser reflection coefficient is high and $\hat{A}_\perp$ is small. When dissipation in the skin layer increases because of collective instabilities then $\hat{A}_\perp$ will also increase. Thus the observed dc magnetic field will probably occur after the electron and ion motion causes strong dissipation to make $\hat{A}_\perp$ significant.

The physical origin of the dc magnetic field lies with the electrons which are expelled in the z direction from the surface $z=0$ by the ponderomotive force of the laser pulse and pile up as $1+n_0$ derived earlier following (7). This generates an average electron current in the z direction so long as the laser pulse is increasing in amplitude. Thus, the sign of B_θ is such that it is created by a normal current flowing in the negative z direction which corresponds to the direction of the magnetic field observed in the simulation of Wilks *et al.* [4]. After the laser pulse saturates in amplitude the motion of the plasma-vacuum interface given by (12) causes the ion and electron densities to increase and drive the current to generate the magnetic field.

The transverse currents in the plasma also result from the quiver motion of the electrons due to the transverse electric field. The electrons perform the well known figure-of-eight orbits in the r-z plane. The amplitude of this quiver motion rises with the rising amplitude of the driving laser pulse leading to an average current because of the inexact cancellation of the orbital motion in the successive half cycles of the laser field. Furthermore, because of spatial gradients in the transverse electric field, there is a spatial gradient in the amplitudes of the figure-of-eight orbits leading to average diamagneticlike currents familiar in the drift theory of charged particles in a magnetized plasma. Thus, the plasma in the very strong oscillating field of a laser exhibits some of the features of the familiar gyrotropic plasma. The neglect of electron ion collisions in our analysis is justified by the reduced Coulomb cross section of the electrons at relativistic quiver velocities.

The dc magnetic field in the Wilks *et al.* [4] simulation peaks well behind the plasma-vacuum interface (their Fig. 4) whereas the present analysis would indicate the peak to occur within the skin layer at the plasma-vacuum interface. This discrepancy may be due to the hot electrons with relativistic velocities perpendicular to the plasma-vacuum surface observed by Wilks *et al.* [4] [their Fig. 1(a)]. These electrons result from kinetic effects not included in the fluid model. They could also arise from the fact that in the phase which includes ion motion the plasma surface is dimpled by the laser radiation pressure. The p-polarized incident beam then is no longer strictly normal to the plasma surface and there may be a component of the incident electric field which is normal to the plasma surface and drives electrons into the plasma.

Finally, it must be recognized that the characteristic time for the dc magnetic field itself is the laser pulse time so that the process outlined above is really a down conversion of the laser frequency by the factor ϵ^2.

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