Experiments and Particle-in-Cell Simulation on Self-Oscillations and Period Doubling in Thermionic Discharges at Low Pressure

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The mechanism of different discharge modes, self-oscillations, and the period-doubling route to chaos is studied by comparing experimental results from a filament cathode discharge with particle-in-cell simulations. The self-oscillation process invokes ion trapping by charge exchange, double layer formation, and ion depletion. The exhausting of resources which underlies the period-doubling route is identified with incomplete ion refilling.

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Discharges with filament cathodes at pressures of about 10^{-1} Pa are in widespread use in magnetic box devices [1], double plasma arrangements [2], or ion sources [3]. Their inherent hysteresis of the $I(U)$ characteristic [4,5] has attracted much interest from the viewpoints of catastrophe theory [6] and nonlinear dynamics [7]. Under certain discharge conditions thermionic discharges perform low-frequency oscillations [8], which can be driven to chaos either by varying the discharge parameters [9] or by applying external modulation [10]. Routes to chaos via intermittency [11] and period doubling [7,12] were observed. Cheung and Wong [7] have discussed the role of ion dynamics and electron neutral collisions in period-doubling scenarios. In the present paper we intend to identify the detailed interplay of plasma processes, which lead to period-doubling transition to chaos.

Our investigations are performed in a magnetic box device with filament cathode [12] (Fig. 1). This device has been selected as a typical representative of this class of discharge. The discharge is operated in argon at $p = 0.03$–0.3 Pa. The anode voltage can be modulated by additionally applying periodic pulses of a few kHz frequency. The spatiotemporal evolution of plasma parameters and plasma potential is measured with movable Langmuir and emissive probes.

The general behavior of this discharge is compiled in Fig. 2. The $I(U)$ characteristic shows the well-known hysteresis curve. The upper branch represents the "temperature-limited mode" (TLM); the lower branch is the "anode-glow mode" (AGM) in the classical terminology for thermionic discharges [13,14]. In our case the states are similar to the above-mentioned collisional discharges despite the fact that the mean free path for electron and ion collisions with neutrals is comparable to the plasma dimensions. For these low pressure thermionic discharges the AGM is established by producing ions in the anode sheath and trapping them by charge exchange in the potential well of the virtual cathode. The resulting plasma potential distribution [Fig. 2(b)] shows a cathodic plasma, which is close to the cathode potential, and an anode layer, where the plasma potential increases to anode potential. From the discharge current we estimate the electron density to be of the order $n_e \approx 10^{14}$ m^{-3}. In the TLM, the plasma potential distribution is homogeneous and close to anode potential [Fig. 2(a)] except for a cathode sheath which is not accessible by emissive probes. The plasma parameters $n_e = 10^{16}$ m^{-3} and $T_e = 2$ eV are obtained from Langmuir probes. Self-oscillations of the discharge current with large amplitudes [Fig. 2(d)] are observed in the AGM close to the right hysteresis point. Their typical frequency lies between 1–2 kHz and depends on the detailed discharge parameters. The plasma potential increases during a current spike [Figs. 2(c),2(d)]. Potential and current spikes have a fast rising edge and a more gradual decay.

Large amplitude oscillations have been studied extensively via computer simulations for thermionic plasma converters and Q machines [8,15,16]. In our simulations

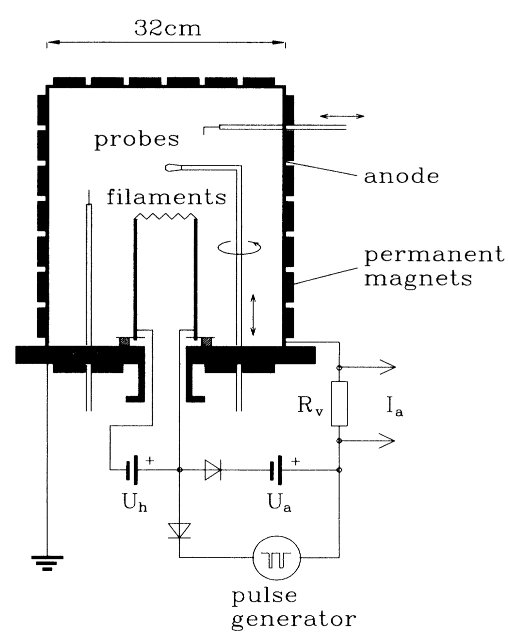

FIG. 1. Magnetic box discharge with filament cathode. A pulse generator is used for external modulation. The diagnostic tools are movable Langmuir and emissive probes.

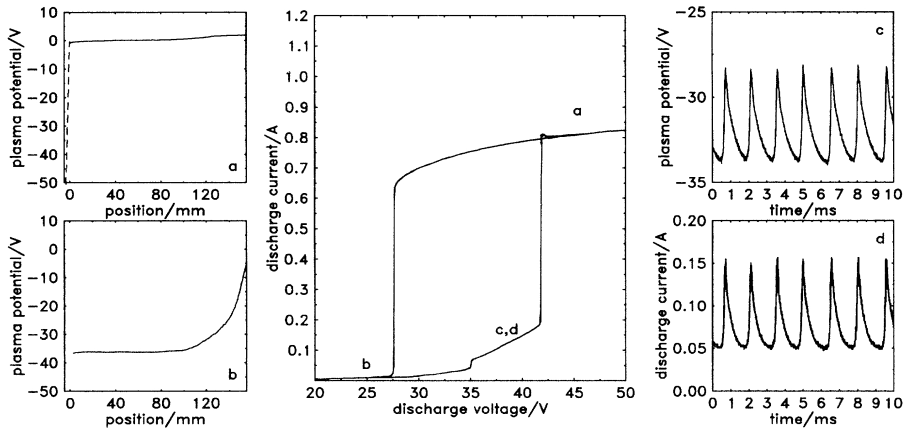

FIG. 2. Experimental results. Center panel: $I(U)$ hysteresis curve. Plasma potential distribution of the (a) temperature-limited mode and (b) anode-glow mode. The potential contours are recorded along the chamber axis. (c),(d) Experimentally obtained self-oscillations of plasma potential and discharge current.

the PIC-MCC code PDP1 [17] is used, which has one space dimension but three velocity components, and includes elastic, inelastic, charge exchange, and ionizing collisions. The self-spiking process is studied at a more realistic mass ratio ($m_i/m_e=10000$) than reported in [18]. The simulation parameters are chosen very similar to laboratory conditions (i.e., gas pressure $P=0.06$ Pa, discharge voltage $U_d=20$ V, length $L=15$ cm, filament temperature $T_f=2300$ K, collision cross sections for argon). The simulation starts with an empty electrode gap, and ion trapping by charge exchange fills the cathodic plasma. In the absence of ion losses the simulation always shows current spikes [Fig. 3(a)] and an increase of plasma potential. Including ion losses, self-spiking is only observed above a critical discharge voltage. The evolution of electron and ion phase space as well as the potential distribution during a single spike are given in Fig. 3(d). The sequence of events can be described by a filling phase (panels 1 and 2), during which ions are produced in the anode sheath and are trapped in the cathodic plasma, while the potential distribution is of AGM type. This cathodic plasma gradually expands towards the anode. Once the cathodic plasma becomes ion rich, the plasma potential in the cathodic plasma rises (panel 3). An electron hole [19] forms and transforms into a double layer (panel 4). By the double layer's electric field, electrons as well as ions of the cathodic plasma are accelerated in opposite directions. We always observe electrostatic waves at the plasma frequency which travel towards the anode. Ions streaming into the cathode sheath reduce the negative space charge and further enhance the current (panel 4). The accelerated ions represent an appreciable loss of ions in the cathodic plasma on the high potential side of the double layer which ultimately leads to its destruction. Finally, the current pulse is quenched by the formation of a negative potential dip [Fig. 3(c), panel 5].

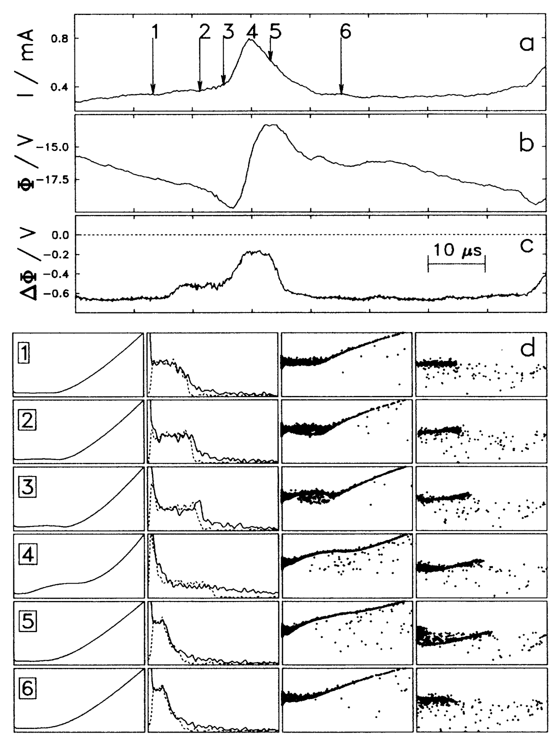

FIG. 3. PIC simulation of self-spiking process. (a) Discharge current, (b) plasma potential at $x=7.5$ cm, and (c) minimum plasma potential. The dashed line marks cathode potential. For (a) to (c), the horizontal axis is the time. (d) From left to right: Plasma potential (scale $\Phi=-22$–0 V), density distribution (dotted lines n_i, straight lines n_e, scale $n_{e,i}=0$–2.67×10^{12} cm^{-3}), electron phase space (scale $v_e=+2.5\times10^6$ m/s), and ion phase space (scale $v_i=\pm2.5\times10^4$ m/s) for the instants marked in (a). Cathode position is on the left ($x=0$ cm) and anode on the right ($x=15$ cm).

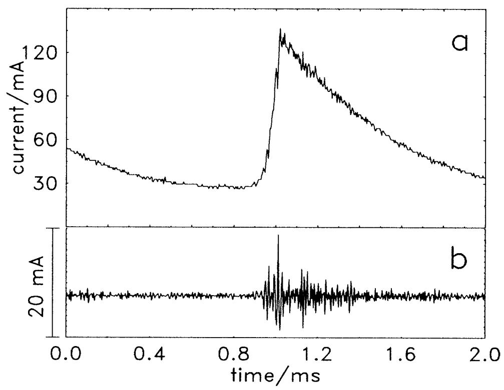

FIG. 4. Comparison of experimentally obtained current-spike (a), and the high pass filtered signal (b). The medium-frequency burst ($f \approx 50$ kHz) accompanies the rising edge of the current spike.

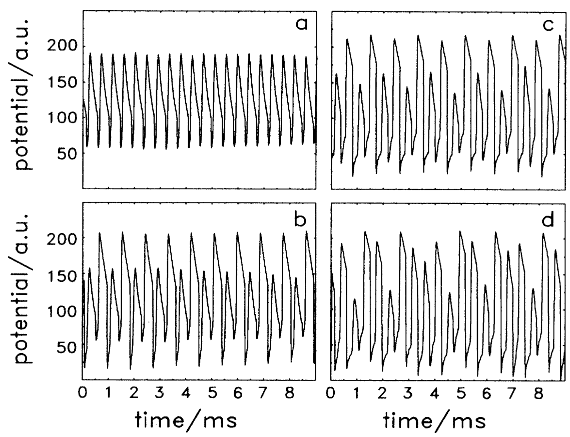

FIG. 5. Response of the plasma potential to external pulse modulation of the discharge experiment. By increasing the pulse width the system undergoes a period-doubling route to chaos. (a) Period $p=1$, (b) $p=2$, (c) $p=4$, and (d) chaos.

The cathodic plasma becomes electron rich, and the refilling starts again (panel 6). The repetition time of the current spikes is determined by the time scale for refilling the cathodic plasma. Hence, the spike frequency is strongly dependent on the neutral gas pressure. The nature of the described oscillations strongly resembles the potential relaxation instability [20]. In addition, similar dynamic potential structures were observed in particle-in-cell (PIC) simulations of positively biased Q machines [16].

Experimental evidence of a low-frequency instability which is associated with the current spikes is given in Fig. 4. The discharge current shows fluctuations with a typical frequency of 50 kHz which start at the rising edge of the current pulse and last for about 400 μs. We have also tried to detect the high-frequency fluctuations at the plasma frequency. But in contrast to observations in an arrangement with a longitudinal magnetic field [18], no substantial hf signals were found between 30–1400 MHz, where one would expect electron plasma waves.

In the experiment, there are two ways of driving the system from regular to chaotic motion. The first is by increasing the driver amplitude in mode-locked states [21]. In the second case, the dc discharge conditions are chosen on the AGM branch where self-spiking is not yet excited. Short pulses of about 2 kHz repetition frequency, which exceed the hysteresis point transiently, are applied. Consequently, the plasma potential as well as the discharge current response follow a period-doubling route to chaos (Fig. 5). Figure 6 shows the corresponding bifurcation diagram and 3D embedded phase space reconstructions [22] of the period doubled and the chaotic state. In [12] the equivalence of pulse and sine wave modulation has been demonstrated. In addition, the internal control parameter was identified to be the number of newly generated ions per period.

The shape of the chaotic attractor (Fig. 6) closely resembles the logistic mapping [23], which, roughly speaking, describes dynamical systems that undergo a period-doubling scenario because of exhausting their resources. To understand period doubling in the present plasma situation, it is therefore necessary to answer the following question: Why is a strong current pulse, or plasma potential enhancement, followed by a weak one?

This suppression mechanism is revealed by observing that the driver frequency has to be chosen close to twice the natural spiking frequency, which would appear at slightly increased dc operating voltage. This idea is fur-

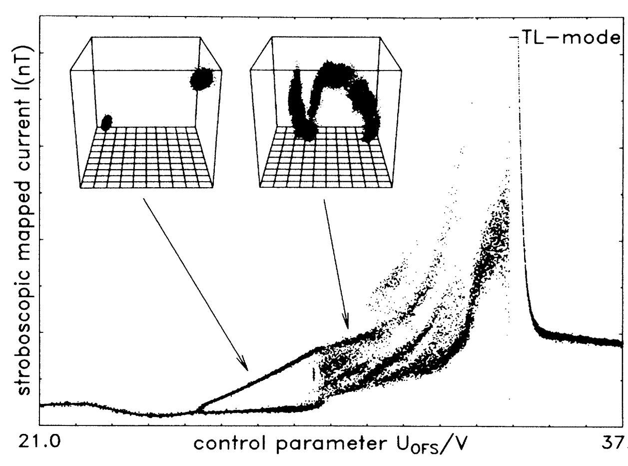

FIG. 6. Experimental bifurcation diagram and phase space reconstruction {coordinate axis $= [I(nT), I(n+1)T), I((n+2)\times T)]$} for the period-doubled and chaotic state. The control parameter is the dc offset U_{OFS} in sine modulation, which was used to vary the effective pulse width ($P=2$ at $U_{OFS}=26.7$ V, chaos at $U_{OFS}=29.6$ V). Note the transition to the TLM after passing a critical control parameter value $U_{OFS}=33.5$ V.

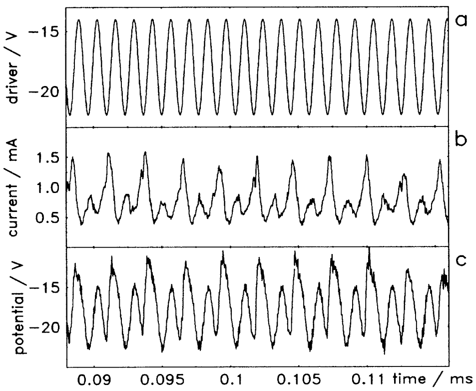

FIG. 7. PIC simulation of a period-doubled state. (a) External driver voltage, (b) discharge current response, and (c) potential response at $x=7.5$ cm.

ther corroborated by the fact that the self-spiking state can easily be mode locked [10,21]. In addition, for high modulation amplitude within the mode-lock two state, the phase space attractor is nearly identical with Fig. 6. First PIC simulations of the mode-lock two state [18] show that the discharge current forms a strictly alternating series of strong and weak current pulses (Fig. 7). For such a period-two state, the forced self-spiking mechanism occurs during the strong current pulse, whereas during a weak pulse no plasma reorganization occurs. Evidently, the formation of the double layer is subjected to a threshold condition, which may be related to the Buneman-type instabilities [24]. As a consequence, we suggest that the efficient depletion of ions from the cathodic plasma during a spike and the incomplete refilling by trapping charge-exchange ions forms the mechanism that gives the system a noninvertible dynamical return map resembling the logistic mapping. The measured quadratic shape of the return map immediately yields the possibility of generating period 4, 8, etc. and chaotic states.

To summarize, the connections between the mechanism of the AGM, the self-spiking process, and the period-doubling route have been established by comparing the behavior of a typical thermionic filament discharge with PIC simulations. The period-doubling scenario is intimately related to the ion depletion from the cathodic plasma during a strong current spike. Period doubling is also related to the chaos transition within the mode-lock two state.

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