Communicating with Chaos

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The use of chaos to transmit information is described. Chaotic dynamical systems, such as electrical oscillators with very simple structures, naturally produce complex wave forms. We show that the symbolic dynamics of a chaotic oscillator can be made to follow a desired symbol sequence by using small perturbations, thus allowing us to encode a message in the wave form. We illustrate this using a simple numerical electrical oscillator model.

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Much of the fundamental understanding of chaotic dynamics involves concepts from information theory, a field developed primarily in the context of practical communication. Concepts from information theory used in chaos include metric entropy, topological entropy, Markov partitions, and symbolic dynamics [1]. On the other hand, because of their exponential sensitivity, chaotic systems are often said to evolve randomly. This terminology is partially justified if one regards the information obtained by *detailed* observation of the chaotic orbit as being less significant than the statistical properties of the orbits. The object of this Letter is to show that we can use the close connection between the theory of chaotic systems and information theory in a way that is more than purely formal. In particular, we show that the recent realization that chaos can be controlled with *small* perturbations [2] can be utilized to cause the symbolic dynamics of a chaotic system to track a prescribed symbol sequence, thus allowing us to encode any desired message in the signal from a chaotic oscillator. The natural complexity of chaos thus provides a vehicle for information transmission in the usual sense. Furthermore, we argue that this method of communication will often have technological advantages.

Specifically, assume that there is an electrical oscillator producing a large amplitude chaotic signal that one wishes to use for communication. The so-called double scroll electrical oscillator [3] yields a chaotic signal consisting of a seemingly random sequence of positive and negative peaks. If we associate a positive peak with a 1, and a negative peak with a 0, the signal yields a binary sequence. Furthermore, we can use *small* control perturbations to cause the signal to follow an orbit whose binary sequence represents the information we wish to communicate. Hence the chaotic power stage that generates the wave form for transmission can remain simple and efficient (complex chaotic behavior occurs in simple systems), while all the complex electronics controlling the output remains at the low-power microelectronic level.

The basic strategy is as follows. First, examine the free-running (i.e., uncontrolled) oscillator and extract from it a symbolic dynamics that allows one to assign symbol sequences to the orbits on the attractor. Typically, some symbol sequences are never produced by the free-running oscillator. The rules specifying allowed and disallowed sequences are called the *grammar*. Methods for determining the grammar (or an approximation to it) of specific systems have been considered in several theoretical [4] and experimental [5] works. (In the engineering literature, a similar concept exists in the context of constrained communication channels.) The next step is to choose a code whereby any message that can be emitted by the information source can be encoded using symbol sequences that satisfy suitable constraints imposed by the dynamics in the presence of the control. (The construction of codes with such constraints is a standard problem in information theory [6], and will be discussed in the context of communicating with chaos, along with the required generalizations, in a longer paper [7].) The code cannot deviate much from the grammar of the free-running oscillator because we envision using only tiny controls that cannot grossly alter the basic topological structure of the orbits on the attractor. Once the code is selected, the next problem is to specify a control method whereby the orbit can be made to follow the symbol sequence of the information to be transmitted. Finally, the transmitted signal must be detected and decoded.

We now present a simple numerical example illustrating how the preceding strategy is carried out. Figure 1(a) is a schematic diagram of the electrical circuit producing the so-called double scroll chaotic attractor [3]. The nonlinearity comes from a nonlinear negative resistance represented by the voltage v_R in Fig. 1. (Different realizations of the negative resistance are possible; we have constructed one using an operational amplifier circuit, and are designing an experiment using this oscillator to demonstrate information transmission using chaos.) The differential equations describing the double scroll system are

$$C_1\dot{v}_{C_1}=G(v_{C_2}-v_{C_1})-g(v_{C_1})\,,$$
$$C_2\dot{v}_{C_2}=G(v_{C_1}-v_{C_2})+i_L\,,$$
$$L\dot{i}_L=-v_{C_2}\,.$$

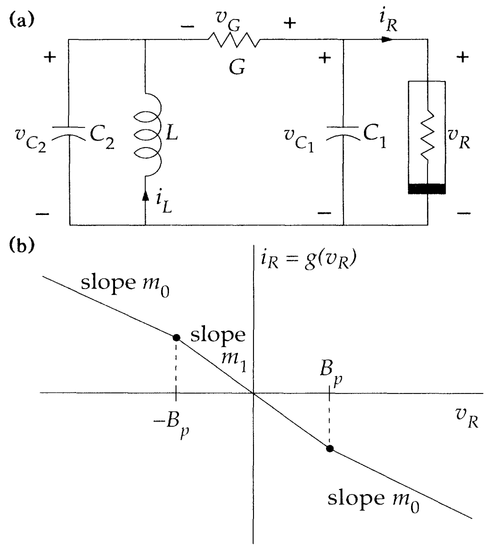

FIG. 1. Double scroll oscillator: (a) electrical schematic and (b) nonlinear negative-resistance i-v characteristic g.

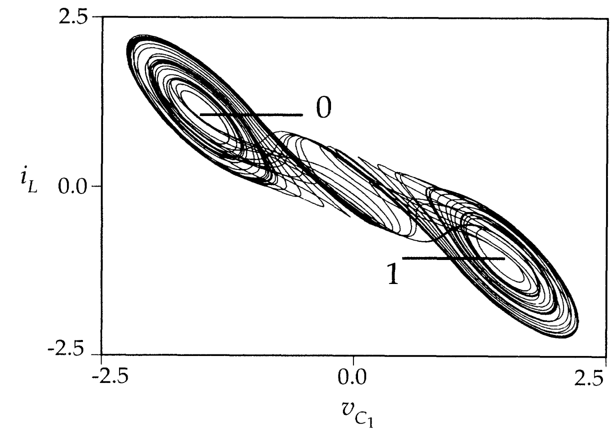

FIG. 2. Double scroll oscillator state-space trajectory projected on the i_L-v_{C_1} plane showing the two branches of the surface of section.

The negative-resistance i-v characteristic g is shown in Fig. 1(b). For our example, we use the normalized parameter values used by Matsumoto [3]: $C_1 = \frac{1}{9}$, $C_2 = 1$, $L = \frac{1}{7}$, $G = 0.7$, $m_0 = -0.5$, $m_1 = -0.8$, and $B_p = 1$. For a Poincaré surface of section (see Fig. 2), we take the surfaces $i_L = \pm GF$, $|v_{C_1}| \leq F$, where $F = B_p(m_0 - m_1)/(G + m_0)$, so that these half planes intersect the attractor with edges at the unstable fixed points at the center of the attractor lobes. Figure 2 shows a trajectory of the double scroll system with the two branches of the surface of section labeled 0 and 1. (The plane surfaces are edge-on in the picture.) The intersection of the strange attractor with the surface of section is approximately a single straight line segment on each of the two branches. Let x denote the distance along this straight line segment from the fixed point at the center of the respective lobe, $x = (F - |v_{C_1}|)\cos\theta + |v_{C_2}|\sin\theta$, where θ is the angle that the line segment makes with the i_L-v_{C_1} plane. Because absolute values are used in defining x, we can use the same x coordinate for both lobes of the attractor.

To construct a description of the symbolic dynamics of the system, we run the computer simulation without control. When the free-running system state point passes through the surface of section, we record the value of the generalized coordinate x (restricted to 1000 discrete bins for the computer simulation), and then record the symbol sequence that is generated by the system after the state point crosses through the surface. Suppose the system generates the binary symbol sequence $b_1 b_2 b_3 \ldots$. We represent this by the real number $0.b_1 b_2 b_3 \ldots$, so that each symbol sequence corresponds to the real number $r = \sum_{n=1}^{\infty} b_n 2^{-n}$, and symbols that occur at earlier times are given greater weight. We refer to the number r, specifying the future symbol sequence, as the *symbolic state* of the system. This defines a function mapping the state-space coordinate x on the surface of section to the symbolic coordinate r. This function $r(x)$ (which we call the *coding function*) is shown in Fig. 3. (The function gives actual symbol sequences when referring to the 0 lobe, and the bitwise complement when referring to the 1 lobe.) Because the oscillator is only approximately described by a binary sequence, multiple values of x lead to the same future symbol sequence. (We only need to track one of them. More sophisticated techniques both for symbol assignment and symbol sequence ordering are discussed in the longer paper [7].) Because the intersection of the attractor with the surface of section is only approximately one dimensional, there is a slight uncertainty in the symbolic state for some values of x; this uncertainty is indicated by the shading in the regions between the upper and lower bounds on the value of r in Fig. 3. Observations of the time wave form produced by the oscillator suggest that the grammar is simple: Any sequence of

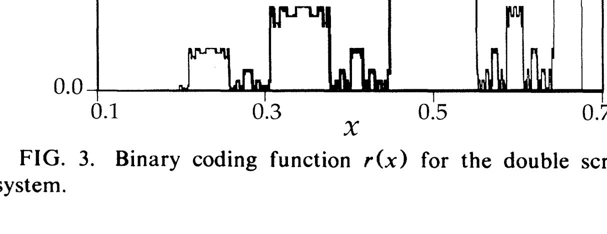

FIG. 3. Binary coding function $r(x)$ for the double scroll system.

binary symbols is allowed, except there can never be less than two oscillations of the same polarity. (We do not discuss the full grammar here, but instead adopt this simple grammar for simplicity of description.) This no-single-oscillation rule leads to a very simple coding: Insert an extra 1 after every block of 1's in the binary stream to be transmitted, and an extra 0 after every block of 0's. This altered data stream now satisfies the constraints of the grammar, and is uniquely decodable: Simply remove a 1 from every block of 1's upon reception, and a 0 from every block of 0's. Thus k oscillations of a given polarity represent $k-1$ information bits.

We now discuss how we control the system to follow a desired binary symbol sequence. Say the system state point passes through branch 0 of the surface of section (shown in Fig. 2) at $x=x_a$, and next crosses the surface of section (on either branch 0 or 1) at $x=x_b$. Because we have previously determined the function $r(x)$, we can use the stored values to find the symbolic state $r(x_a)$. We then convert the number $r(x_a)$ to its corresponding binary sequence truncated at some chosen length N, and store this finite-length symbol sequence in a *code register*. As the system state point travels towards its next encounter with the surface of section at $x=x_b$, we shift the sequence in the code register left, discarding the most significant bit (the leftmost bit), and insert the first desired information code bit in the now empty least significant slot (the rightmost slot) of the code register. We then convert this new symbol sequence to its corresponding symbolic state r_b'. Now, when the system state point crosses the surface of section at $x=x_b$, we use a simple search algorithm to find the nearest value of the coordinate x that corresponds to the desired symbolic state r_b'; call this x_b'. By construction, $|r(x_b)-r(x_b')| \leq 2^{-N}$. [If $r(x)$ is continuous, as in the Lorenz system, for example, this search can be replaced by a more efficient local derivative projection to find the desired value of x.] Now let $\delta x = x_b - x_b'$. Because we have chosen the branches of the surface of section at constant values of the inductor current i_L, the deviation δx in the generalized coordinate corresponds to a deviation in the voltages v_{C_1} and v_{C_2} across the two capacitors in Fig. 1. We thus apply a vector correction parallel to the surface of section (at constant i_L) along the attractor cross section to put the orbit at $x=x_b'$. This small correcting voltage perturbation is given by $\delta v_{C_1} = \pm\,\delta x \cos(\theta)$, $\delta v_{C_2} = \pm\,\delta x \sin(\theta)$, where the + signs are used for lobe 1 of the attractor, and the − signs for lobe 0. We plan to do this experimentally with current pulse generators connected in parallel with each capacitor. (Many methods of applying control perturbations are possible, but this one is particularly straightforward.) On each successive pass through the surface of section, a new code bit is shifted into the code register, and we repeat the procedure to correct the state-space coordinates, and thus the symbolic state, of the system. The coded information sequence, because it is shifted through the code register, does not begin to appear in the output wave form until N iterations of the procedure, where N is the length of the code register. If the symbol sequence is coming from a properly coded discrete ergodic information source, the process of shifting the information sequence through the code register can be viewed as locking the symbolic dynamics of the oscillator to the information source. Thus, there is a short transient phase during which the symbolic dynamics of the oscillator is being locked to the information source, and the symbolic dynamics of the oscillator is always N bits behind the information source.

Figure 4 shows an encoded wave form for the double scroll system produced by the described technique. This wave form corresponds to the voltage wave form $v_G(t)$ across the passive conductance G. If the conductance G is replaced by a transmission channel of the same impedance, the signal produced can be transmitted through the channel. We have represented each letter of the Roman alphabet by the five-bit binary number for its location in the alphabet, and added the extra bits to satisfy the no-single-oscillation constraint to encode the word "chaos." We have applied the technique to first bring the system to a periodic orbit about lobe 1 of the attractor, then to execute the writing of the word, and then to bring the system back to a periodic orbit about lobe 0. The trajectory shown in Fig. 2 is actually the encoded trajectory, but this is not apparent in the figure because the controlled trajectory approximates a possible natural trajectory. The root-mean-squared amplitude of the control signal over the writing of the word was of order 10^{-3} in the normalized units. The control probably cannot be made much smaller using this simple technique, primarily because the one-dimensional approximation in the surface of section causes the coding function to be slightly inaccurate. This control amplitude, though already very small compared to the oscillator signal voltages, does not

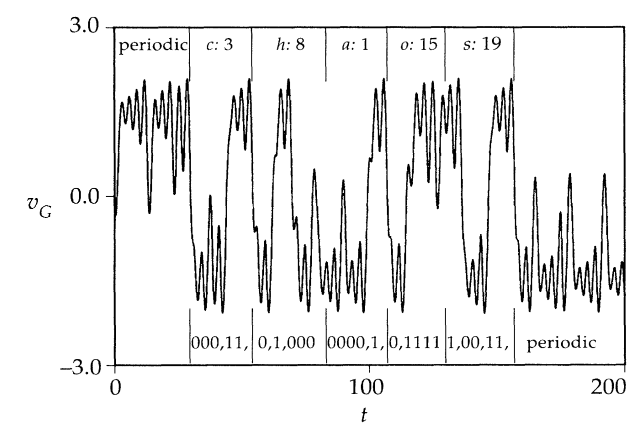

FIG. 4. Controlled $v_G(t)$ signal for the double scroll system encoded with "chaos." Each letter is shown at the top of the figure, along with its numerical position in the alphabet. Shown at the bottom are the corresponding binary code words. Extra bits (indicated by commas) are added to satisfy the constraints imposed by the grammar.

appear to be a fundamental limit, and we are developing control techniques to reduce it.

We conclude with some comments concerning the scope, application, and theoretical significance of our technique.

(1) Since we envision the transmitted signal to be a single scalar, its instantaneous value does not specify the full system state of the chaotic oscillator. If more state information is needed to extract the symbol sequence, time delay embedding [8] can be used. As our example using the double scroll equations shows, however, time delay embedding is not always necessary.

(2) Because our control technique uses only small perturbations [9], the dynamical motion of the system is approximately described by the equations for the uncontrolled system. Knowing the equations of motion greatly simplifies the task of removing noise [10] from a received signal. The basic bipolar nature of the signal in Fig. 4 implies that the message can still be extracted for noise amplitudes that are significant, but not too large compared to the signal. We consider the effects of additive noise on the detection of chaos signals quantitatively in the longer paper [7].

(3) Signals that are generated by chaotic dynamical systems and carry information in their symbolic dynamics have an interesting and possibly useful property: More than one encoded symbol can be extracted from a single sample of the trajectory if time delay embedding is used. This is done by using the state-space partition for a higher order iterate of the return map [7] of the system.

(4) Much of the theory needed to understand information transmission using the symbolic dynamics of chaotic systems already exists [11]. For example, because the topological entropy [12] of a dynamical system is the asymptotic growth exponent of the number of finite symbol sequences that the system can generate (given the best state-space partition), the channel capacity of a chaotic system used for information transmission is given by the topological entropy. The types of channel constraints that arise with a chaotic system will be discussed in a longer paper [7], along with other theoretical considerations.

(5) We emphasize that the particular methods for control and coding used in our double scroll example were chosen for simplicity, and that other more optimal methods are possible. Also, the double scroll oscillator itself was chosen because it is simple, and a large body of research is available about its dynamics. It is not intended as an example of a practical oscillator for communication wave form synthesis. It may be possible to use a higher-dimensional radio-frequency band-limited chaotic system for improved performance (higher information rate and better noise immunity), roughly analogous to the use of complex signaling constellations in classical communication systems. We are now developing more practical high-speed symbolic control techniques that could be used at higher bit rates than an implementation of the straightforward example given here.

(6) There has been much discussion of the role of chaos in biological systems, and we speculate that the control of chaos with tiny perturbations may be important for information transmission in nature.

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[1] For a pedagogical discussion emphasizing the role of concepts from information theory in chaos, see R. Shaw, Z. Naturforsch. **36A**, 80 (1981).

[2] (a) E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990); (b) T. Shinbrot, E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **65**, 3215 (1990).

[3] Several articles about this circuit appear in Proc. IEEE **75**, No. 8 (1987): e.g., T. Matsumoto, p. 1033.

[4] P. Cvitanović, G. Gunaratne, and I. Procaccia, Phys. Rev. A **38**, 1503 (1988); P. Grassberger, H. Kantz, and U. Moenig, J. Phys. A **22**, 5217 (1989).

[5] D. P. Lathrop and E. J. Kostelich, Phys. Rev. A **40**, 4028 (1989); L. Flepp, R. Holzner, E. Brun, M. Finardi, and R. Badii, Phys. Rev. Lett. **67**, 2244 (1991).

[6] R. E. Blahut, *Principles and Practice of Information Theory* (Addison-Wesley, Reading, MA, 1988); C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (University of Illinois Press, Chicago, 1963).

[7] S. Hayes, C. Grebogi, and E. Ott (to be published).

[8] F. Takens, in *Dynamical Systems and Turbulence,* edited by D. A. Rand, Springer Lecture Notes in Mathematics Vol. 898 (Springer-Verlag, New York, 1981), p. 366.

[9] Our control technique can also be used to target a chaotic system [2(b)] in state space. The relationship between symbolic states and state-space coordinates is established by the coding function $r(x)$.

[10] E. J. Kostelich and J. A. Yorke, Phys. Rev. A **38**, 1649 (1988); S. M. Hammel, Phys. Lett. A **148**, 421 (1990). It is easier to filter noise that is introduced in the communication channel than it is to filter noise that is present in the chaotic oscillator itself.

[11] A very different concept is that of using the synchronization of two chaotic systems [L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990)] for secure communications. In this case a small information-bearing signal is masked by a large chaotic signal. Several papers discussing this appear in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing* (IEEE, New York, 1992). This mechanism is not, however, based on the information theoretic formalism of chaos.

[12] R. L. Adler, A. G. Konheim, and M. H. McAndrew, Trans. Am. Math. Soc. **114**, 309 (1965).