## Comment on "Elastic String in a Random Potential"

In a recent Letter [1], Dong, Marchetti, Middleton, and Vinokur studied numerically the dynamics of an elastic string driven through a random medium by an external force $F$. The transition from a pinned to a moving string at a threshold force $F_T$ is characterized by a set of scaling exponents. The aim of this Comment is to show that, due to an incorrect procedure, their finding of a roughness exponent $\chi=0.97\pm0.05$ is inconclusive. Our simulation [2,3] of related models yields $\chi\simeq1.25$. A roughness exponent $\chi>1$ implies that the linear elasticity theory adopted in Ref. [1] is not self-consistent for a driven flux line in the plane at the depinning transition.

Using the notation of Ref. [1], the string is directed along the $z$ axis with the transverse displacement given by a single-valued function $x(z)$ defined on integer $z$. Denoting by $x(k)$ the Fourier transform of $x(z)$, we adopt the definition of the roughness exponent $\chi$ in $(1+1)$ dimensions through the expression $\langle x(k)x(k')\rangle \sim k^{-1-2\chi}\delta(k+k')$ for sufficiently small $k$. [This is the expression one usually uses in an analytical calculation of $\chi$. For a string of finite length, $x(k)$ is well defined.] In Ref. [1], the height-height correlation function

$$C_x(z)=\langle\overline{[x(z_0+z)-x(z_0)]^2}\rangle \tag{1}$$

was used to measure $\chi$, where the overbar and the angular brackets denote spatial and disorder averages, respectively. It was assumed that, after an initial transient period, $C_x(z)$ saturates to $C_x(z)\sim z^{2\chi}$. The latter relation, however, does not hold for $\chi>1$. In fact, $C_x(z)$ is bounded from above by $C_x(1)z^2$ in all cases. This follows immediately from the expression

$$C_x(z)=\sum_{i=0}^{z-1}\sum_{j=0}^{z-1}\langle\overline{\Delta x(z_0+i)\Delta x(z_0+j)}\rangle , \tag{2}$$

with $\Delta x(i)=x(i+1)-x(i)$, and the inequality

$$|\langle\overline{\Delta x(z_0+i)\Delta x(z_0+j)}\rangle|\leq\langle\overline{[\Delta x(z_0]^2}\rangle\equiv C_x(1) .$$

One can, however, determine $\chi$ from $C_x(z)$ using either the finite-size scaling form $C_x(z,L)\simeq L^{2\chi}\Phi(z/L)$ in the saturated regime or the early-time behavior $C_x(z,t) \simeq z^{2\chi}\Psi(z/t^{\beta/\chi})$ [4]. In the latter case the string is assumed to be straight at $t=0$ and $\beta$ is the dynamical roughening exponent. For $\chi>1$, $\Phi(y)\sim y^2$ and $\Psi(y) \sim y^{2-2\chi}$ for small $y$, so that $C_x(z)$ increases as $z^2$ for sufficiently small $z$, with an amplitude which grows either as $L^{2\chi-2}$ or as $t^{2\beta(1-1/\chi)}$, as the case may be. Using these scaling forms, we obtained $\chi\simeq1.25$ for a driven elastic string subjected to either random-field (RF) or random-bond (RB) disorder at the depinning transition in $(1+1)$ dimensions. The error bar on $\chi$ is about 1% its value for a RF model with discrete $x$ (Ref. [2]) and 7% with continuous $x$ (Ref. [3]). Our result for $\chi$ supports a recent suggestion by Narayan and Fisher [5] that the driven depinning transition for RF and RB disorder belong to the same universality class, but disagrees with their other claim [5] that $\chi=(4-D)/3$ is *exact* for the more general case of a $(D+1)$-dimensional interface at the driven depinning transition [6].

An important consequence of our result $\chi>1$ is that the mean-square fluctuation $C_x(1)$ of the local slope $dx/dz$ is no longer bounded in the limit $t,L\to\infty$; hence the description of a driven single flux line in the plane by an elastic string is not valid at the depinning transition. Given the anisotropy (in the sense of an orientation-dependent threshold force) which is usually present in real flux line systems, the depinning transition may very well belong to the universality class of an interface pinned by directed percolation clusters as discussed in Ref. [7].

The work is supported in part by the Deutsche Forschungsgemeinschaft under SFB 166 and 341.

Heiko Leschhorn
Fakultät für Physik und Astronomie
Ruhr-Universität Bochum
D-4630 Bochum, Federal Republic of Germany

Lei-Han Tang
Institut für Theoretische Physik
Universität zu Köln
Zülpicherstrasse 77
D-5000 Köln 41, Federal Republic of Germany

Received 25 February 1993
PACS numbers: 74.60.Ge, 05.60.+w, 68.10.−m

[1] M. Dong, M. C. Marchetti, A. A. Middleton, and V. Vinokur, Phys. Rev. Lett. **70**, 662 (1993).
[2] H. Leschhorn (to be published).
[3] L.-H. Tang (unpublished).
[4] F. Family and T. Vicsek, J. Phys. A **18**, L75 (1985).
[5] O. Narayan and D. S. Fisher, Harvard report, 1993 (to be published).
[6] T. Nattermann, S. Stepanow, L.-H. Tang, and H. Leschhorn, J. Phys. II (France) **2**, 1483 (1992).
[7] L.-H. Tang and H. Leschhorn, Phys. Rev. A **45**, R8309 (1992); S. V. Buldyrev, A.-L. Barabási, F. Caserta, S. Havlin, H. E. Stanley, and T. Vicsek, *ibid.* **45**, R8313 (1992).