Parity-Induced Suppression of the Coulomb Blockade of Josephson Tunneling

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(Received 23 September 1992)

We study the Josephson current through a superconducting grain coupled by two tunnel junctions to macroscopic superconducting leads. As a result of the Coulomb blockade we predict the critical Josephson current to have a threshold behavior when the superconducting gap is suppressed below the charging energy by a magnetic field. This effect is a direct consequence of a discrimination by the superconducting ground state between odd and even number of electrons on the grain. The critical current is periodically modulated by the voltage of a gate electrostatically coupled to the grain. We find a characteristic change in this modulation at the threshold.

PACS numbers: 73.20.Dx, 73.40.Gk

It is well known that the electrostatic Coulomb energy can suppress charge fluctuations on a grain coupled by tunnel junctions to macroscopic leads. The number of electrons forming the quantized grain charge can be controlled by the potential V_g of an external electrode (gate). For a nonsuperconducting grain, extra electrons are added to the grain one by one when the gate voltage changes [1]. For the superconducting grain the situation is different. If the charging energy is smaller than the magnitude of the superconducting gap Δ (as was implicitly assumed in [2]), the ground state of the grain has to contain an even number of electrons at any gate voltage. As a result the energies of two "even" charge states differing by $2e$ can become degenerate for special values of V_g. Because of this "Coulomb resonance" Cooper pair tunneling between the superconducting leads and the grain is deblocked and the suppression of the Josephson current through the grain is lifted [2]. In this Letter we point out that the discrimination in favor of ground states with an even number of electrons is not universal but depends in an essential way on the relative magnitude of the gap Δ in the single particle excitation spectrum of the grain and the charging energy, $e^2/2C$ (C is the total capacitance between the grain and the rest of the system). For a sufficiently large gap, $\Delta > e^2/2C$, charge states with an odd number of electrons are always energetically unfavorable and hence excluded at zero temperature [3, 4]. In contrast, when $\Delta < e^2/2C$, a lower Coulomb energy for the "odd" charge state can compensate for the extra energy required to put the unpaired electron in an excited quasiparticle state. Hence for small Δ there is no restriction on the "parity" of the ground state of the superconducting grain, which may contain either an even or an odd number of electrons. The magnitude of the superconducting gap can be decreased by a magnetic field, which makes it possible to study experimentally the transition at $\Delta = e^2/2C$ from a parity-selective to a parity-independent ground state of the superconducting grain. It is natural to call this transition a "parity effect" and we will show that it manifests itself in a drastic drop in the critical Josephson current. The modulation pattern of the critical Josephson current $J_c = J_c(V_g)$ is also affected. At small Δ the period is half of that found at large gaps.

The electrostatic energy of a grain with a discrete charge Ne is equal to

$$E_C(N) = (Ne)^2/2C - \varphi_g Ne, \qquad (1)$$

where φ_g is the potential induced on the grain by the gate voltage V_g [5]. Because of the superconducting gap and pairing of electrons in the condensate, ground states of even and odd number of electrons have energies differing by Δ. (This idea was introduced a long time ago in nuclear physics [6]). Hence depending on the parity of the number of electrons, there are two sequences of discrete energies shifted by Δ. As illustrated in Fig. 1(a), for a wide gap a degeneracy of the ground state may result in charge fluctuations between $Ne = 2ne$ and $N'e = (2n+2)e$. As the gate voltage is varied, degeneracy occurs periodically with period $\delta\varphi_g = 2e/C$. When two even charge states are in resonance, the minimum excitation energy of a state with different parity is equal to $\delta E = \Delta - e^2/2C$. The behavior of the system changes drastically when this energy becomes negative; the resonant states now have different parity; see Fig. 1(b). The degeneracy condition $E_C(2n \pm 1) = E_C(2n) + \Delta$ determines two sets of values of φ_g. The two sets are shifted by an amount $\delta\tilde{\varphi}_g = e/C - 2\Delta/e$ with respect to each other, but they both have the same period $\delta\varphi_g$; see Fig. 2. At $\Delta \ll e^2/2C$ all points of degeneracy are equally spaced and the modulation period of the function $J_c = J_c(\varphi_g)$ becomes e/C.

We now turn to calculating the Josephson current through the system, assuming first that Δ is above the threshold, $\Delta > e^2/2C$. In this case the effect of the Coulomb energy on the tunneling of a Cooper

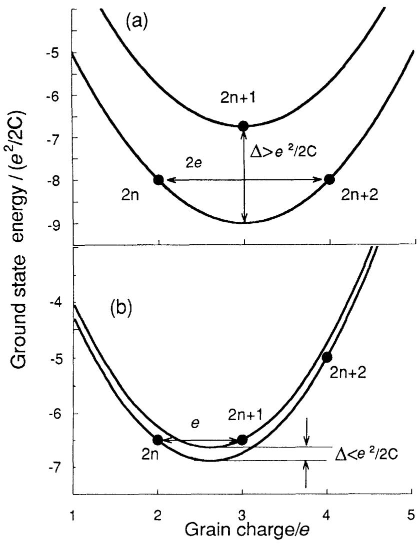

FIG. 1. Ground state energy as a function of the grain charge for a fixed value of the charging energy $e^2/2C$ and two different values of the superconducting gap Δ. Because of pairing in the superconducting state, there are two branches corresponding to an even and odd number of electrons on the grain. In (a) $\Delta > e^2/2C$ and the gate voltage has been adjusted to bring two charge states in the even branch, $2n$ and $2n+2$, in resonance. The lowest charge state in the odd branch is always separated by a gap from the ground state. In (b) $\Delta < e^2/2C$ and the gate voltage has a different value which brings states of different parity, $2n$ and $2n+1$, in resonance. Sweeping the gate voltage through the resonance changes the parity of the ground state.

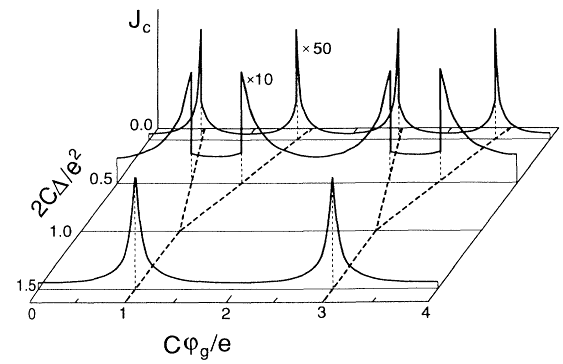

FIG. 2. Critical Josephson current J_c through a grain at zero temperature as a function of gate-induced voltage φ_g. Results for three different values of the superconducting gap are shown: $\Delta = 0.1\times$, $0.5\times$, and $1.5\times e^2/2C$. When $\Delta > e^2/2C$ the maximum current always occurs when two charge states differing by $2e$ are in resonance. Such resonances appear periodically in gate-induced voltage with period $\delta\varphi_g = 2e/C$ (thick dashed lines; in the figure only two resonant values are shown). Below a threshold value $\Delta = e^2/2C$ two sets of resonances corresponding to charge fluctuations $Ne \leftrightarrow (N+1)e$ occur and the current peaks split (thick dashed lines). In the limit $\Delta \ll e^2/2C$ all these resonances, where the critical current has maxima, fall into a single sequence with period e/C. For this calculation the conductances of the two links were taken to be equal, $G_1 = G_2 = 0.1G_q$, where $G_q = e^2/\hbar$.

pair between leads and the grain can be accounted for by a straightforward generalization of the standard Ambegaokar-Baratoff expression [7] for the Josephson current through each of the two junctions. Hence the generalized Josephson coupling energy for a single junction can be written as

$$E_{1(2)} = E^0_{1(2)} F\left(\frac{E_C(2n) - E_C(2n+1)}{\Delta}\right), \quad (2)$$

where $E^0_{1(2)} = \hbar J_{1(2)}/2e$ is related [8] to the corresponding critical current $J_{1(2)} = \pi G_{1(2)}\Delta/2e$ through a junction in the absence of charging effects, and the factor

$$F(x) \equiv \frac{1}{\pi^2}\int_{-\infty}^{\infty} d\xi_k \int_{-\infty}^{\infty} d\xi_p \frac{\Delta}{\epsilon_k\epsilon_p}\frac{1}{\epsilon_k+\epsilon_p - x\Delta} \quad (3)$$

differs from unity in the presence of charging effects ($x > 0$). Here $\epsilon_{k(p)} = (\xi^2_{k(p)} + \Delta^2)^{1/2}$ are quasiparticle excitation energies and we have for simplicity assumed Δ to be the same in the leads and grain; G_1, G_2 are conductances of the junctions. Close to the Coulomb resonance, where $E_C(2n) \approx E_C(2n+2)$, the charging energy in the intermediate state with one extra quasielectron on the grain is $E_C(2n+1) - E_C(2n) \approx -e^2/2C$ [cf. Fig. 1(a)] and one finds from (2) and (3) that the Josephson energy $E_{1(2)}$ is *increased* by a factor $F(e^2/2C\Delta) > 1$ due to charging effects.

The Josephson energy of the entire system may be expressed in terms of the phases ϕ_1, ϕ_2, and ψ of order parameter in the left and right leads and in the grain, respectively:

$$E_J = -E_1\cos(\psi - \phi_1) - E_2\cos(\psi - \phi_2). \quad (4)$$

The phase ψ should be viewed as a dynamic variable whose value is not fixed by the external conditions. In other words its value cannot be obtained simply by minimizing the Josephson energy (4) for given ϕ_1, ϕ_2. Rather, the restrictions on the charge fluctuations imposed by the electrostatic part of the Hamiltonian $\hat{H} = \hat{E}_C(N) + \hat{E}_J(\psi)$ induces quantum fluctuations in ψ [2].

The Josephson current is significantly modified by charging effects if the Josephson energy is small compared to the charging energy. In the following we assume that $E_J \ll e^2/2C$, and use perturbation theory in the small parameter $2CE_J/e^2$. Because resonances occur periodically in φ_g, it is enough to consider the interval $\varphi_g^{(n)} - e/C < \varphi_g < \varphi_g^{(n)} + e/C$, where $\varphi_g^{(n)} = (2n+1)e/C$. We follow the standard procedure [2] to find the ground state energy E_{gr} as a function of the phase difference $\phi = \phi_1 - \phi_2$, from which the Josephson current can then be obtained as $J = (2e/\hbar)(\partial E_{gr}/\partial\phi)$. The result is

$$J(\varphi_g,\phi) = \frac{2e}{\hbar}\frac{E_1E_2\sin\phi}{4[e^2(\varphi_g-\varphi_g^{(n)})^2+E_J^2(\phi)/4]^{1/2}}, \quad (5)$$

where $E_J(\phi) = \left[E_1^2+E_2^2+2E_1E_2\cos\phi\right]^{1/2}$.

The critical current J_c is determined as the maximum of (5) with respect to ϕ and is a periodic function of gate voltage, $J_c = J_c(\varphi_g)$, with sharp maxima at $\varphi_g = \varphi_g^{(n)}$; see Fig. 2 and the right inset in Fig. 3. These maxima occur because of the resonant charge fluctuations $2ne \leftrightarrow (2n+2)e$ which lift the Coulomb blockade of Cooper pair tunneling so that the critical current is only weakly dependent on the Coulomb energy and limited by the weaker tunnel junction, which we from now on assume to be the junction labeled "1,"

$$J_c(\varphi_g) = \frac{e}{\hbar}E_1. \quad (6)$$

The maximum critical current would be equal to exactly half its value in the absence of the Coulomb blockade [2] only if we neglect the effect of charging energy on the Josephson energy E_1; see Eqs. (2) and (3).

Away from resonances, at $|\varphi_g-\varphi_g^{(n)}| \gg E_2$, the Josephson effect is strongly suppressed due to the Coulomb blockade. If $\varphi_g^{(n)} < \varphi_g < \varphi_g^{(n+1)}$ the critical current is

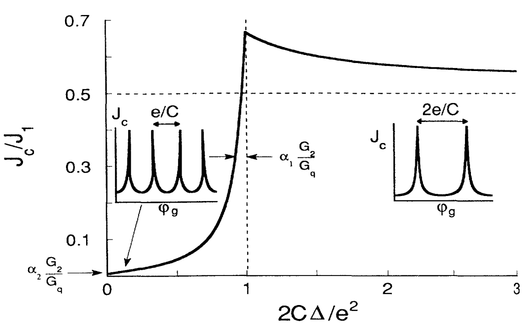

FIG. 3. The critical Josephson current J_c through a grain as a function of the superconducting gap Δ. For each value of Δ we plot here the maximum critical current that can be obtained by adjusting the gate-induced voltage φ_g. These maxima appear periodically in φ_g as shown in Fig. 2. The period is $2e/C$ if $\Delta > e^2/2C$ while maxima appear periodically with exactly half that period in the limit $\Delta \ll e^2/2C$; see insets. The current has been normalized to J_1, the critical current through the weakest of the two links between grain and leads in the absence of charging effects (the same parameters as in Fig. 2 were used; $G_1 = G_2 = 0.1G_q$, where $G_q = e^2/\hbar$). The value of J_c in the asymptotic limit $\Delta \gg e^2/2C$ (horizontal dashed line) is $J_1/2$. Deviations of the calculated current (full curve) from the asymptotic value are due to the effect of charging on Cooper pair tunneling from lead to grain and follow from a straightforward modification of the standard Ambegaokar-Baratoff result; see text. The critical current has a threshold at $\Delta = e^2/2C$. The proportionality constants α_1 and α_2 are of order 1 and 0.1, respectively.

$$J_c(\varphi_g) = \frac{e}{\hbar}\frac{E_1E_2}{2e|\varphi_g-\varphi_g^{(n)}|} + \frac{e}{\hbar}\frac{E_1E_2}{2e|\varphi_g-\varphi_g^{(n+1)}|}. \quad (7)$$

It follows from (6) and (7) that the peak-to-valley ratio of $J_c(\varphi_g)$ is e^2/CE_2.

The two last equations were obtained for the limit of zero temperature. It is easy to check that Eq. (6) for the peak values is valid at $k_BT \ll E_J$ [9], whereas Eq. (7) for the off-peak values is valid only at much lower temperatures, $k_BT \ll CE_J^2/e^2$. An increase of temperature suppresses the Josephson current.

When Δ decreases, approaching $e^2/2C$ from above, the peak critical current becomes larger. This "Coulomb stimulation" of the maximum critical current is due to a *negative* charging energy $E_C(2n+1)-E_C(2n) \approx -e^2/2C$ that enhances E_1 and E_2 through the factor F of (3). The resulting increase in the peak values of J_c can be as large as about 30%, as shown in Fig. 3. As long as Δ is above the threshold even slightly, the degeneracy of the ground state at $\varphi_g = \varphi_g^{(n)}$ allows charge fluctuations between $Ne = 2ne$ and $N'e = (2n+2)e$ and the resonance condition for the Josephson current is fulfilled. Hence Eq. (6) remains valid and the modulation period of the function $J_c = J_c(\varphi_g)$ is $2e/C$; cf. Fig. 2.

Immediately below the threshold, the resonance between two even states can no longer be achieved. When the gate voltage is tuned towards a resonance, the critical current grows as long as the ground state corresponds to an even number of electrons on the grain. However, at $\tilde{\varphi}_{g\pm}^{(n)} = \varphi_g^{(n)} \pm (e/2C - \Delta/e)$ the odd electron enters the grain, and the critical current drops abruptly. As a result, the peaks in the function $J_c(\varphi_g)$ split; see Fig. 2. Outside the gap separating two adjacent maxima of the critical current, Eq. (5) is still valid if $\Delta \lesssim e^2/2C$ and gives the dependence of the maximum value of J_c on Δ; see Fig. 3. It follows from Eq. (7) with $\varphi_g = \tilde{\varphi}_g^{(n)}$ that the maximum value drops by a factor of 2 when Δ is decreased by $\sim (G_2/G_q)e^2/2C \ll e^2/2C$ below the threshold.

At even smaller Δ, when the critical current is suppressed significantly, it is possible to find the whole function $J_c(\varphi_g)$ using the perturbation theory. In addition to the tunneling of Cooper pairs through the grain, there is a number of other channels not taken into account in Eq. (5). These channels correspond to the transfer of a Cooper pair between leads by a sequential tunneling of the two electrons through the grain [10]. In these tunneling processes the grain energy varies only by Δ, if $\varphi_g = \tilde{\varphi}_g^{(n)}$. The new channels dominate the critical current at small $\Delta \lesssim 0.15e^2/2C$. One finds in this limit the maximum value of the critical current to be

$$J_c = \gamma\frac{G_1G_2}{G_q}\frac{\Delta}{e}, \quad \gamma \approx 0.1. \quad (8)$$

A comparison of Eqs. (8) and (6) shows that the peak value of the critical current far below the threshold is only a small fraction of its value above it; normalized to the critical Josephson current in the absence of charging effects its value is $\alpha_2(G_2/G_q)$, with $\alpha_2 \approx 0.06$. The calculated maximum critical Josephson current, including contributions from all channels, is shown for a range of Δ in Fig. 3.

At finite temperatures thermal fluctuations tend to suppress the Josephson effect. The suppression is strong for $k_BT > \hbar J_c/2e$. We have shown that due to parity effects in the Coulomb blockade the value of J_c changes sharply at $\Delta = e^2/2C$. This is why temperatures corresponding to the Josephson energy right above this threshold, $k_BT \sim \pi(e^2/C)(G_1/G_q)$, would not significantly affect the peak values of J_c if $\Delta > e^2/2C$, while simultaneously suppressing the Josephson current below the threshold.

In conclusion, the Josephson current through a superconducting grain is sensitive to the relation between the superconducting gap Δ and the charging energy $e^2/2C$. At large Δ, a gate voltage can bring two states with charge differing by $2e$ to a resonance. Below the threshold, at $\Delta < e^2/2C$, only states with charge differing by a single e can be brought into resonance. The different nature of the charge fluctuations above and below the threshold is reflected in the dependence of the critical Josephson current on Δ and on the gate voltage V_g. Above the threshold, the resonances appear periodically as V_g is varied. The resonant value of the critical current is of the same order of magnitude as in the absence of the Coulomb blockade ($C \to \infty$) and depends on the ratio of the superconducting gap and the charging energy. Below the threshold, the Coulomb blockade suppresses J_c at any gate voltage. As a result the critical current drops abruptly when Δ is decreased below the threshold value, for instance by applying a magnetic field. Finally, we notice that the Josephson effect reflects the equilibrium properties of the system. Observation of $2e$ periodicity and a threshold behavior of the critical current would therefore be evidence of the parity effect in the ground state. In this respect the measurement of the Josephson current is different from studying the current at finite bias voltage [4], which provides information about nonequilibrium states.

This work was supported by NSF through Grant No. DMR-9117341, by the Swedish Natural Research Council (NFR), and by the Swedish Research Council for Engineering Science (TFR). We are grateful for discussions with T. Claeson, P. Delsing, T. Eiles, D. Haviland, L. Kuzmin, and M. Tuominen. L.G. acknowledges the hospitality of the Department of Applied Physics at the Chalmers University of Technology/University of Göteborg.

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[9] The characteristic temperature E_J/k_B defined by this relation should be compared with the temperature [4] $T^* \simeq \Delta/k_B \ln(\nu\Omega\Delta)$ at which the distinction between the even and odd states is masked by thermal fluctuations (here Ω is the grain volume and ν is the density of states for the grain material). The lowest temperature provides the restriction for the applicability of Eq. (6). For the typical case $G/G_q \sim 0.1$ and an Al grain with $\ln(\nu\Omega\Delta) \sim 10$ these two temperatures are both close to 300 mK.

[10] Depending on the gate voltage the dominating channel can involve the sequence of three virtual states with excess charges $-e,0,-e$ (electron channel) or $+e,0,+e$ (hole channel). The two "mixed" electron-hole channels can never dominate.