Nonlinear Theory of Gyroharmonic Radiation from Spatiotemporally Modulated Electron Beams

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A multimode nonlinear particle simulation code is used to find the saturated efficiency for power transfer into modes of a cylindrical waveguide carrying a spatiotemporally modulated gyrating electron beam. For a TE_{51}-mode fifth harmonic 94-GHz harmonic converter using a 150-kV, 6.7-A cold beam, this code predicts a conversion efficiency of 57% when a linearly tapered guide magnetic field is used, and 70% when a nonlinear taper is used. Efficiency in the linear taper case is shown to be insensitive to beam axial velocity spread, while both cases show negligible power flow into competing modes.

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Efficient, high-power rf sources are in demand for many applications, such as drivers for next generation electron-positron colliders, as sources for fusion plasma heating and control, and as amplifiers for advanced mm-wave radar systems. Extensions of proven technologies are being undertaken to meet these demands. Thus advanced klystrons [1] and gyroklystrons [2] show promise as 50-MW sources above 10 GHz for driving next generation colliders, cavity gyrotrons [3] have generated 940 kW at 140 GHz for plasma heating, and 35-GHz gyrotron traveling-wave amplifiers have shown 32% bandwidth and 30% efficiency at output levels of 25 kW [4]. This Letter reports first results on the nonlinear theoretical properties of a recently proposed [5] alternative mechanism to satisfy these demands, namely, gyroharmonic radiation from a spatiotemporally modulated gyrating electron beam. Prior to the work reported here, it was speculated that harmonic conversion based on this process could be highly efficient, that radiation into competing modes could be small, and that a moderate axial velocity spread on the electron beam could be tolerated [5,6]. Results are given here that confirm these speculations.

The harmonic conversion process as described here is fundamentally different from that operating in gyroresonance devices such as the gyrotron. The latter depends upon phase bunching induced near gyroresonance by the rf fields, a process that is *second order* in the amplitude of the fields. Harmonic conversion operates without induced phase bunching, since a spatiotemporally modulated beam can transfer energy efficiently to radiation fields without additional modulation; this process is *first order* in the amplitude of the fields. This distinction was pointed out in treatments of the linear aspects of the process [5,6], but the crucial issues of mode competition and maximum harmonic conversion efficiency could not be addressed prior to development of the theory outlined here.

Linear theory for the harmonic conversion process was published initially for rectangular waveguides [5,6]. The waveguides were taken to contain a spatiotemporally modulated gyrating beam in a static axial magnetic field having a current density that could be characterized as

$$\mathbf{J}(r,\phi,z,t) = -e\int_{-\infty}^{\infty} du \int_0^{\infty} dw\, w \int_0^{2\pi} d\varphi \left[\hat{\mathbf{e}}_\theta \frac{w}{\gamma} + \hat{\mathbf{e}}_z \frac{u}{\gamma}\right] \delta(\theta-\theta_0+\xi z - pt) f_0(u,w,\varphi,r,\phi)\,, \tag{1}$$

where r and ϕ are the radial and azimuthal coordinates, and u and w are components of a beam electron's momentum (divided by the electron rest mass m) along and across the z axis. The relativistic Lorentz factor is related to the momenta by $\gamma^2 = (u^2+w^2+c^2)/c^2$, and a uniform axial static magnetic field $\hat{\mathbf{e}}_z B_0$ is imposed. Unit vectors $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_z$ are along the gyrating particles' angular and axial momenta. The distribution function for the beam electrons is $f_0(u,w,\varphi,r,\phi)$, where φ is the azimuth angle in momentum space. The delta function in the integrand of Eq. (1) gives the equilibrium current density its spatiotemporal character: An individual particle moves on a helix of axial pitch number ξ, and the helix rotates with an angular frequency p. This equilibrium is taken to model the beam produced by a cyclotron autoresonance accelerator [7] that is driven at angular frequency p. It was shown [5] that $\xi = \gamma(p-\Omega)/u$, where $\Omega = eB/m\gamma$ is the gyrofrequency.

In general, the linear theory showed that a beam described by Eq. (1) will couple to many rectangular waveguide modes at a frequency $\omega = sp$, where s is the harmonic number. However, maximum power growth will occur only for a mode whose group velocity c^2k_z/ω equals the mean axial beam velocity U/γ, where k_z is the axial wave number. Below, we refer to this as the *match-*

ing condition. For a cold beam the power flow into the TE_{mn} mode when matching is observed has been shown to be [6]

$$P_{mn}(L)=\frac{Q_{mn}Z_{TE}}{A}\left[\frac{W}{U}I_0LK_s(k_{mn}\rho)\right]^2, \qquad (2)$$

when no signal is present at $z=0$. In Eq. (2), Q_{mn} is a dimensionless geometric coupling factor that reflects symmetry-based selection rules at each harmonic; $Z_{TE}=120\pi(\omega/k_zc)\ \Omega$; A is the waveguide cross-sectional area; W and U are the transverse and axial normalized electron momenta; I_0 is the dc beam current; and $K_s=J_s'(k_{mn}\rho)$, the derivative with respect to its argument of the sth-order Bessel function; $k_{mn}^2=(\omega/c)^2-k_z^2$; and $\rho=W/\Omega\gamma$ is the gyration radius. Equation (2) predicts that the power flow will grow quadratically with both the interaction length and beam current. Such growth will cease when matching fails due to depletion of electron energy, once moderate wave growth occurs. It was argued [5,6] that cumulative power growth could be maintained throughout the nonlinear regime if the axial magnetic field (or the waveguide dimension) was tapered, so as to preserve matching. In this way, it was claimed, all transverse energy initially on the beam could be transferred to the wave. This speculation did not take into account any details of the nonlinear dynamics between the particles and the large amplitude fields in a superposition of allowed modes, nor indeed any variations in transverse particle momentum brought about from the axial gradient of a tapered B field. It was based simply upon the assertion, for a cold beam, that all particles would have the same history, and thus could be "milked" of all their transverse energy when matching was present all along their path.

Recent extension [8] of the linear theory to cylindrical waveguides has shown that the harmonic power flow follows Eq. (2), with $Q_{mn}^{-1}=J_m^2(x_{mn}')(1-m^2/x_{mn}'^2)$, where x_{mn}' is the nth zero of $J_m'(x)$. For cylindrical waveguides, the following additional features were shown to arise if the distribution of electron guiding centers in the beam is axisymmetric: (i) Power growth at a harmonic frequency $\omega=sp$ is absent, except for modes having an azimuthal mode index $m=s$. (ii) A beam with a uniform distribution of guiding centers will have an initial power growth rate no less than 90% that of a beam with no guiding center spread if the ratio of the outer guiding center radius to the waveguide radius is less than $0.8944/x_{m1}'$. (iii) Diminution in initial sth harmonic power growth due to axial velocity spread will be less than 10% the growth rate of a cold beam if the relative spread is less than $48/N$%, where N is the number of interaction guide wavelengths. (iv) Power transfer from a cold beam to TM modes is absent when matching prevails, as a result of exact cancellation between contributions from the axial and transverse field components. Items (i) and (iv) are a direct result of the higher symmetry of cylindrical waveguide, as compared to rectangular. These points guided the formulation of a theory and attendant particle simulation code to model the nonlinear behavior of this interaction.

A slow-time-scale formulation in three dimensions developed originally for modeling the steady-state operation of gyroamplifiers [9] has been extended to include the coupling between allowed modes. In this formulation, the electromagnetic field is expanded as a superposition of unperturbed TE and TM modes of a cylindrical waveguide. By averaging Maxwell's equations over a fundamental wave period, a series of slow-time-scale equations is derived for the spatial evolution of the amplitude and phase of each waveguide mode as driven by the electron beam in the assigned axial guide magnetic field. In general, the guide magnetic field is axisymmetric but nonuniform. The axial magnetic field profile may either be prescribed by a requirement for maintaining matching as electron energies deplete or it may be externally specified. Coupling of waveguide modes occurs through mutual nonlinear interactions with the ensemble of beam particles. This slow-time-scale averaging allows multimode coupling to be treated in this problem because the frequencies of the competing modes are all integral multiples of the fundamental pump frequency p, and because the time average is taken over the period of this fundamental frequency. (This then corresponds to averaging over m periods for the mth harmonic.) The slow-time-scale field equations are integrated simultaneously with the three-dimensional Lorentz force equations, but no such averaging is performed for the orbit equations.

The governing equations for the slowly varying amplitude $A_{mn}(z)$ and axial wave number $k_z(z)$ for TE_{mn} modes were previously derived [9]. For an axisymmetric distribution of guiding centers, these quantities are given by

$$\frac{d^2A_{mn}}{dz^2}+\left[\left(\frac{mp}{c}\right)^2-k_{mn}^2-k_z^2\right]A_{mn}=2\mu_0I_0mpC_{mn}k_{mn}\left\langle\frac{U(0)}{\langle U(0)\rangle}\frac{W}{|U|}J_0(k_{mn}r_g)J_m'(k_{mn}\rho)(-\sin\Phi_m)\right\rangle,$$
$$\frac{1}{A_{mn}}\frac{d}{dz}[k_zA_{mn}^2]=2\mu_0I_0mpC_{mn}k_{mn}\left\langle\frac{U(0)}{\langle U(0)\rangle}\frac{W}{|U|}J_0(k_{mn}r_g)J_m'(k_{mn}\rho)(\cos\Phi_m)\right\rangle, \qquad (3)$$

where the detuning parameter is $\Phi_m=mpt-\int_0^z dz'k_z(z')-m\theta$, with θ the electron gyration phase, r_g the guiding center coordinate, k_{mn} the cutoff wave number, and C_{mn} a geometrical factor. The angular brackets denote an ensemble average over the initial distribution function of the electrons. For TM_{mn} modes the following equations are obtained:

$$\frac{d^2A_{mn}}{dz^2}+\left[\left(\frac{mp}{c}\right)^2-k_{mn}^2-k_z^2\right]\left(1+\frac{k_{mn}^2}{k_z^2}\right)A_{mn}=2\mu_0I_0mpC_{mn}k_{mn}\left\langle\frac{U(0)}{\langle U(0)\rangle}\frac{W}{|U|}J_0(k_{mn}r_g)\frac{mJ_m(k_{mn}\rho)}{k_{mn}\rho}\left[1-\frac{k_{mn}^2\rho U}{mk_zW}\right](\cos\Phi_m)\right\rangle . \quad (4)$$

$$\frac{1}{A_{mn}}\frac{d}{dz}\left[\left(1+\frac{k_{mn}^2}{k_z^2}\right)k_zA_{mn}^2\right]=2\mu_0I_0mpC_{mn}k_{mn}\left\langle\frac{U(0)}{\langle U(0)\rangle}\frac{W}{|U|}J_0(k_{mn}r_g)\frac{mJ_m(k_{mn}\rho)}{k_{mn}\rho}\left[1-\frac{k_{mn}^2\rho U}{mk_zW}\right](\sin\Phi_m)\right\rangle .$$

The particle orbit equations [9] complete the formulation. These equations are not restated here to conserve space. The coupled set of equations are integrated in z and the Poynting flux at each location z is found. The input boundary conditions at $z=0$ for the particles are chosen to model an axisymmetric monoenergetic beam entering the interaction waveguide with a spatiotemporally modulated gyration angle $\theta=\theta_0-\xi z+pt$, as indicated in Eq. (1). Axial momentum spread is introduced through a Gaussian pitch-angle distribution. The boundary conditions at $z=0$ on the radiation field are $dA_{mn}/dz=0$ and $k_z(0)^2=m^2p^2/c^2-k_{mn}^2$; $A_{mn}(0)$ is taken to be at the noise level.

For a cold beam satisfaction of the matching condition corresponds to vanishing of the factor in square brackets on the right-hand side of Eq. (4). As a result, coupling is absent between a cold beam and all TM modes, as was stated above on the basis of linear theory. A numerical evaluation of Eq. (4) under conditions where matching is not strictly observed did not yield significant power flow into TM modes. It can then be concluded that only TE modes need to be considered in this problem.

To illustrate, numerical results of evaluating Eq. (3) and the coupled orbit equations are shown in Figs. 1–3 for fifth harmonic operation at 94 GHz, where the TE_{51} mode interacts with a 150-kV, 6.667-A beam. A beam velocity ratio $\alpha=W/U=2.0$ is selected. The pump frequency $p/2\pi=18.8$ GHz. For these parameters, matching sets the waveguide radius $R=0.34$ cm, the cutoff frequency $ck_{51}/2\pi=90.1$ GHz, and the initial axial magnetic field $B_0(0)=B_{gr}=7.983$ kG. Linear theory indicates that the most dangerous competing modes in this case are TE_{41} and TE_{61} at the fourth and sixth harmonics, respectively. For a uniform magnetic field $B(z)=B_{gr}$, the fifth harmonic saturated efficiency was found to not exceed 10%. Tapering the magnetic field allows the saturated efficiency to greatly exceed this value, as is shown in the figures. Efficiency is plotted in Fig. 1 for a cold beam in a linearly tapered B field when the TE_{41}, TE_{51}, and TE_{61} modes are simultaneously excited. The particular taper that proved to maximize the power transfer is $B(z)=B_{gr}$ for $z<z_0$, and $B(z)=B_{gr}[1-a(z-z_0)/R]$ for $z\geq z_0$, where $z_0=0.34$ cm and $a=0.002$. The TE_{51} mode is seen to saturate at a distance of 28.3 cm with a peak efficiency of 57%, corresponding to an output power of 570 kW; negligible power transfer into the competing modes is found. In Fig. 2 the influence of axial electron momentum spread is shown. Saturated efficiency, for the same B-field profile used in the cold beam case (Fig. 1), is seen to drop from 57% to 40% as the fractional axial momentum spread $\delta u/u$ increases from zero to 15%. But the linearly tapered B-field profile is not necessarily the optimum for achieving the highest power transfer. Figure 3 shows results for a cold beam when the profile is chosen

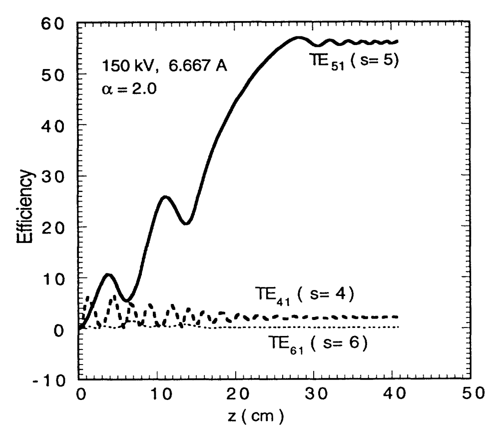

FIG. 1. Efficiency vs interaction distance for fifth harmonic conversion using a cold beam in a linearly tapered B field. $B=B_{gr}=7.983$ kG for $z<R$, and $B(z)=B_{gr}[1-0.002(z-R)/R]$ for $z>R$, with the waveguide radius $R=0.34$ cm.

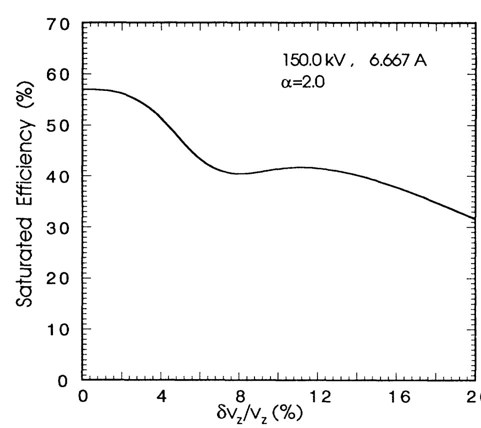

FIG. 2. Maximum efficiency for fifth harmonic conversion as a function of axial velocity spread $\delta u/u$ for the linear B-field taper of Fig. 1.

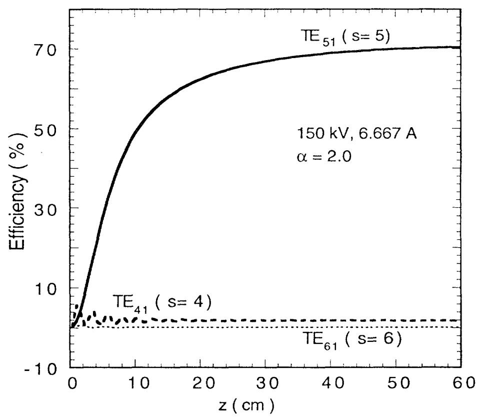

FIG. 3. Efficiency vs interaction length for fifth harmonic conversion using a cold beam in a nonlinear B-field taper. $B = B_{gr}$ for $z < 2.2R$, and $B(z)$ tapered to maintain exact phase matching for $z > 2.2R$. $R = 0.34$ cm.

to preserve exact phase matching along the entire interaction length. Here a saturated efficiency of 70.5% is found for the fifth harmonic TE_{51} mode at a distance of 50.3 cm, with again negligible power in the competing modes. Examination of the electron distribution at the point of maximum power transfer shows that residual momentum is essentially in the axial component, and that the particles all have the same energy, regardless of their gyration phase. Saturation is due to the nearly complete depletion of the transverse energy of the beam, mainly by transfer to the radiation fields, but (to a small extent) by transfer in the down-tapered magnetic field to axial particle motion. The particle phases remain favorable for power flow into the radiation fields throughout the interaction, and thus do not exhibit trapping in an unfavorable phase that could otherwise limit the interaction efficiency.

In conclusion, a multimode nonlinear theory has been developed to describe the transfer of power from a spatiotemporally modulated electron beam to the fields of cylindrical waveguide in a nonuniform axially symmetric guide magnetic field. This theory is applicable to a wide range of gyroresonant traveling wave interactions. For a beam with an axisymmetric distribution of guiding centers, coupling is shown to be absent for all TM modes and for TE modes whose azimuthal mode index is not equal to the harmonic number. Power transfer to a phase-matched mode has been shown to occur with high efficiency: The 70% value predicted for fifth harmonic conversion corresponds to transfer of 88% of the initial transverse beam energy to radiation—a value that is (to our knowledge) unprecedented for high-power millimeter-wave harmonic generation. Negligible power is transferred to competing modes when the B-field taper is judiciously chosen. Conditions have been found where the power transfer is relatively insensitive to axial momentum spread on the beam. Overall harmonic conversion efficiency for a device based on the theory outlined here could approach 100%, since a single stage depressed collector should be capable of recovering the residual beam power.

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