Methods for Conditioning Electron Beams in Free-Electron Lasers

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The operation of free-electron lasers can be severely limited by the axial velocity spread of the beam electrons. In this Letter we propose methods for reducing the axial velocity spread in electron beams by redistributing the electron energy via interaction with an axially symmetric, slow, TM waveguide mode. In the first method, the energy redistribution is correlated with the electrons' betatron amplitude, while in the second method it is correlated with the electrons' synchrotron amplitude. Reductions by more than a factor of 40 in the rms axial velocity spread have been obtained in simulations.

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Many coherent radiation generation mechanisms are based on the longitudinal bunching of electron beams. These sources include traveling-wave tubes and free-electron lasers (FELs) [1-7]. The degree to which an electron beam can be bunched is a strong function of the beam quality. The two independent contributions to the electron beam quality are the intrinsic energy spread and the emittance, both of which lead to a spread in the axial electron velocity and limit the operating wavelength, gain, and efficiency of the device [7-10]. A method for conditioning, i.e., reducing the axial beam velocity spread, was recently proposed in which the beam was propagated through a periodic array of focusing, drift, defocusing, drift channels and microwave cavities excited in the non-axially-symmetric TM_{210} mode [11].

In this Letter we propose an alternative conditioning method which redistributes the electrons' energy according to their betatron amplitude by using the electric field of an axially symmetric, slow, TM waveguide mode. In addition, a second method is briefly described in which the energy redistribution is correlated with the electrons' synchrotron amplitude.

We first obtain the restriction on the operating FEL wavelength due to emittance and energy spread. In the FEL mechanism, the resonance condition is $\omega - v_z(k + k_w) \simeq 0$, where $\omega = ck$ is the frequency, v_z is the axial electron velocity, $k_w = 2\pi/\lambda_w$, and λ_w is the wiggler wavelength. This condition implies that the beam's axial velocity spread δv_z should satisfy $\delta\beta_z \ll \lambda/2L$, where $\delta\beta_z = \delta v_z/c$, λ is the radiation wavelength, and L is the interaction length (e-folding length) of the radiation field in the low gain (high gain) regime. The axial velocity spread can be written as $\delta\beta_z = [(1+a_w^2/2)\delta\gamma/\gamma - \varepsilon_n^2/2r_b^2]/\gamma^2$ [see Eq. (2)], where $\gamma = 1 + E/m_0c^2$ is the relativistic factor, E is the beam energy, $\delta\gamma/\gamma$ is the fractional intrinsic beam energy spread, ε_n is the normalized emittance [12], a_w is the wiggler strength parameter, and r_b is the radius of the matched electron beam. Using this expression for $\delta\beta_z$ we find the restriction on the operating wavelength,

$$\lambda \gg (\pi)^{1/2} \left(\frac{L}{Z_R} \right)^{1/2} f^{-1/2} \frac{r_b}{\gamma} \left| \frac{\varepsilon_n^2}{2r_b^2} - \left(1 + \frac{a_w^2}{2} \right) \frac{\delta\gamma}{\gamma} \right|^{1/2}, \tag{1}$$

where $Z_R = \pi r_s^2/\lambda$ is the vacuum Rayleigh length associated with a Gaussian beam with minimum spot size r_s and $f = r_b^2/r_s^2$ is the filling factor.

The inequality in (1) can be simplified by noting that, in both the low and high gain regimes of the FEL, $L \simeq Z_R$ for $f \simeq 1$ [13-16]. That is, in the low gain regime, $L \simeq Z_R$ and $f \simeq 1$ are required to minimize diffraction effects and maximize gain. In the high gain optically guided regime, it can be shown that the e-folding length is approximately equal to the vacuum Rayleigh length, i.e., $L \simeq Z_R$, for $f \simeq 1$. In many cases, electron beam quality is limited by the emittance contribution and not the energy spread term, i.e., $\delta\gamma/\gamma \ll \frac{1}{2}(\varepsilon_n/r_b)^2$. In this case, the wavelength limit in FELs can be simply written as $\lambda \gg \varepsilon_n/\gamma$. It is clear that electron beam quality, in particular $\delta\beta_z$, limits the operating wavelength of FELs.

To analyze our conditioning methods we first consider the electron trajectories in a planar wiggler with parabolic pole faces. These orbits consist of rapidly varying (wiggler-period scale length) and slowly varying (betatron-period scale length) terms [17]. For the wiggler field components used in [17], the approximate electron orbits are $x = x_f + x_s$ and $y = y_f + y_s$. The fast components of the orbit are $x_f(z) = -(a_w/\gamma k_w)\,[1 + k_w^2(x_s^2 + y_s^2)/4]\cos(k_w z)$, $y_f(z) \simeq 0$, where $a_w = |e|B_w/k_w m_0 c^2$, $k_w = 2\pi/\lambda_w$, and λ_w is the wiggler period, while the slow components are $x_s(z) = x_0\cos(K_\beta z + \theta_{0x})$, $y_s(z) = y_0 \times \cos(K_\beta z + \theta_{0y})$, where (x_0, y_0) are the maximum amplitude of the betatron oscillations in the x and y directions, respectively, $K_\beta = (k_\beta^2 - k_p^2)^{1/2}$, $k_\beta = a_w k_w/2\gamma \ll k_w$ is the betatron wave number, $k_p^2 = (\omega_p^2/c^2)/2\gamma^3$, $\omega_p^2 = 4\pi|e|^2 n_0/m_0$, n_0 is the beam density, and θ_{0x}, θ_{0y} are constant phases. We have assumed that $a_w/\gamma \ll k_w r_b \ll 1$ and $\gamma k_p r_b \ll 1$, which implies that $v/\gamma \ll 1$ where $v = I/$

$17000\beta_z = \omega_p^2 r_b^2/4c^2$ is Budker's parameter and I is the current in amperes.

In the highly relativistic limit, the axial particle velocity normalized to the speed of light is given by $\beta_z \simeq 1 - 1/2\gamma^2 - (\beta_x^2+\beta_y^2)/2$. As pointed out in Ref. [17], the square of the perpendicular velocity, averaged over the wiggler period, is independent of z. Substituting the fast and slow orbits into the expression for β_z and setting $\gamma=\gamma_0+\delta\gamma$, where $\delta\gamma$ is the electron's energy deviation term, we find that $\beta_z=\beta_{0z}+\delta\beta_z$, with $\beta_{0z}=1-(1+a_w^2/2)/2\gamma_0^2$, and

$$\delta\beta_z=(1+a_w^2/2)\delta\gamma/\gamma_0^3-k_\beta^2 r_0^2/2\,. \qquad (2)$$

Here γ_0 is the gamma associated with the reference electron, traveling along the z axis without a betatron oscillation, $r_0^2=x_0^2+y_0^2$, and terms varying on the wiggler wavelength scale have been neglected. The space charge contribution to β_z is of order ν/γ_0^3 and is neglected. The normalized beam emittance, for a matched beam in the focusing fields of the wiggler, is $\varepsilon_n=\gamma_0 k_\beta r_b^2$. Note that the emittance contribution to the velocity spread in (2), i.e., $k_\beta^2 r_0^2/2$, is independent of propagation distance. It will be assumed that the axial velocity spread due to emittance initially dominates the velocity spread caused by the intrinsic energy spread.

The proposed conditioning field is an axially symmetric, slow, TM waveguide mode (Fig. 1) with axial electric field

$$E_z=-E_0 I_0(k_\perp r)\cos\psi\,, \qquad (3)$$

together with the associated transverse electric and magnetic fields, where E_0 is the maximum electric field amplitude on axis, $k_\perp$ is the transverse wave number, k is the axial wave number, $\omega=c(k^2-k_\perp^2)^{1/2}$ is the frequency, $\psi=kz-\omega t$ is the phase, and I_n is the modified Bessel function of order n. The axial phase velocity of the traveling wave is matched to the axial beam velocity, $\beta_{ph}=\omega/ck=\beta_0$, where $\beta_0=(1-1/\gamma_0^2)^{1/2}$ is the normalized axial velocity of the reference electron. To maintain synchronism between the conditioning field and the electrons, the axial and transverse wave numbers must satisfy $k=(\omega/c)\gamma_0(\gamma_0^2-1)^{-1/2}=(\omega/c)/\beta_0$ and $k_\perp=(\omega/c)(\gamma_0^2-1)^{-1/2}=(\omega/c)/\gamma_0\beta_0$, respectively.

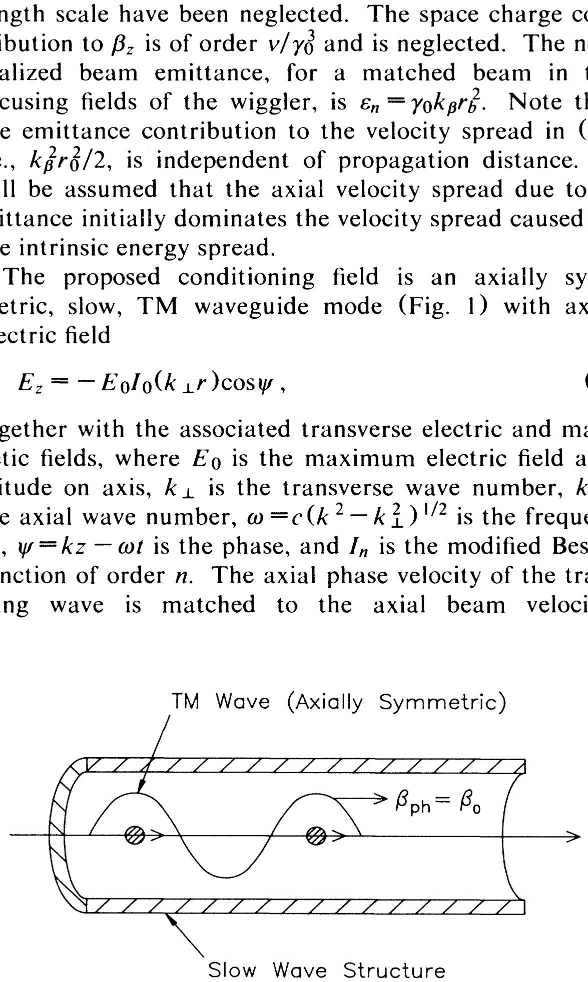

FIG. 1. Schematic of the beam conditioning configuration showing the axially symmetric, slow, TM wave conditioning properly phased electron pulses. The phase velocity of the TM wave is β_{ph} and the axial velocity of the reference electron is β_0 (both normalized to the speed of light). An inductively generated electric field may be employed to cancel the accelerating component of the TM field.

In our conditioning methods, the beam electrons are given an energy increment which cancels out the emittance contribution to the axial velocity spread. To reduce the velocity spread to zero, the conditioning field must give all the individual electrons a different fractional energy increment, $\delta\gamma_c/\gamma_0$, given by

$$\frac{\delta\gamma_c}{\gamma_0}=\frac{\gamma_0^2 k_\beta^2 r_0^2}{2(1+a_w^2/2)}=\frac{\varepsilon_n^2 r_0^2}{2(1+a_w^2/2)r_b^4}\,. \qquad (4)$$

In the first method, the energy increment is proportional to the square of the betatron amplitude and the electron pulse length remains approximately constant. Our results show that the degree of beam conditioning in the first method can be significantly improved by removing the accelerating component of the TM field. A straightforward way to achieve this is to introduce an inductively generated axial electric field in addition to the waveguide mode. This uniform inductive field may be generated by an azimuthal magnetic field that is confined to the interior of a cylindrical shell of high-permeability and high-resistivity material, e.g., a ferrite.

The rate of change of electron energy and phase with respect to z is, respectively,

$$\gamma'=a_0\frac{\omega}{c}\left[I_0(k_\perp r)\cos\psi-1+\frac{k}{k_\perp}r' I_1(k_\perp r)\sin\psi\right]\,, \qquad (5)$$

$$\psi'=(\omega/c)\delta\beta_z/\beta_0^2\,, \qquad (6)$$

where the prime denotes a derivative with respect to z and $a_0=|e|E_0/m_0 c\omega$.

Conditioning requires that the spreading of the electron phase (expansion of the prebunched electron beam) be limited. An upper bound on the phase change is obtained by integrating Eq. (6) with only the emittance term in $\delta\beta_z$, $|\psi(z)-\psi(0)|<kk_\beta^2 r_b^2 z/2$. If the phase remains small, i.e., $|\psi|\ll kr/\sqrt{2}\gamma_0$ and $|\psi|\ll kr/2\gamma_0^2 r'$, Eq. (5) simplifies to $\gamma'=a_0(\omega/c)(kr/2\gamma_0)^2$, for $k_\perp r\ll 1$ and $\psi\simeq 0$. The transverse dynamics of the electrons is governed by only the wiggler field and the conditioning field can be neglected if $k_\beta^2\gg[a_0 k^2/(2\gamma_0^3)]\sin\psi$ and $k_\beta\gg a_0 k/2\gamma_0$. Substituting the slowly varying orbits into the expression for γ' and integrating, we obtain $\gamma=\gamma_0+\delta\gamma$, where $\delta\gamma=[(kr_0)^2/8\gamma_0^2]a_0(\omega/c)z$. Making use of Eq. (2), the effective fractional axial energy spread is

$$\frac{\delta\gamma_z}{\gamma_0}\equiv\gamma_0^2\delta\beta_z=\left[\frac{(1+a_w^2/2)k^3 a_0 z}{4\gamma_0^3}-\frac{\varepsilon_n^2}{r_b^4}\right]\frac{r_0^2}{2}\,, \qquad (7)$$

provided $k_\beta z=n\pi$, $n=1,2,\ldots$. From Eq. (7) it follows that complete conditioning of the beam is achieved at $k_\beta z=n\pi$, provided the normalized strength of the

TABLE I. Simulation parameters for conditioning a 10 MeV (example 1) and a 1 MeV (example 2) electron beam.

Electron beam	Example 1	Example 2
Energy, E	10 MeV	1 MeV
Emittance, ε_n	3.5×10^{-3} cm rad	3.5×10^{-3} cm rad
rms radius, r_b	0.14 cm	0.14 cm
Initial axial energy spread	2.3×10^{-4}	2.4×10^{-4}
Final (min.) axial energy spread	5.8×10^{-6}	7.9×10^{-6}
Wiggler		
Strength parameter, a_w	0.175	0.175
Period, λ_w	3.14 cm	3.14 cm
Betatron period, λ_β	754 cm	104 cm
Conditioning field		
Wavelength, λ	2 cm	2 cm
Strength parameter, a_0	0.1	2×10^{-3}
Electric field, E_0	160 kV/cm	3.2 kV/cm
Interaction length, $\sim\lambda_\beta/2$	375 cm	53 cm

waveguide field is

$$a_0=\frac{4\gamma_0^5(k_\beta/k)^3}{n\pi(1+a_w^2/2)}=\frac{\gamma_0^2}{2\pi n}\frac{a_w^3}{1+a_w^2/2}\left(\frac{\lambda}{\lambda_w}\right)^2. \quad (8)$$

A full scale particle simulation of an electron beam in a wiggler and conditioning field has been carried out. The fully relativistic Lorentz force equations were integrated for 10^3 particles, using the standard leapfrog algorithm [18].

The first beam conditioning method is illustrated with two examples, a 10 MeV and a 1 MeV electron beam; see Table I. For the 10 MeV example, the axial velocity spread of the beam in the conditioning fields will reach a minimum at $z\simeq\pi/k_\beta=375$ cm, beyond which it increases to its original value. To maintain the minimum spread the conditioning field is adiabatically removed at $z\lesssim\pi/k_\beta$. Figure 2 shows the evolution of the fractional axial energy spread for 100 randomly selected electrons as a function of distance along the waveguide. The convergence of the trajectories in Fig. 2 with propagation distance indicates that the spread in axial velocity of the electrons is significantly reduced by the conditioning field. Figure 3 shows the root mean square (rms) beam axial energy spread, $\gamma_0^2(\delta\beta_z)_{\rm rms}$, as a function of distance. In this illustration the spread is reduced by a factor of $\sim$40. For the 1 MeV example, the rms spread in the axial velocity is observed to be reduced by a factor of approximately 30. In both examples, the required value of the conditioning field is in excellent agreement with the analytical prediction in Eq. (8). For a waveguide diameter of 1 cm the power in the conditioning field is $\sim$10

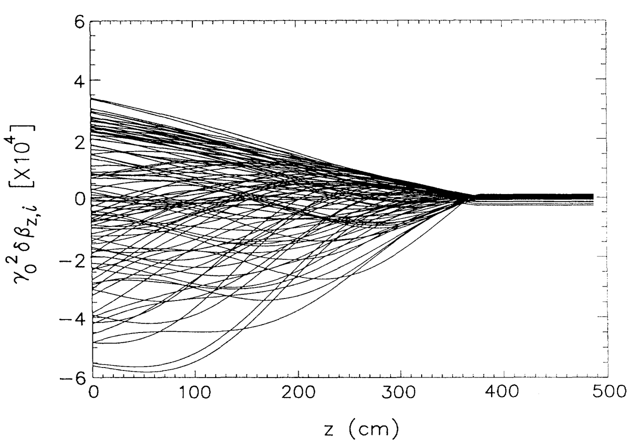

FIG. 2. Fractional axial energy spread, $\gamma_0^2\delta\beta_{z,i}$, where $i=1$–100, vs distance along the conditioning waveguide. The curves represent 100 particles chosen randomly from a distribution of 10^3 particles. In this figure, the conditioning field is adiabatically turned off at $\simeq$375 cm.

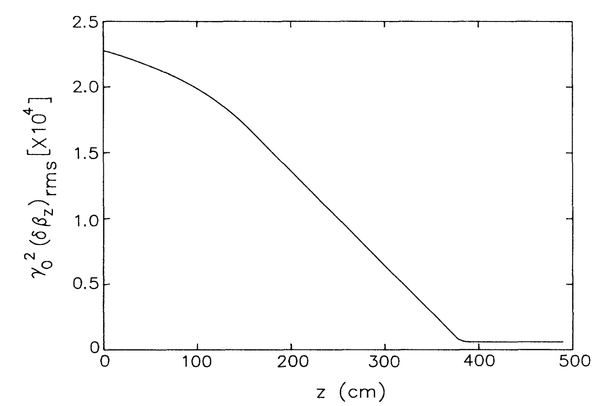

FIG. 3. Root mean square (rms) fractional axial energy spread vs distance. The spread is reduced by a factor of $\sim$40.

MW and $\sim$4 kW for examples 1 and 2, respectively. For comparison, we have performed simulation studies using the beam parameters for the 54 MeV example in Ref. [11]. Our results indicate a reduction in the rms spread in the axial velocity by a factor ≥ 30 using a 100 MW conditioning field.

In the second method, the energy redistribution is correlated with the electrons' synchrotron amplitude. This method can be viewed as a rotation of the electron distribution in phase space (ψ, ψ'). The beam axial velocity spread and length are proportional to the spread in $\delta\psi'$ and $\delta\psi$, respectively. Under certain conditions the beam distribution undergoes a rotation in phase space such that the axial velocity spread decreases at the expense of its length. Here, the electrons interact with the wiggler field and the TM waveguide field in Eq. (3) and the electron pulse is phased such that $\psi = \pi/2 + \delta\psi$, where $|\delta\psi| \ll 1$ and $k_\perp r \ll 1$. The electrons' synchrotron oscillations are described by the pendulum equation, $\delta\psi'' + K_s^2 \delta\psi \simeq 0$, where $K_s \simeq (a_0/\gamma\gamma_z^2)^{1/2}\omega/c$ is the synchrotron wave number. The beam's axial velocity spread reaches a minimum after propagating a distance $L_s = \pi/2K_s$ ($\frac{1}{4}$ of a synchrotron period). By this method, we have obtained reductions of $\gtrsim 10$ in the rms axial velocity spread in unoptimized simulation studies.

In conclusion, in this paper two methods are proposed for dramatically reducing the electron axial velocity spread in FELs. A reduction in axial velocity spread can improve the gain and efficiency of FELs as well as traveling-wave tubes. The beam conditioning field is that of an axially symmetric, slow, TM waveguide mode. In the first method, a reduction in the velocity spread by a factor of 40 was obtained. In the second method, reductions of $\gtrsim 10$ in the velocity spread have been obtained.

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