Predictive Neutrino Spectrum in Minimal SO(10) Grand Unification

K. S. Babu
Bartol Research Institute, University of Delaware, Newark, Delaware 19716

R. N. Mohapatra
Department of Physics, University of Maryland, College Park, Maryland 20742
(Received 31 August 1992)

We show that in minimal SO(10) models where the fermions have Yukawa couplings to only one (complex) **10** and one **126** of Higgs scalars the standard model doublet contained in the **126** receives an induced vacuum expectation value at tree level. In addition to correcting the bad asymptotic mass relations $m_d=m_e$ and $m_s=m_\mu$, this also leads to a predictive neutrino spectrum. We find that (i) the ν_e-ν_μ mixing angle lies in the range $\sin\theta_{e\mu}=0$–0.3, (ii) $\sin\theta_{e\tau}\simeq 3|V_{td}|\simeq 0.05$, (iii) $\sin\theta_{\mu\tau}\simeq 3|V_{cb}|=0.12$–$0.16$, and (iv) $m_{\nu_\tau}/m_{\nu_\mu}\gtrsim 10^3$, implying that ν_μ-ν_τ oscillations should be accessible to forthcoming experiments.

PACS numbers: 12.15.Ff, 12.10.Dm, 14.60.Gh

It is quite possible that the deficit of solar neutrinos reported in the chlorine, Kamiokande, SAGE, and GALLEX experiments [1] is an indication that the neutrinos have masses and mixings very much like the quarks. The observed deficit can be explained in terms of neutrino oscillations in two different ways: (i) long wavelength vacuum oscillation and (ii) resonant matter oscillation [the Mikheyev-Smirnov-Wolfenstein (MSW) effect [2]]. Assuming a two-flavor ν_e-ν_μ oscillation, in the former case, the neutrino masses and mixing angle should satisfy $\Delta m^2\sim 10^{-10}$ eV^2 and $\sin^2 2\theta_{e\mu}\simeq 0.75$–$1$. In the case of MSW there are two allowed windows that fit all of the experimental data [3]: (a) the small-mixing-angle nonadiabatic solution, which requires $\Delta m^2\simeq(0.3$–$1.2)\times 10^{-5}$ eV^2 and $\sin^2 2\theta_{e\mu}\simeq(0.4$–$1.5)\times 10^{-2}$ and (b) the large-angle solution with $\Delta m^2\simeq(0.3$–$5)\times 10^{-5}$ eV^2 and $\sin^2 2\theta_{e\mu}\simeq 0.5$–$0.9$. In all these cases, barring an unlikely scenario of near mass degeneracy among neutrinos, either ν_μ or ν_τ should have mass in the 10^{-5}–10^{-3} eV range. A natural explanation for the origin of such tiny neutrino masses is the seesaw mechanism. The solar neutrino puzzle indicates that the $B-L$ scale is in the 10^{12}–10^{16} GeV range.

All of the observations above, viz., nonzero neutrino masses, the seesaw mechanism, and a high $B-L$ scale, fit rather naturally in grand unified models based on the gauge group SO(10). In its nonsupersymmetric version, experimental constraints from proton lifetime and the weak mixing angle $\sin^2\theta_W$ require that SO(10) break not directly into the standard model, but at least in two steps. In a two-step breaking scheme, the left-right symmetric intermediate scale is around 10^{12} GeV. In supersymmetric (SUSY) SO(10) there is no need for an intermediate scale; SO(10) can break directly to the standard model at around 10^{16} GeV.

To confront SO(10) models with the solar neutrino data, one must make precise predictions of the neutrino masses and mixing angles. This requires, however, detailed information of the Dirac neutrino mass matrix as well as the Majorana matrix. In grand unified theories (GUTs), it is possible to relate the quark masses with the lepton masses. In SO(10) models, the charge $-\frac{1}{3}$ quark mass matrix is related to the charged lepton matrix and the neutrino Dirac mass matrix is related to the charge $\frac{2}{3}$ quark matrix. However, the heavy Majorana matrix (and hence the light neutrino spectrum) is arbitrary.

In this Letter we show that in a class of minimal SO(10) models both the Dirac and the Majorana neutrino matrices get related to the charged fermion sector, leading to a predictive neutrino spectrum. We use a simple Higgs system with one (complex) **10** and one **126** that have Yukawa couplings to fermions. The **10** is needed for quark and lepton masses; the **126** is needed for the seesaw mechanism. Crucial to the predictivity of the neutrino spectrum is the observation that the standard model doublet contained in the **126** receives an induced vacuum expectation value (VEV) at tree level. In its absence, one would have the asymptotic mass relations $m_b=m_\tau$, $m_s=m_\mu$, $m_d=m_e$, the last two of which are in disagreement with observations. The induced VEV of the standard doublet of **126** corrects these bad relations and at the same time also relates the Majorana neutrino mass matrix to observables in the charged fermion sector, leading to a predictive neutrino spectrum.

We shall consider non-SUSY SO(10) breaking to the standard model via the $SU(2)_L\times SU(2)_R\times SU(4)_C\equiv G_{224}$ chain as well as SUSY SO(10) breaking directly to the standard model. The breaking of SO(10) via G_{224} is achieved by either a **54** or a **210** of Higgs. The **210** also breaks the discrete D parity; the **54** preserves it. D parity is a local discrete Z_2 subgroup of SO(10); under D, a fermion field f transforms into its charge conjugate f^c. Breaking of D parity at the GUT scale makes the seesaw mechanism natural [4]. The second stage of symmetry breaking goes via the **126**. Finally, the electroweak symmetry breaking proceeds via the **10**. In SUSY SO(10), the first two symmetry breaking scales coalesce into one.

In the fermion sector, denoting the three families belonging to **16**-dimensional spinor representation of SO(10) by ψ_a, $a=1$–3, the complex **10**-plet of Higgs by H, and the **126**-plet of Higgs by Δ, the Yukawa couplings can be written down as

$$L_Y = h_{ab}\psi_a\psi_b H + f_{ab}\psi_a\psi_b\bar{\Delta} + \text{H.c.} \tag{1}$$

Note that since the **10**-plet is complex, one other coupling $\psi_a\psi_b H^*$ is allowed in general. In SUSY SO(10), the requirement of supersymmetry prevents such a term. In the non-SUSY case, we forbid this term by imposing a $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry, which may be needed anyway in order to solve the strong CP problem.

The **10** and **126** of Higgs have the following decomposition under G_{224}: $\mathbf{126} \to (1,1,6)+(1,3,10)+(3,1,\overline{10})+(2,2,15)$, $\mathbf{10} \to (1,1,6)+(2,2,1)$. Denote the (1,3,10) and (2,2,15) components of $\Delta(\mathbf{126})$ by Δ_R and Σ, respectively, and the (2,2,1) component of $H(\mathbf{10})$ by Φ. The VEV $\langle \Delta_R^0 \rangle \equiv v_R \sim 10^{12}$ GeV breaks the intermediate symmetry down to the standard model and generates Majorana neutrino masses given by f_{v_R}. Φ contains two standard model doublets which acquire VEV's denoted by κ_u and κ_d with $\kappa_{u,d} \sim 10^2$ GeV. κ_u generates charge $\frac{2}{3}$ quark as well as Dirac neutrino masses, while κ_d gives rise to $-\frac{1}{3}$ quark and charged lepton masses.

Within this minimal scheme, we have found new contributions to the fermion mass matrices which are of the right order of magnitude to correct the bad relations $m_\mu = m_s$ and $m_e = m_d$. To see this, note that the scalar potential contains, among other terms, a crucial term

$$V_1 = \lambda\Delta\bar{\Delta}\Delta H + \text{H.c.} \tag{2}$$

Such a term is invariant under the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry. It will be present in the SUSY SO(10) as well, arising from the **210** F term. This term induces VEV's for the standard doublets contained in the Σ multiplet of **126**. The VEV arises through a term $\bar{\Delta}_R\Delta_R\Sigma\Phi$ contained in V_1.

We can estimate the magnitudes of the induced VEV's of Σ (denoted by v_u and v_d along the up and down directions) assuming the survival hypothesis to hold:

$$v_{u,d} \sim \lambda(v_R^2/M_{\Sigma_{u,d}}^2)\kappa_{u,d}\,. \tag{3}$$

Suppose $M_U \sim 10^{15}$ GeV, $M_I \sim 3\times10^{12}$ GeV, and $M_\Sigma \sim 10^{14}$ GeV, consistent with survival hypothesis; then v_u and v_d are of order 100 MeV, in the right range for correcting the bad mass relations. We emphasize that there is no need for a second fine tuning to generate such induced VEV's. In the SUSY version with no intermediate scale, the factor v_R^2/M_Σ^2 is not a suppression, so the induced VEV's can be as large as $\kappa_{u,d}$.

We are now in a position to write down the quark and lepton mass matrices of the model:

$$\begin{aligned} &M_u = h\kappa_u + f v_u, \quad M_d = h\kappa_d + f v_d\,, \\ &M_\nu^D = h\kappa_u - 3 f v_u, \quad M_l = h\kappa_d - 3 f v_d\,, \\ &M_\nu^M = f v_R\,. \end{aligned} \tag{4}$$

Here M_ν^D is the Dirac neutrino matrix and M_ν^M is the Majorana mass matrix.

Before proceeding, we should specify the origin of CP violation in the model. We shall assume that it is spontaneous or soft; that will keep the number of parameters at a minimum. The Higgs sector described above already has enough structure to generate realistic CP violation either softly or spontaneously. The Yukawa coupling matrices h and f in this case are real and symmetric. Although there will be three different phases in the VEV's (one common phase for κ_u and κ_d and one each for v_u and v_d), only two combinations enter into the mass matrices, as the overall phase can be removed from each sector. We shall bring these two phases into v_u and v_d and henceforth denote them by $v_u e^{i\alpha}$ and $v_d e^{i\beta}$.

In the basis where the matrix h is diagonal, there are thirteen parameters in all, not counting the superheavy scale v_R: three diagonal elements of the matrix $h\kappa_u$, six elements of $f v_u$, two ratios of VEV's $r_1 = \kappa_d/\kappa_u$ and $r_2 = v_d/v_u$, and the two phases α and β. These thirteen parameters are related to the thirteen observables in the charged fermion sector, viz., nine fermion masses, three quark mixing angles, and one CP violating phase. The light neutrino mass matrix will then be completely specified, up to an overall scale.

The relations of Eq. (4) hold at the intermediate scale M_I where quark-lepton symmetry and left-right symmetry are intact. There are calculable renormalization corrections to these relations below M_I. The quark and charged lepton masses as well as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements run between M_I and low energies. The neutrino masses and mixing angles, however, do not run below M_I, since the right-handed neutrinos have masses of order M_I and decouple below that scale. The predictions in the neutrino sector should then be arrived at by first extrapolating the charged fermion observables to M_I.

We shall present results for the non-SUSY SO(10) model with the G_{224} intermediate symmetry. We fix the intermediate scale at $M_I = 10^{12}$ GeV and use the one-loop standard model renormalization group equations to track the running of the gauge couplings between M_Z and M_I. For SUSY SO(10), the results are similar, we shall postpone details to a forthcoming longer paper.

To compute the renormalization factors, we choose as low energy inputs the gauge couplings at M_Z to be $\alpha_1(M_Z)=0.01688$, $\alpha_2(M_Z)=0.03322$, $\alpha_3(M_Z)=0.11$. For the light quark (running) masses, we choose values listed in Ref. [5]. The top-quark mass will be allowed to vary between 100 and 200 GeV. Between 1 GeV and M_Z, we use two-loop QCD renormalization group equations for the running of the quark masses and the $\mathrm{SU}(3)_C$ gauge coupling [5], treating particle thresholds as step functions. From M_Z to M_I, the running factors are computed semianalytically both for the fermion masses and for the CKM angles by using the one-loop renormalization group equations for the Yukawa couplings and keeping the heavy top-quark contribution [6]. The running factors, defined as $\eta_i = m_i(M_I)/m_i(m_i)$ [$\eta_i = m_i(M_I)/m_i(1\text{ GeV})$ for light quarks (u,d,s)] are $\eta(u,c,t) = (0.273, 0.286, 0.506)$, $\eta(d,s,b) = (0.279, 0.279, 0.327)$, $\eta(e,\mu,\tau) = 0.960$ for the case of $m_t = 150$ GeV. The (common) running factors for the CKM angles (we follow the parametrization advocated by the Particle Data Group) S_{23} and S_{13} is 1.081 for $m_t = 150$ GeV. The Cabibbo angle S_{12} and the KM phase $\delta_{\rm KM}$ are essentially unaltered.

Let us first analyze the mass matrices of Eq. (4) in the limit of CP conservation. We shall treat spontaneous CP violation arising through the phases of the VEV's $v_u e^{i\alpha}$ and $v_d e^{i\beta}$ as small perturbations. This procedure will be justified *a posteriori.* In fact, we find that realistic fermion masses, in particular the first family masses, require these phases to be small.

We can rewrite the mass matrices M_l, M_ν^D, and M_ν^M of Eq. (4) in terms of the quark mass matrices and three ratios of VEV's—$r_1 = \kappa_d/\kappa_u$, $r_2 = v_d/v_u$, $R = v_u/v_R$:

$$
\begin{aligned}
M_l &= \frac{4r_1r_2}{r_2-r_1}M_u - \frac{r_1+3r_2}{r_2-r_1}M_d\,,\\
M_\nu^D &= \frac{3r_1+r_2}{r_2-r_1}M_u - \frac{4}{r_2-r_1}M_d\,,\\
M_\nu^M &= \frac{1}{R}\frac{r_1}{r_1-r_2}M_u - \frac{1}{R}\frac{1}{r_1-r_2}M_d\,.
\end{aligned}
\tag{5}
$$

In a basis where M_u is diagonal, M_d is given by $M_d = VM_d^{\rm diag}V^T$, where $M_d^{\rm diag} = {\rm diag}(m_d, m_s, m_b)$ and V is the CKM matrix. One sees that M_l of Eq. (5) contains only physical observables from the quark sector and two parameters r_1 and r_2. From the trace of M_l, one obtains $r_1 \simeq (m_\tau + 3m_b)/4m_t$ (valid for $r_2 \gg m_b/m_t$). Since $|m_b| \simeq |m_\tau|$ at the intermediate scale to within 30% or so, depending on the relative sign of m_b and m_τ, r_1 will be close to either m_b/m_t or to $m_b/2m_t$. Note also that if $r_2 \gg r_1$, M_l becomes independent of r_2, while M_ν^D retains some dependence. This implies that r_2 can only be loosely constrained from the charged fermion sector.

We do the fitting as follows. For a fixed value of r_2, we determine r_1 from the ${\rm Tr}(M_l)$ using the input values of the masses and the renormalization factors discussed above. M_l is then diagonalized numerically. There will be two mass relations among charged fermions. Since the charged lepton masses are precisely known at low energies, we invert these relations to predict the d-quark and s-quark masses. The s-quark mass is sensitive to the muon mass; the d mass is related to the electron mass. This procedure is repeated for other values of r_2. For each choice, the light neutrino masses and the leptonic CKM matrix elements are then computed using the seesaw formula. $r_2 \sim 4m_s/m_c \sim \pm 0.4$ leads to a qualitatively different result since there are cancellations then in M_ν^D.

Numerical results for the three different cases are presented below. The input values of the CKM mixing angles are chosen for all cases to be $S_{12} = -0.22$, $S_{23} = 0.052$, $S_{13} = 6.24\times10^{-3}$. Since $\delta_{\rm KM}$ has been set to zero for now, we have allowed for the mixing angles to have either sign. Not all signs result in acceptable quark masses though. Similarly, the fermion masses can have either sign, but these are also restricted. The most stringent constraint comes from the d-quark mass, which has a tendency to come out too small. Acceptable solutions are obtained when θ_{23}, θ_{13} are in the first quadrant and θ_{12} in the fourth quadrant.

Solution 1.— Input:

$$m_u(1\text{ GeV}) = 3\text{ MeV}\,,$$

$$m_c(m_c) = 1.22\text{ GeV},\quad m_t = 150\text{ GeV}\,,$$

$$m_b(m_b) = -4.35\text{ GeV},\quad r_1 = -1/51.2,\quad r_2 = 2.0\,.$$

Output:

$$m_d(1\text{ GeV}) = 6.5\text{ MeV},\quad m_s(1\text{ GeV}) = 146\text{ MeV}\,,$$

$$(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R(2.0\times10^{-2}, 9.9, -2.3\times10^4)\text{ GeV}\,,$$

$$V_{\rm KM}^{\rm lepton} = \begin{pmatrix} 0.9488 & 0.3157 & 0.0136 \\ -0.3086 & 0.9349 & -0.1755 \\ -0.0681 & 0.1623 & 0.9844 \end{pmatrix}. \tag{6}$$

Solution 2.— Input:

$$m_u(1\text{ GeV}) = 3\text{ MeV}\,,$$

$$m_c(m_c) = 1.22\text{ GeV},\quad m_t = 150\text{ GeV}\,,$$

$$m_b(m_b) = -4.35\text{ GeV},\quad r_1 = -1/51,\quad r_2 = 0.2\,.$$

Output:

$$m_d(1\text{ GeV}) = 5.6\text{ MeV},\quad m_s(1\text{ GeV}) = 156\text{ MeV}\,,$$

$$(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R(7.5\times10^{-3}, 2.0, -2.8\times10^3)\text{ GeV}\,,$$

$$V_{\rm KM}^{\rm lepton} = \begin{pmatrix} 0.9961 & 0.0572 & -0.0676 \\ -0.0665 & 0.9873 & -0.1446 \\ 0.0584 & 0.1485 & 0.9872 \end{pmatrix}. \tag{7}$$

Solution 3.— Input:

$$m_u(1\text{ GeV}) = 3\text{ MeV}\,,$$

$$m_c(m_c) = 1.27\text{ GeV},\quad m_t = 150\text{ GeV}\,,$$

$$m_b(m_b) = -4.35\text{ GeV},\quad r_1 = -1/51.1,\quad r_2 = 0.4\,.$$

Output:

$$m_d(1\text{ GeV}) = 6.1\text{ MeV},\quad m_s(1\text{ GeV}) = 150\text{ MeV}\,,$$

$$(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R(4.7\times10^{-2}, 1.4, -5.0\times10^3)\text{ GeV}\,,$$

$$V_{\rm KM}^{\rm lepton} = \begin{pmatrix} 0.9966 & 0.0627 & -0.0541 \\ -0.0534 & 0.9858 & 0.1589 \\ 0.0633 & -0.1555 & 0.9858 \end{pmatrix}. \tag{8}$$

Solution 1 corresponds to choosing $r_1 \sim m_b/m_t$. Since r_2 is large, the Dirac neutrino matrix is essentially M_u, which is diagonal; so is the Majorana matrix. All the leptonic mixing angles arise from the charged lepton sector. Note that the predictions for m_d and m_s are within the range quoted in Ref. [7]. The ratio $m_s/m_d=22$ is within the allowed range from chiral perturbation theory estimates: $15\leq m_s/m_d\leq 25$ [8]. The mixing angle $\sin\theta_{\nu_e\text{-}\nu_\mu}$ relevant for solar neutrinos is 0.30, close to the Cabibbo angle. Such a value may already be excluded by a combination of all solar neutrino data taken at the 90% C.L. (but not at the 95% C.L.) [3]. Actually, within the model, there is a more stringent constraint. Note that the ν_μ-ν_τ mixing angle is large; it is approximately $3|V_{cb}|\simeq 0.16$. For that large a mixing, constraints from ν_μ-ν_τ oscillation experiments imply [9] that $|m^2_{\nu_\tau}-m^2_{\nu_\mu}|\leq 4$ eV^2. Solution 1 also has $m_{\nu_\tau}/m_{\nu_\mu}\simeq 2.3\times 10^3$, requiring that $m_{\nu_\mu}\leq 0.9\times 10^{-3}$ eV. This is a factor of 2 too small for ν_e-ν_μ MSW oscillation for the solar puzzle (at the 90% C.L.), but perhaps is not excluded completely, once astrophysical uncertainties are folded in. If ν_τ mass is around 2×10^{-3} eV, ν_e-ν_τ oscillation may be relevant; that mixing angle is $\simeq 3|V_{td}|\simeq 6\%$. It would require the parameter $R=v_u/v_R\sim 10^{-16}$ or $v_R\sim 10^{16}$ GeV for $v_u\sim 1$ GeV. Such a scenario fits very well within SUSY SO(10).

Solution 2 differs from 1 in that r_2 is smaller; $r_2=0.2$. The ratio $m_s/m_d=27.8$ is slightly above the limit in Ref. [8]. The 1-2 mixing in the neutrino sector is large in this case, so it can cancel the Cabibbo-like mixing arising from the charged lepton sector. As we vary r_2 from around 0.2 to 0.6, this cancellation becomes stronger, the ν_e-ν_μ mixing angle becoming zero for a critical value of r_2. For larger r_2, the solution will approach solution 1. The ν_μ-ν_τ mixing angle is still near $3|V_{cb}|$, so as before, $m_{\nu_\tau}\lesssim 2$ eV. From the ν_τ/ν_μ mass ratio, which is 1.4×10^3 in this case, we see that $m_{\nu_\mu}\leq 1.5\times 10^{-3}$ eV. This is just within the allowed range [3] (at 95% C.L.) for small-angle nonadiabatic ν_e-ν_μ MSW oscillation, with a predicted count rate of about 50 solar neutrino units for the Gallium Experiment. Note that there is a lower limit of about 1 eV for the ν_τ mass in this case. Forthcoming experiments should then be able to observe ν_μ-ν_τ oscillations. A ν_τ mass in the 1–2 eV range can also be cosmologically significant; it can be at least part of the hot dark matter. In SUSY SO(10), ν_e-ν_τ oscillation (the relevant mixing is about $3|V_{td}|\simeq 5\%$) could account for the solar neutrino puzzle.

Solution 3 corresponds to choosing $r_1=0.4$. $m_s/m_d=24.6$ is within the allowed range. However, the mass ratio ν_τ/ν_μ is $\sim 3.6\times 10^3$, and $\sin\theta_{\mu\tau}\simeq 3|V_{cb}|$ so ν_e-ν_μ oscillation cannot be responsible for solar MSW. As in other cases, ν_e-ν_τ MSW oscillation with a 6% mixing is a viable possibility.

Let us now reinstate the *CP* violating phases α and β in the VEV's perturbatively. Small values of the phases are sufficient to account for realistic *CP* violation in the quark sector. We shall present details for the case of solution 2 only; others are similar. We also tried to fit all the charged fermion masses and mixing angles for large phases, but found no consistent solution.

First we make a basis transformation to go from the basis where M_u is diagonal to one where the matrix $h\kappa_u$ is diagonal. It is easier to introduce phases in that basis. For $\alpha=3.5°$, $\beta=4.5°$, the *CP* violating parameter J for the quark system [7] is $J\simeq 1\times 10^{-5}$, which is sufficient to accommodate ϵ in the neutral K system. The leptonic *CP* violating phases are correspondingly small; e.g., the analog of J is $J_l\simeq 7\times 10^{-5}$. These small phases modify the first family masses slightly, but the effect is less than 10%. Our predictions for the neutrino mixing angles are essentially unaltered.

In summary, we have presented a class of minimal SO(10) models where the light neutrino masses and mixing angles are predicted. Our approach does not use any sort of family symmetry and has manifest stability under radiative corrections.

We thank Kari Enqvist for the invitation to the 1992 Neutrino Program at NORDITA where this work began. K.S.B. wishes to acknowledge the Aspen Center for Physics, where part of this work was done, for its hospitality. We also thank Stuart Raby and Lincoln Wolfenstein for discussions. R.N.M. wishes to thank Gerry Gilbert and Lee-Dai Gyu for introducing him to Mathematica. K.S.B. was supported in part by the Department of Energy Grant No. DE-FG02-91ER406267 and R.N.M. was supported by the National Science Foundation Grant No. PHY-9119745.

[1] R. Davis *et al.*, in *Proceedings of the Twenty-First International Cosmic Ray Conference,* edited by R. J. Protheroe (University of Adelaide Press, Adelaide, 1990), Vol. 12, p. 143; K. S. Hirata *et al.*, Phys. Rev. Lett. **65**, 1297 (1990); A. I. Abazov *et al.*, Phys. Rev. Lett. **67**, 3332 (1991); P. Anselman *et al.*, Phys. Lett. B **285**, 376 (1992).

[2] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheyev and A. Yu Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)].

[3] X. Shi, D. N. Schramm, and J. Bahcall, Phys. Rev. Lett. **69**, 717 (1992); S. A. Bludman, N. Hata, D. C. Kennedy, and P. Langacker, Pennsylvania Report No. UPR-0516T, 1992 (to be published).

[4] D. Chang and R. N. Mohapatra, Phys. Rev. D **32**, 1248 (1985).

[5] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[6] K. S. Babu, Z. Phys. C **35**, 69 (1987); K. Sasaki, Z. Phys. C **32**, 149 (1986); B. Grzadkowski, M. Lindner, and S. Theisen, Phys. Lett. B **198**, 64 (1987).

[7] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).

[8] D. Kaplan and A. Manohar, Phys. Rev. Lett. **56**, 2004 (1986).

[9] N. Ushida *et al.*, Phys. Rev. Lett. **57**, 2897 (1986).