Gravitational Smearing of Minimal Supersymmetric Unification Predictions

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(Received 20 October 1992)

The prediction for α_s in the minimal supersymmetric SU(5) grand unified theory is generically subject to uncertainties arising from a gravitationally induced dimension-five operator. Unless the coefficient of this operator is small, the correlation between α_s and the mass scale which governs proton decay to $K\nu$ is destroyed. Furthermore, a reduction of the experimental uncertainty in α_s would not provide a significant test of the theory.

PACS numbers: 12.10.Gq, 04.60.+n, 11.30.Pb

Of the eighteen parameters in the standard model, only the weak mixing angle has been successfully predicted to a high level of accuracy. Simple supersymmetric versions of grand unified theories (GUTs) use the experimental value of the strong coupling α_s to predict [1] $\sin^2\theta_W = 0.233$, in full agreement with the value obtained from experiments at the CERN e^+e^- collider LEP [2]. We choose [3] instead to use the very precise measurement of $\sin^2\theta_W$ and predict α_s, which may then be compared to various experimental determinations. Over the past few years several groups [3–6] have studied what information may be extracted from this unification of gauge couplings. It is frequently stated that to further test GUTs, the strong coupling should be measured more precisely [5, 7]. Some claim that this would determine the scale of superpartner masses, others that such improved accuracy would help pin down the proton decay rate in the minimal model [5].

In this Letter we first recall how the α_s prediction in the minimal model can be made independent of most threshold corrections, and then demonstrate that it *is* subject to corrections arising from higher-dimension operators [8–10] induced at the Planck scale. We show that these corrections constitute a fundamental, generic source of uncertainty in the prediction of α_s which cannot be removed without some assumptions about physics at the Planck scale, that is, beyond the scope of the grand unified theory.

While we only consider the minimal model explicitly, similar results will hold in most models. Logarithmic threshold corrections to the prediction for α_s may arise from each nondegenerate ("split") SU(5) multiplet, and some such corrections will be present in every GUT [11]. The minimal number of representations whose states are not degenerate is two: one contains the superheavy (mass M_X) and the light gauge particles; the other contains the superheavy (mass $M_{\rm tr}$) and the light members of the multiplet responsible for spontaneous electroweak symmetry breaking. In addition, the remnants Σ of the representation which breaks the GUT group to $SU(3)\times SU(2)\times U(1)$ have masses M_Σ which generically differ from those of the Goldstone bosons eaten by the superheavy gauge particles. It is well known that (at one-loop order) the prediction for α_s does not depend on M_X. Recently it was pointed out [5, 6] that there is also no dependence on M_Σ—a fact overlooked in the previous threshold analysis [11]. This raises the interesting possibility that, in certain simple GUT models, the only significant dependence of the α_s prediction on the superheavy sector is through $M_{\rm tr}$, which in these models controls the rate of proton decay. Since α_s increases with $M_{\rm tr}$, an improved experimental upper limit on α_s could reduce the upper bound on $M_{\rm tr}$. In that case, super-Kamiokande could definitively test this theory [5].

To one-loop order, and *in the absence of gravitational corrections*, gauge coupling unification is embodied in the three renormalization-group equations relating the values of the gauge couplings at the Z mass, $\vec{\alpha}^{-1} \equiv \vec{\alpha}^{-1}(m_Z) \equiv (\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1})$, and the common gauge coupling α_G at the GUT scale M_G:

$$\vec{\alpha}^{-1} = \alpha_G^{-1}\vec{1} - \sum_a \vec{\beta}_a \ln\left(\frac{M_a}{M_G}\right) . \qquad (1)$$

Here $\vec{1} \equiv (1,1,1)$ and $\vec{\beta}_a \equiv \vec{b}_a/(2\pi)$ where $\vec{b}_a$ are the three beta-function coefficients for the particle labeled by a. The sum extends over all particles in the model, and M_a denotes the mass threshold at which each is integrated out. (We neglect electroweak-breaking effects in the supersymmetric mass spectrum, and treat the top quark as being degenerate with the Z. These effects are small relative to the dominant uncertainties in the experimental inputs and in the gravitational corrections.) All of the standard model particles are already present at the initial scale m_Z; we then include the second Higgs doublet at m_{H_2}, the squarks at an average mass $m_{\tilde{q}}$, the sleptons at their average mass $m_{\tilde{l}}$, the W-inos at $m_{\tilde{w}}$, the gluinos at $m_{\tilde{g}}$, the higgsinos at $m_{\tilde{H}}$, the color-triplet component of the **5** of Higgs bosons at $M_{\rm tr}$, the non-Goldstone-boson members of the **24** of Higgs bosons at M_Σ, and finally the superheavy gauge bosons and their superpartners ("X")

at M_X. The GUT scale is the highest mass threshold, above which all particles fill complete SU(5) multiplets. The experimental inputs are derived from [2] $s^2 \equiv \sin^2\theta_W = 0.2325 \pm 0.0008$ and $1/\alpha = 127.9 \pm 0.1$. The two-loop contributions to (1) and the conversion from the $\overline{\rm MS}$ (modified minimal subtraction) scheme into the $\overline{\rm DR}$ (dimensional reduction) scheme [12] are incorporated, using typical values for all the parameters, by adding a term $\vec{\Delta}_2 = (0.65, 1.09 + 2/12\pi, 0.55 + 3/12\pi)$ to the right-hand side of (1).

We concentrate on the predictions of (1) for α_s and $M_{\rm tr}$, and ignore the prediction for α_G. Therefore we consider (well-known) linear combinations of (1) which do not involve α_G. Define for convenience (up to an irrelevant overall normalization) one projection vector $\vec{P}_1$ by requiring $\vec{P}_1 \cdot \vec{1} = 0$ and $\vec{P}_1 \cdot \vec{\beta}_X = 0$, and another projection vector $\vec{P}_2$ by requiring $\vec{P}_2 \cdot \vec{1} = 0$ and $\vec{P}_2 \cdot \vec{\beta}_{\rm tr} = 0$. We choose

$$\vec{P}_1 = (-1, 3, -2), \qquad \vec{P}_2 = (5, -3, -2). \tag{2}$$

The dot product of $\vec{P}_1$ with (1) will be independent of α_G and of M_X and M_Σ, and therefore will furnish a simple expression for α_s in terms of the low-energy parameters and the Higgs-triplet mass. The dot product of $\vec{P}_2$ with (1) will be independent of α_G and of $M_{\rm tr}$ and the light Higgs sector, and will relate the masses of the superheavy gauge multiplet and the superheavy **24**. Finally, the unification scale m_G enters (1) only through the combination $(\ln m_G)\sum_a \vec{\beta}_a \propto \vec{1}$ and so it, too, is projected out; thus any other scale may be used in these dot products, and we choose that scale for convenience to be m_Z.

Note that in the dot product with $\vec{P}_1$ both the X *and* the Σ were projected out. The reason is simple. The $\vec{\beta}$ for any complete SU(5) multiplet, and in particular $\vec{\beta}_{\bf 24}$, contributes equally to the running of all gauge couplings, so it is proportional to $\vec{1}$ and hence orthogonal to $\vec{P}_1$. The $\vec{\beta}_{\rm GB}$ for the Goldstone mode components of the **24** is proportional to the $\vec{\beta}_X$ of the superheavy X since they carry the same quantum numbers, and so it too is orthogonal to $\vec{P}_1$. Therefore their difference $\vec{\beta}_{\bf 24} - \vec{\beta}_{\rm GB} = \vec{\beta}_\Sigma$ also satisfies $\vec{P}_1 \cdot \vec{\beta}_\Sigma = 0$ and the Σ does *not* make a threshold contribution to this equation. Similarly, both the Higgs doublet *and* the triplet are projected out in the dot product with $\vec{P}_2$. The Σ could contribute if it were split, which is not the case in the minimal model but would be the case in most extensions. An example of such a contribution in the minimal model is provided by the gauginos: they would also be projected out since they carry quantum numbers complementary to those of the X, but their masses are widely split by renormalization-group running so they make a significant (and calculable) contribution to the predictions for α_s.

To obtain specific predictions, we need the mass spectrum of the model. In the minimal model the weak-scale masses are determined to a good approximation by the four mass parameters m_0 (the common scalar mass), $m_{1/2}$ (the common gaugino mass at the GUT scale), μ (the coupling of the two Higgs doublets in the superpotential) and m_{H_2}. For our purposes the following simplified spectrum will suffice [5, 13]: $m_{\tilde q} \simeq \sqrt{m_0^2 + 6m_{1/2}^2}$, $m_{\tilde l} \simeq \sqrt{m_0^2 + 0.4m_{1/2}^2}$, $m_{\tilde g} \simeq 2.7m_{1/2}$, $m_{\tilde w} \simeq 0.8m_{1/2}$, and $m_{\tilde H} \simeq \mu$. By applying the projections (2) to (1) and including the two-loop term $\vec{\Delta}_2$ we find

$$\frac{2}{\alpha_s} + \frac{6}{5\pi}\ln\frac{M_{\rm tr}}{m_Z} = f_1(s^2, m_0, m_{1/2}, \mu, m_{H_2}) \tag{3}$$

and

$$\frac{2}{\alpha_s} + \frac{6}{\pi}\ln\frac{M_\Sigma}{m_Z} + \frac{12}{\pi}\ln\frac{M_X}{m_Z} = f_2(s^2, m_0, m_{1/2}), \tag{4}$$

where

$$f_1 = \frac{3(6s^2-1)}{5\alpha} - \frac{3}{20\pi}\ln\frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 0.4m_{1/2}^2} - \frac{2}{\pi}\ln\frac{2.7}{0.8} + \frac{4}{5\pi}\ln\frac{\mu}{m_Z} + \frac{1}{5\pi}\ln\frac{m_{H_2}}{m_Z} - 1.52$$
$$\simeq 27.9 + 0.4\sigma + \frac{1}{\pi}\ln\frac{\mu^{4/5}m_{H_2}^{1/5}}{m_Z}, \tag{5}$$

$$f_2 = \frac{3(1-2s^2)}{\alpha} - \frac{3}{4\pi}\ln\frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 0.4m_{1/2}^2} - \frac{2}{\pi}\ln(2.7\cdot 0.8) - \frac{4}{\pi}\ln\frac{m_{1/2}}{m_Z} + 1.13 + \frac{1}{\pi}$$
$$\simeq 206.2 - 0.6\sigma - \frac{3}{4\pi}\ln\frac{m_0^2 + 6m_{1/2}^2}{m_0^2 + 0.4m_{1/2}^2} - \frac{4}{\pi}\ln\frac{m_{1/2}}{m_Z}, \tag{6}$$

and $\sigma \equiv (s^2 - 0.2325)/0.0008$.

Equation (4) can be viewed as a prediction for M_Σ. It shows that one can raise the X mass by lowering the mass of the Σ without affecting the prediction for α_s. This point will be crucial to showing that the gravitational corrections can be large. But since there are no experimental consequences of a light Σ, we focus for now on (3). To avoid excessive fine-tuning and retain the motivation for supersymmetric unification, we restrict m_0, $m_{1/2}$, μ, and m_{H_2} to lie below 1 TeV. (Our results are not sensitive to the exact value of this cutoff.) As we vary these four parameters and vary s^2 within 1 standard deviation of its central value (namely, $|\sigma| \leq 1$), the prediction for α_s as a function of $M_{\rm tr}$ ranges between the two black curves in Fig. 1.

We now turn to the effects of quantum gravity on these predictions. In the absence of a specific and predictive theory of quantum gravity, we can only estimate these effects by including higher-dimension operators that would arise in an effective theory once the Planck-scale degrees of freedom have been integrated out. Following Hill [8] and Shafi and Wetterich [9], we restrict our attention to the dominant dimension-five operator

$$\delta\mathcal{L} = \frac{c}{2\hat{M}_P}\,\mathrm{tr}\,(GG\Sigma)\,, \tag{7}$$

where $G \equiv G_a T^a$ is the field-strength tensor of the SU(5) gauge field and the generators are normalized to $\mathrm{tr}\ T^aT^b = \frac{1}{2}\delta^{ab}$. We assume that the mass scale suppressing such nonrenormalizable operators is the *reduced* Planck mass $\hat{M}_P \equiv (8\pi G_N)^{-1/2} \simeq 2.4\times 10^{18}$ GeV, since this is the combination which enters quantum-gravity calculations. We have also conservatively included a factor of 1/2 to account for the two identical operators GG. The remaining coefficient c is unknown without further assumptions; we have no reason to think it is less than ~ 1 in magnitude. SU(5) is broken by the vacuum expectation value $\langle\Sigma\rangle \equiv vT^0$ where $T^0 = \mathrm{diag}(2,2,2,-3,-3)\,/2\sqrt{15}$. This breaking induces superheavy masses $M_X = \sqrt{\frac{5}{6}}g_5 v$ from the covariant derivative $D_\mu\Sigma = \partial_\mu\Sigma_a T^a + ig_5 X_{\mu a}\Sigma_b[T^a,T^b]$, a triplet mass $M_{\rm tr} = \sqrt{\frac{5}{12}}\lambda_5 v$ from the term $\frac{3}{5}M_{\rm tr}\overline{H}_5H_5 + \lambda_5\overline{H}_5\Sigma H_5$ (after a fine-tuning to make the doublets light), and a Σ mass $M_\Sigma = \frac{1}{2}\sqrt{\frac{5}{12}}\lambda_{24}v$ from the term $\frac{1}{5}M_{\rm tr}\,\mathrm{tr}\,\Sigma^2 + \frac{1}{3}\lambda_{24}\,\mathrm{tr}\,\Sigma^3$. It also modifies the kinetic terms of the standard-model gauge bosons through $\delta\mathcal{L}$:

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}(FF)_{\rm U(1)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{-1}{2\sqrt{15}}\right)\right] - \frac{1}{2}\,\mathrm{tr}\,(GG)_{\rm SU(2)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{-3}{2\sqrt{15}}\right)\right] - \frac{1}{2}\,\mathrm{tr}\,(GG)_{\rm SU(3)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{1}{\sqrt{15}}\right)\right]. \tag{8}$$

Consequently the three gauge couplings are not degenerate at the GUT scale. The first term in the equations for unification (1) must be replaced by $\alpha_G^{-1}\vec{1} \longrightarrow \alpha_G^{-1}(\vec{1} + \vec{\epsilon})$ where $\vec{\epsilon} \equiv (cv/2\hat{M}_P)(-1/2\sqrt{15}, -3/2\sqrt{15}, 1/\sqrt{15})$. We have absorbed the sign of v into c, so v and $\lambda_{5,24}$ are by definition positive.

By applying the projection operators to the modified unification equations, we obtain

$$\frac{2}{\alpha_s} + \frac{6}{5\pi}\ln\frac{M_{\rm tr}}{m_Z} - \sqrt{\frac{12}{5}}\frac{c}{2}\frac{v}{\hat{M}_P}\frac{1}{\alpha_G} = f_1(s^2, m_0, m_{1/2}, \mu, m_{H_2}) \tag{9}$$

and

$$\frac{2}{\alpha_s} + \frac{9}{\pi}\ln\frac{5}{12} + \frac{12}{\pi}\ln g_5 + \frac{6}{\pi}\ln\lambda_{24} + \frac{18}{\pi}\ln\frac{v}{m_Z} = f_2(s^2, m_0, m_{1/2})\,. \tag{10}$$

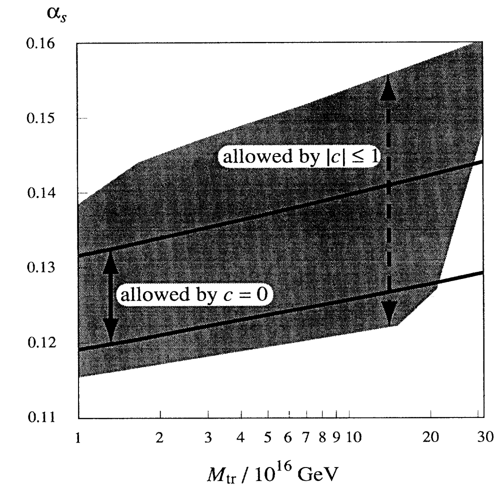

FIG. 1. The prediction in the minimal supersymmetric SU(5) model of α_s as a function of the color-triplet mass $M_{\rm tr}$. The region between the two black curves accounts for the possible variation of the light superpartner masses and the second Higgs doublet mass between 100 GeV and 1 TeV, and for the variation of $\sin^2\theta_W$ between 0.2317 and 0.2333. It does not incorporate any Planck-scale corrections, nor does it place any restriction on $\lambda_{5,24}$. The shaded region adds the gravitational corrections, with the restrictions that $0.1 \leq \lambda_{5,24} \leq 3$ and that $|c| \leq 1$.

In (9) we use the zeroth-order expression for $\alpha_G = g_5^2/4\pi \simeq 1/25$ in the coefficient of $\vec{\epsilon}$. Equation (10) has no direct gravitational contributions, since it turns out that $\vec{P}_2\cdot\vec{\epsilon} = 0$; we have merely rewritten (4) using the above expressions for the superheavy masses. The magnitude of the gravitational smearing may be readily estimated from (9). If $v \sim 2\times 10^{17}$ GeV and $|c| \sim 1$ then the prediction for α_s is corrected by $\sim 10\%$.

To be more precise, we study the predictions of the two equations (9) and (10) in the five unknowns $\{\alpha_s, M_{\rm tr}, \lambda_5 = \sqrt{\frac{12}{5}}M_{\rm tr}/v, \lambda_{24}, c\}$. A nonzero c couples the two equations and makes an exact analytic solution impossible. Instead, they may be solved analytically to a good approximation ($\sim\pm 1.5\%$ in α_s and $\sim\pm 30\%$ in $M_{\rm tr}$), or numerically to a high precision. The analytic expressions are

$$\alpha_s \simeq 0.132 \left[1 - 0.024\sigma - 0.02 \ln \frac{\mu^{4/5} m_{H_2}^{1/5}}{m_Z} + 0.025 \ln \frac{M_{\rm tr}}{3\times 10^{16}\,{\rm GeV}} - 0.025\, c\,(1-0.1\sigma)\left(\frac{m_{1/2}}{m_Z}\right)^{-2/9} \lambda_{24}^{-1/3}\right] \tag{11}$$

and

$$M_{\rm tr} \simeq (3\times 10^{16}\,{\rm GeV})\,\lambda_5\,(1-0.1\sigma)\,\lambda_{24}^{-1/3}\left(\frac{m_{1/2}}{m_Z}\right)^{-2/9}. \tag{12}$$

[The coefficient of the last term in (11) should be changed from -0.025 to -0.04 for large values (0.14–0.15) of α_s in order to achieve the desired accuracy.] Numerically, one subtracts (10) from (9) and solves the resulting equation for $M_{\rm tr}$. The solution(s) can then be used to find the corresponding α_s. We allow $\lambda_{5,24}$ to vary between 0.1 and 3, and also let c vary between -1 and 1. For each such choice of λ_5, λ_{24}, and c, we obtain numerically a region of allowed $(\alpha_s, M_{\rm tr})$ values as before. The overlap of all these regions is shown as the gray area in Fig. 1.

Figure 1 represents the region allowed in the minimal supersymmetric [14] SU(5) model by our present knowledge of s^2, our suspicions about the ranges of superpartner masses, our assumptions about the scalar couplings in the superpotential [15], and our ignorance of the true theory at the GUT scale. The domain of predictions for α_s is greatly increased by the possible Planck-scale corrections, and the correlation between α_s and the parameter $M_{\rm tr}$ relevant to proton decay is largely blurred away.

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and in part by NSF Grant No. PHY-90-21139.

Note added.—After this work was completed, we learned of some similar work in a paper by Langacker and Polonsky [16].

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[14] We note in passing that the nonsupersymmetric minimal SU(5) model cannot be resurrected by these gravitational effects. The analog of (10) shows that M_Σ is proportional to v^{-22} (compare with v^{-3} in the supersymmetric version). For gravitational effects to be relevant we need to raise v from its typical value of 10^{14} GeV to above 10^{16} GeV, which would require an intolerably low M_Σ.

[15] How large can the gravitational smearing become if we relax the lower bound of 0.1 on λ_{24}? For $c \geq 0$ there is always a (unique) solution to (9) and (10), so one could lower the prediction for α_s as far as desired by lowering λ_{24}. However, for $c < 0$ there may be 2, 1, or 0 solutions. For large λ_{24} two solutions exist, one above $M_{\rm tr}^{\rm cr} \equiv -14\lambda_5\alpha_G \tilde{M}_P/\pi c$ and one below. (We have chosen the lower one in the above estimates and in the figure.) These two merge into $M_{\rm tr}^{\rm cr}$ at some critical value of λ_{24} which depends on the other parameters, while for smaller λ_{24} no solutions exist. Thus the largest corrections occur when $M_{\rm tr} = M_{\rm tr}^{\rm cr}$, in which case the gravitational correction term in (9) becomes exactly $84/5\pi$. Such a correction raises even the lowest possible uncorrected α_s value to above 0.17. We learn that any conceivable future experimental determination of α_s may be accommodated using some (reasonable) value of λ_{24}.

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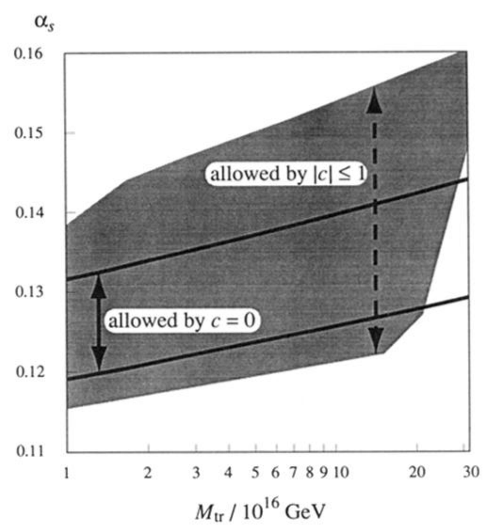

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