

Nonsingularity of the Exact Two-Dimensional String Black Hole

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In the semiclassical limit, the two-dimensional space-time which emerges from string theory is a black hole similar to the Schwarzschild solution. However, we find that in the *exact* space-time, the singularity is replaced by a regular surface of time-reflection symmetry. The maximally extended space-time thus consists of an infinite sequence of asymptotically flat regions linked by wormholes, rather analogous to the Reissner-Nordström space-time except that there are no singularities in this case. We also calculate the mass and temperature associated with this space-time.

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One of the great problems of classical general relativity is that it is plagued by space-time singularities. For example, one of the singularity theorems [1] asserts that if one has a trapped surface in an asymptotically flat space-time with matter obeying the strong energy condition (as classical matter traditionally does), and which satisfies Einstein's equation, then the space-time is geodesically incomplete. A simple corollary of this is that a black hole must be associated with a space-time singularity.

There has been some hope that a satisfactory quantum theory of gravity will resolve the difficulties associated with singularities. The only known realistic candidate for such a theory at present is string theory. Contrary to popular folklore, our understanding of string theory has seen important advances in the past few years. Perhaps most notable among these has been the realization that consistent string theories can be constructed in target space dimensions much lower than the critical dimension. A very successful example is the $c = 1$ matrix model (for reviews see Refs. [2,3]), which nonperturbatively describes strings propagating in a $(1+1)$ -dimensional background. Another major advance has been the discovery of background metrics for these low-dimensional target spaces. These classical solutions are in general curved space-times, and indeed the prototype is a black hole solution in $1+1$ dimensions [4,5].

The dynamics of strings in a curved space-time is governed by the world-sheet conformal invariance of string theory, which in turn is imposed by the vanishing of the β functions for the target space massless fields [6]. In $1+1$ dimensions for the bosonic string, these are the metric, dilaton, and tachyon. However, the β -function equations are only known perturbatively in the inverse string tension α' , and so conformal invariance can only be imposed order by order. The $(1+1)$ -dimensional black hole of Ref. [4] was found by setting the tachyon to zero and solving the lowest order β functions, which have the form of Einstein's equation coupled to a dilaton.

The resulting black hole solution is given by the metric and dilaton [5]

$$ds^2 = -\tanh^2 r dt^2 + dr^2, \quad (1)$$

$$\phi = \phi_0 + \ln \cosh^2 r;$$

or, in Kruskal-type coordinates, as

$$ds^2 = -\frac{du dv}{1-uv}, \quad (2)$$

$$\phi = \phi_0 + \ln |1-uv|.$$

It is clearly seen to possess a causal structure similar to that of the Schwarzschild black hole in four dimensions [5]: with an event horizon at $uv = 0$ and a curvature singularity at $uv = 1$. The Penrose diagram for the maximally extended black hole space-time is shown in Fig. 1.

Witten has managed to find an exact conformal field theory description of this black hole [5], which ensures that conformal invariance is obeyed nonperturbatively to all orders in k , the Kac-Moody level. This descrip-

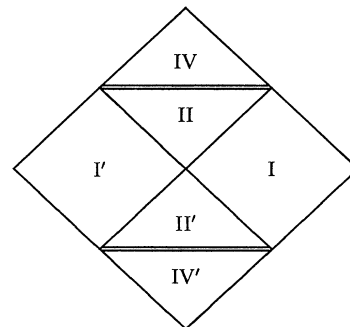


FIG. 1. Penrose diagram for the two-dimensional black hole as discussed by Witten. Regions I, I' are asymptotically flat space-times exterior to the black hole and white hole horizons. Regions II, II' are inside the horizons, while IV, IV' are asymptotically flat regions each containing a naked singularity. The double lines represent the curvature singularities at $uv = 1$.

tion is in the form of a Wess-Zumino-Witten (WZW) model [7] based on the group $G = \text{SO}(2, 1)$, gauged by the subgroup $H = \text{SO}(1, 1)$. The resulting coset space construction G/H maps to a noncompact (1+1)-dimensional gravitational background in which strings propagate. In the semiclassical approximation $k \rightarrow \infty$, the above black hole solution is recovered [5]. However, because of the relationship between k and the central charge c of the $\text{SO}(2,1)/\text{SO}(1,1)$ model given by [8]

$$c = \frac{3k}{k-2} - 1, \quad (3)$$

k takes the value 9/4 for a bosonic string background since $c = 26$ in order to cancel the contribution from the diffeomorphism ghosts. Thus, $1/k$ is quite large, and corrections due to this should not be ignored.

The effective space-time background for general k was first derived by Dijkgraaf, Verlinde, and Verlinde [9], and is

$$ds^2 = 2(k-2) [-\beta(r) dt^2 + dr^2], \quad (4)$$

$$\phi = \phi_0 + \frac{1}{2} \ln |\sinh^2 2r/\beta(r)|,$$

where $\beta(r)$ is given by

$$\beta(r) = \left(\coth^2 r - \frac{2}{k} \right)^{-1}. \quad (5)$$

It reduces to (1) for $k \rightarrow \infty$ (up to an overall conformal factor), and is believed to solve the β -function equations exactly. This has been confirmed by explicit computation to four-loop level in Refs. [10,11].

A few things can be said about the geometry of this space-time for $k > 2$, which we will assume from now on. Apart from considerations of string physics, it is believed that $k > 2$ in order to have a unitary conformal field theory. The coordinate patch (4) and (5) is asymptotically flat and describes the geometry exterior to a horizon at $r = 0$. In fact there are two copies of this region, one for r positive and the other for r negative, which are joined together at $r = 0$ in a way analogous to the Kruskal bridge of the Schwarzschild black hole. This space-time corresponds to regions I and I' of Fig. 1. Also note that the dilaton grows in strength asymptotically.

It is simple to see that the space-time given here is a black hole by a straightforward analogy with the $k \rightarrow \infty$ limit. The black hole horizon, at $r = 0$, is as usual associated with a Hawking temperature. The easiest way to find the temperature [12] is to analytically continue the metric (4) to its Euclidean region, and observe that the conical singularity at $r = 0$ is removed by identifying the Euclidean time coordinate with period 2π . The inverse of the proper period at infinity is then the Hawking temperature, and is given by $T_H = 1/(2\pi\sqrt{2k})$.

One can also estimate the mass of this black hole. As-

suming that the target space action is given by the Einstein action, and that the various higher-order corrections are negligible as one goes towards $r \rightarrow \infty$, one can calculate the Arnowitt-Deser-Misner (ADM) mass by following standard techniques [in particular, using Eq. (3.29) of Ref. [13]]. The result is

$$M = e^{\phi_0} \sqrt{\frac{2}{k}} \left(1 - \frac{2}{k} \right)^{-1}, \quad (6)$$

which agrees with the mass of the semiclassical black hole [5] to order $1/k$.

We can now determine what the space-time represents physically. Outside the horizon the space-time is static, and therefore it represents a black hole in thermodynamic equilibrium with a cloud of tachyons at the Hawking temperature T_H . Since the mass is finite, it seems that the total energy of the radiation is finite. Thus the entire system is gravitationally bound, rather like an isolated "star."

Within the context of gauged WZW models, there exists a duality transformation [9] which maps the above space-time to a new one again of the form (4), but now with

$$\beta(r) = \left(\tanh^2 r - \frac{2}{k} \right)^{-1}. \quad (7)$$

The conformal field theories associated to each of these two dual target spaces are completely equivalent, but their geometries are very different. The new space-time given by (7) is also asymptotically flat, but there is a curvature singularity at $r_c \equiv \text{arctanh} \sqrt{2/k}$. Thus the region $r_c < r < \infty$, which we denote by IV and where $\beta(r) > 0$, is a space-time that is exposed to a naked singularity. There is also a region behind the singularity $0 \leq r < r_c$, where $\beta(r) < 0$, which we call III. It vanishes in the $k \rightarrow \infty$ limit, and so has no analog in Fig. 1.

A new form of the metric which extends over the different regions of the entire space-time has been found in Ref. [14]. It is

$$ds^2 = 2(k-2) \left[-\beta(x) dt^2 + \frac{dx^2}{4(x^2-1)} \right], \quad (8)$$

$$\beta(x) = \left(\frac{x+1}{x-1} - \frac{2}{k} \right)^{-1},$$

with the associated dilaton

$$\phi = \phi_0 + \frac{1}{2} \ln \left| \frac{x^2-1}{\beta(x)} \right|. \quad (9)$$

To recover the above metrics, we have to reparametrize x in the various regions of this extended geometry. By setting $x = \cosh 2r \geq 1$, (8) reduces to the metric (4) with (5), thus yielding the exterior region I. The black

hole horizon is at $x = 1$. By contrast, the dual metric, consisting of regions III and IV, is obtained via the redefinition $x = -\cosh 2r \leq -1$. The singularity is at $x_c \equiv -(k + 2)/(k - 2) < -1$. Last, region II inside the horizon is labeled by $-1 < x < 1$, and a suitable parametrization of it is $x = \cos 2r$.

Note that in addition to being singular at the horizon and singularity, the metric (8) is singular at the point $x = -1$ between them. It is a coordinate singularity as can be seen from the regularity of the scalar curvature

$$R = \frac{2k}{k-2} \frac{(k-2)x + k - 4}{[(k-2)x + k + 2]^2} \tag{10}$$

at that point. Does it correspond to anything special?

From (8), observe that region I has signature $(-+)$, region II has signature $(+-)$, region III has signature $(++)$, while region IV has signature $(-+)$. Thus $x = -1$ appears to mark the boundary between two regions of different space-time signature. But what does it mean to have a region of Euclidean signature between the horizon and singularity of a Lorentzian black hole? Specifically, what happens to a particle [15] when it falls into the black hole and reaches $x = -1$? By studying geodesic motion in this geometry, this boundary turns out to be at finite proper distance from any point at finite x . However, nothing can enter the Euclidean region III as the signature of the space-time is fixed to be Lorentzian.

There is no real physical meaning attached to this "boundary," which follows by finding coordinates which are nonsingular there, e.g.,

$$x = -1 + 2\rho^2 . \tag{11}$$

The metric (8) then takes the form

$$ds^2 = 2(k-2) \left[\frac{k(1-\rho^2)}{2+\rho^2(k-2)} dt^2 - \frac{d\rho^2}{1-\rho^2} \right] . \tag{12}$$

It is flat and nonsingular at $\rho \simeq 0$. Thus an infalling particle has radial coordinate ρ running smoothly through $\rho = 0$, and it sees nothing special at that point. However, there are coordinate singularities at $\rho = \pm 1$, corresponding to two copies of the horizon $x = 1$.

To summarize, we have found that the black hole space-time is disconnected from the singularity by a region of Euclidean signature. A particle falling into the black hole sees no singularity, but emerges into another asymptotically flat black hole space-time. Two copies of region II are glued together at the surface of time-reflection symmetry $x = -1$ to form a wormhole bridging two asymptotically flat space-times. Thus we have an infinite chain of universes connected by timelike wormholes, whose Penrose diagram is shown in Fig. 2. It is reminiscent of the conformal structure of the Reissner-Nordström black hole, except that there are no singularities present in this case. We also have a disjoint asymptotically flat region containing a naked singularity.

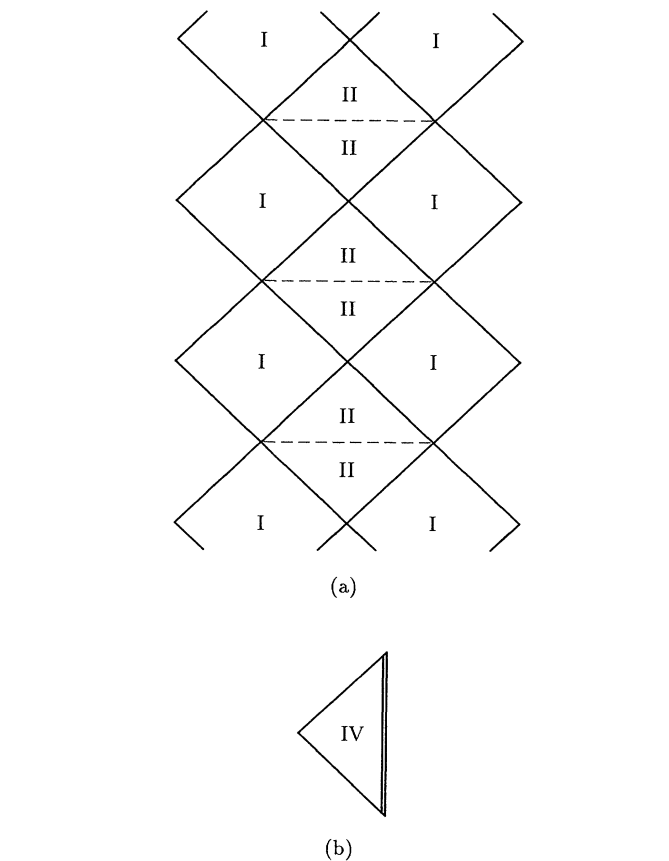


FIG. 2. (a) Penrose diagram for the maximally extended exact black hole geometry. This consists of an infinite sequence of asymptotically flat regions I linked by wormholes II. The dashed lines represent $x = -1$ surfaces. (b) There is also a disjoint asymptotically flat region IV which contains a naked singularity.

totically flat region containing a naked singularity.

Such an unusual situation of a black hole space-time having an event horizon but no singularity has no analog in classical general relativity. The reason for the appearance of a wormhole rather than a curvature singularity may be traced down to the negative scalar curvature of region II inside the horizon. The nonattractive character of gravity there prevents a singularity from forming, but supports a wormhole. Note that this feature results only when one includes the higher-order terms, since the scalar curvature of region II blows up in the $k \rightarrow \infty$ approximation. This reinforces the fact that higher-order corrections are important in regions of high curvature, for instance near the singularity in the semiclassical approximation.

A similar nonsingular geometry has been found in extremely charged black strings in three dimensions [16], which is also an exact string background. One may be tempted to speculate that nonsingular black holes

are generic to string theory, but more examples of such conformally exact metric solutions of string theory are needed to confirm this.

A direct generalization of the two-dimensional spacetime of this Letter to three dimensions [based on the coset space $SO(2,2)/SO(2,1)$] and four dimensions [$SO(3,2)/SO(3,1)$] has also been carried out in Ref. [14] and explicit metrics found. However, the complexity of these metrics makes them rather intractable at this stage. In the $k \rightarrow \infty$ limit, the three-dimensional metric was found to have a bizarre singularity structure [17]. Would this simplify in the more realistic case of finite k ? Of course other coset space constructions may be considered, and a classification of possible G/H coset spaces that yield noncompact Lorentzian target spaces has been done in Ref. [18].

Ultimately, one would be interested in four-dimensional black holes, especially representing known solutions of general relativity within a string framework, and looking for any qualitative differences near the singularities. However, to avoid the presence of singularities, the background fields of string theory have to violate the strong energy condition, which is indeed the case for the dilaton.

We intend to relate further details elsewhere.

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