

Wormhole Cosmology and the Horizon Problem

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We construct an explicit class of dynamic Lorentzian wormholes connecting Friedman-Robertson-Walker spacetimes. These wormholes can allow two-way transmission of signals between spatially separated regions of spacetime and could permit such regions to come into thermal contact. The cosmology of a network of early Universe wormholes is discussed.

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The most convincing evidence for the large-scale smoothness of the observable Universe is provided by the cosmic microwave background (CMB) radiation, which was in thermal equilibrium with matter until the Universe cooled to a temperature of about 4000 K, when the radiation decoupled from matter. From its discovery in 1965, numerous measurements have established the CMB to be consistent with a blackbody spectrum at a temperature $T = 2.735 \pm 0.06$ K, and uniform in all directions (on angular scales from $10''$ to 180°) to about a part in 10^4 [1]. Since this radiation is received from regions which could not have been in causal contact at the time of last scattering, the uniformity in temperature evident in the CMB must be arranged as an initial condition in the big-bang model [2]. This shortcoming of the original big-bang model is known as the horizon problem. There are two ways to solve this problem. One is to postulate that the cosmic scale factor underwent an era of exponential expansion (around 50 e -foldings) such that all the presently observable Universe is in one inflated causal domain. This is the inflationary Universe scenario which has been exhaustively explored in the literature [3]. The second possibility is that the Universe did not necessarily inflate, but has (or had) traversable bridges (wormholes) connecting otherwise causally disconnected regions of spacetime. This Universe with "handles" could alter and perhaps solve the horizon problem. Since this approach has not been studied, we are led to investigate it here, where we envision a network of traversable wormholes existing for a very brief time in the early Universe, arising out of Planck-time quantum metric fluctuations.

The first step in any investigation of wormhole "phenomenology," and a main result of this work, is the construction of specific cosmological wormhole solutions to Einstein's equations [4]. Although our treatment of wormholes will be purely classical, we assume (and we may turn out to be wrong [5]) that quantum field theory permits sufficient violation of the classical weak energy condition to allow the formation of traversable wormholes that can emerge from the Planck era and survive sufficiently long to result in the thermalization of the Universe (described below). In this regard, we explore

plausible scenarios for wormhole evolution [6] while assuming the most trivial ensemble of wormholes; i.e., we assume all wormhole mouths have equal "mass" and are at rest with respect to the cosmic expansion (Hubble flow). In our conclusions we discuss relaxing these last two assumptions.

The wormholes we will consider here result from surgically modified Friedman-Robertson-Walker (FRW) spacetimes. We adopt this technique for convenience's sake only; however, we assume the qualitative features of the wormholes are independent of the details of the construction. To construct them, take two copies of the FRW solution [7] (adopting units where $c = G = 1$)

$$ds^2 = -dt^2 + R^2(t) \left(\frac{dr^2}{1 - \kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \quad (1)$$

and remove from each an identical four-dimensional region of the form $\Omega_{1,2} = \{r_{1,2} \leq a\}$. The resulting spacetime contains two disjoint boundaries $\partial\Omega_{1,2} = \{r_{1,2} = a\}$, which are timelike hypersurfaces. An orientation preserving identification $\partial\Omega_1 \equiv \partial\Omega_2$ yields two FRW spacetimes connected by a wormhole whose throat is located on their mutual boundary $\partial\Omega$ [4]. Here, $a = a(\tau)$ is a function describing the dynamics of the wormhole's throat; the physical radius of the wormhole is equal to aR . The wormhole is spherically symmetric and the boundary layer is just the world volume swept out by its throat. This procedure also leads to a wormhole connecting a single FRW spacetime to itself if one identifies the two background spacetimes; i.e., we have a FRW space with a handle. In this case, the two regions $\Omega_{1,2}$ can be separated by an arbitrarily large distance in an open universe. To insure the modified spacetime is itself a solution of the gravitational field equations requires a proper matching of the metric across the boundary layer. The required junction conditions are most conveniently derived by introducing Gaussian normal coordinates in the neighborhood of the boundary hypersurface and integrating the Einstein equations across the boundary [8]. The most general stress-energy tensor which gives rise to two iden-

tical FRW spaces attached by a wormhole as described here is given by

$$T_\nu^\mu(x) = S_\nu^\mu \delta(\eta) + T^{(+)\mu}_\nu \Theta(\eta) + T^{(-)\mu}_\nu \Theta(-\eta), \quad (2)$$

where $T^{(+)\mu}_\nu = T^{(-)\mu}_\nu$ is one of the standard perfect-fluid source terms leading to (1) and

$$S_\nu^\mu \equiv \lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{\epsilon} d\eta T_\nu^\mu(x) \quad (3)$$

is the surface stress-energy. The Gaussian coordinate η parametrizes the proper distance measured perpendicularly through $\partial\Omega$. Integrating $G_\nu^\mu = 8\pi T_\nu^\mu$ across $\partial\Omega$ and taking the limit as indicated in (3) yields [8] $S_\eta^\eta = S_j^j = 0$, and

$$[\Delta \mathcal{H}_j^i - \delta_j^i \text{Tr}(\Delta \mathcal{H})] = 8\pi S_j^i, \quad (4)$$

where $\Delta \mathcal{H}_j^i \equiv \lim_{\epsilon \rightarrow 0} (\mathcal{H}_j^{(+i)} - \mathcal{H}_j^{(-i)})$ is the “jump” in the extrinsic curvature of the hypersurface in going from the $-\epsilon$ to the $+\epsilon$ “side.” For the case at hand, reflection symmetry implies $\Delta \mathcal{H}_j^i = 2\mathcal{H}_j^{(+i)}$ and the spherical symmetry implies $\mathcal{H}_j^i = \text{diag}(\mathcal{H}_\tau^\tau, \mathcal{H}_\theta^\theta, \mathcal{H}_\theta^\theta)$ and $S_j^i = \text{diag}(-\sigma, P, P)$, where σ, P denote the surface energy and pressure densities. The extrinsic curvature (in any coordinate system) is defined by

$$\mathcal{H}_{i,j} = n_\mu \nabla_{(j)} e_{(i)}^\mu, \quad (5)$$

where the $e_{(i)}$ constitute a set of three independent tangent vectors defined along the intrinsic coordinates ξ^i parametrizing the hypersurface, and n^μ is the outward unit normal ($n_\mu n^\mu = +1$, for timelike surfaces); the background covariant derivative is taken along the j th coordinate direction. The throat proper time and the two angles provide a convenient set of intrinsic coordinates: $\xi^i = (\tau, \theta, \phi)$. Since the position of the throat (i.e., its embedding in the background FRW space) is $X^\mu = (t, a(\tau), \theta, \phi)$, the tangent vectors are simply $e_{(i)}^\mu = dX^\mu/d\xi^i$, and the unit normal is

$$n^\mu = \left[\frac{a'R}{(1 - \kappa a^2)^{1/2}}, \frac{1}{R} [1 - \kappa a^2 + (a'R)^2]^{1/2}, 0, 0 \right].$$

A straightforward calculation of (5) gives

$$\mathcal{H}_\theta^\theta = a'\dot{R} + \frac{1}{aR} [1 + (a'R)^2]^{1/2} \quad (6)$$

and

$$\mathcal{H}_\tau^\tau = \frac{a''R}{[1 + (a'R)^2]^{1/2}} + 2a'\dot{R}, \quad (7)$$

where $a' = da/d\tau$ and $\dot{R} = dR/dt$ [9]. We have set $\kappa = 0$ since the curvature term in (1) is negligible during the early stages of expansion. Note the appearance of the two time parameters τ and t in (6) and (7). We can always eliminate one in favor of the other (e.g., coordinate time t) by using $a' = \dot{a} dt/d\tau$ and

$$d\tau/dt = [1 - (\dot{a}R)^2]^{1/2}. \quad (8)$$

Physically, this amounts to calibrating the clocks attached to the throat in terms of the comoving clocks. Doing so, and substituting (6) and (7) into (4), yields the wormhole's equations of motion:

$$-2\pi\sigma = \frac{\pm \dot{a}\dot{R} + (aR)^{-1}}{[1 - (\dot{a}R)^2]^{1/2}} \quad (9a)$$

and

$$4\pi P = \frac{\pm \ddot{a}R \pm 3\dot{a}\dot{R} + (aR)^{-1}}{[1 - (\dot{a}R)^2]^{1/2}} \pm \frac{(\dot{a}R)^2 [\ddot{a}R + \dot{a}\dot{R}]}{[1 - (\dot{a}R)^2]^{3/2}}. \quad (9b)$$

The plus (minus) sign corresponds to the two possible solutions of \dot{a} in terms of a' . The usual redundancy between the Einstein equations and the conservation of stress-energy T_ν^μ leads to a similar relation between the wormhole structure equations (9a) and (9b) and the surface stress tensor (3). The jump in the field equation $G_j^\eta = 8\pi T_j^\eta$, together with (4), implies [8]

$$\nabla_m S^{mj} = T^{(-)\eta j} - T^{(+)\eta j} = 0, \quad (10)$$

where ∇_m denotes the covariant derivative intrinsic to the hypersurface. Thus, the surface stress-energy tensor is conserved. The derivation of (9a) and (9b) together with (10) constitutes the most important formal result of this Letter.

With the wormhole equations of motion at hand, we now proceed to solve them. As a first example, consider the comoving case, $\dot{a} = a' = 0$. Then (cf. [6])

$$\sigma = -1/2\pi aR, \quad (11)$$

and $P = -\frac{1}{2}\sigma$. With a constant throat function a , the wormhole's radius simply grows in direct proportion to the background scale factor R . This behavior is also confirmed by (8), which implies $t = \tau$. These two time scales coincide if and only if the wormhole throat is coupled to the expansion. The surface energy density is negative, as indeed it must be: The violation of the weak energy condition (WEC) at the wormhole throat is expected on general grounds. In simple terms, the spacetime region surrounding the throat acts as a diverging lens which defocuses light and particle geodesics as they traverse the wormhole. This path divergence reflects a gravitational repulsion at the throat and signals the presence there of a localized negative energy density. Although no classical stress-energy tensor is known to violate the WEC, quantum fields in curved backgrounds tend to develop localized negative energy densities [10,11], and suggest these wormholes are driven by quantum field fluctuations, their backreaction on the metric [12], and the quantum fluctuations in the metric as envisioned, for example, in the spacetime foam [13]. According to (11), the comoving wormholes can exist for any value of the fixed throat function a . The magnitude of the surface energy density decreases for increasing scale factors. While these comoving wormholes do solve the structure equations, it may be more likely for FRW wormholes to evolve in time in such a way that they decouple from the expansion. Thus we are led to consider decoupled dynamic wormholes, where the full nonlinearity of the structure equations (9a) and (9b) comes into play.

The strategy for solving the general time-dependent

case involves selecting a relevant scale factor R , a surface energy density σ , and an equation of state $P=P(\sigma)$ subject to (10). First let us assume a power law expansion $R(t)=R_i(t/t_i)^p$. Then the choice for $a(t)$ that most simplifies (9a) and (9b) is $a(t)=a_i(t/t_i)^{1-p}$, which renders all the denominators constant. While these solutions are *ad hoc*, they are the simplest ones and are not implausible, and hopefully they capture the qualitative dynamics of more general wormhole physics, since they describe wormhole throats expanding faster than the Hubble flow. Here R_i and a_i are the scale factor and throat function at some initial time t_i . Now both P and σ go like t^{-1} and the equation of state is in general nonlinear. The relevant power law for early Universe applications is $p=\frac{1}{2}$, i.e., a radiation dominated expansion. For this case the surface energy density is $\sigma=\epsilon_0/a^2$, where ϵ_0 is a constant with units of mass given below. Then the *exact* solution of (9a) and (9b) has

$$a(t)=a_i(t/t_i)^{1/2}, \quad (12)$$

and an equation of state as given below. Note that the wormhole throat function is related to the background expansion: $\dot{a}/a=\dot{R}/R$. Moreover, the wormhole motion is decoupled from the expansion since $(1/aR)d(aR)/dt > \dot{R}/R$. The connection between proper and cosmic time implied by (8) is $t=\text{const} \times \tau$, where the constant of proportionality is greater than 1, and depends on a_i , R_i , and t_i .

For each t_i , there is an upper bound on the size spectrum of initial wormhole throat radii which follows if we assume no point on the throat can move faster than the speed of light [14]: $|\beta| < 1$ where $\beta \equiv 2\dot{a}R/c$ is a constant for the class of solutions treated here (with $p=\frac{1}{2}$). For radiation dominated expansion, this condition implies

$$a_i R(t_i) \leq (aR)_{\text{max}} \equiv ct_i, \quad (13)$$

where $R(t_i)$ is the scale factor for radiation dominance evaluated at some initial time t_i [15]. The constant ϵ_0 appearing in the surface energy density is given by (cf. [6]) (restoring c and G)

$$\epsilon_0 = -\frac{ca_i^2}{4\pi G t_i} \left(\frac{\beta + \beta^{-1}}{(1 - \beta^2)^{1/2}} \right) \quad (14)$$

and is negative, as for the comoving case.

The equation of state for the $p=\frac{1}{2}$ case is given by

$$P = -\frac{\sigma}{2} \left(\frac{1 \pm 2\beta^2}{1 \pm \beta^2} \right), \quad (15)$$

with sign choices as in (9a) and (9b). Note specifically that in the $\beta \rightarrow 0$ limit $P = -\sigma/2$ which corresponds to comoving wormholes in agreement with (11) and the subsequent discussion. Other interesting limits are $\beta=1$, on choosing the plus sign in (15) where $P = -3\sigma/4$; while for $\beta=1/\sqrt{2}$, the negative sign solution yields $P=0$, corresponding to pressureless negative energy “dust”; and $\beta \rightarrow 1$ where P diverges.

Thus far, we have dealt with aspects of a single comov-

ing or decoupled wormhole in a FRW background. It is clear, however, that the above construction can be applied an indefinite number of times and will lead to an explicit solution of Einstein's equation with an arbitrary number of wormholes. In fact, this construction is an application of the connected sum operation, familiar from algebraic topology. In general, we let \mathcal{M}_1 and \mathcal{M}_2 be two spacetimes and remove from each the interiors of the four-dimensional disks $D_1 \subset \mathcal{M}_1$ and $D_2 \subset \mathcal{M}_2$, and then identify the boundary sets up to homeomorphism $h: \partial D_1 \rightarrow \partial D_2$. The resulting manifold defines the connected sum of \mathcal{M}_1 and \mathcal{M}_2 : $\mathcal{M}_1 \# \mathcal{M}_2$ [16]. The operation $\#$ is both commutative and associative. This latter property implies that *multiwormhole* solutions of Einstein's equation can be constructed unambiguously.

Let us end with a discussion (and some speculations) on the applications of our results. To be of interest, wormholes need only stay open long enough for the radiation to traverse the throat. For the class of solutions treated here, a time scale is set by the frequency of the radiation traversing the throat, $\Delta t \sim 1/\nu$, and a length scale by $\lambda \sim a_i R_i$. This follows since our wormholes have zero-length throats, that is, a particle going down the wormhole mouth in one region comes out the other end instantaneously (see Fig. 1). Of course, we do not claim all wormholes will be of this form, but this still serves as a useful estimate of the wormhole lifetime needed to initiate thermal contact at a fixed frequency. With these reservations [17] we now proceed with a purely illustrative example. We assume a number density $n(t_{\text{Pl}})$ of Planck-sized wormholes (of radius l_{Pl}) at the Planck time, and we let $n(t_{\text{Pl}}) = \gamma n_{\text{Pl}} (\gamma = \text{const})$ [18] where $n_{\text{Pl}} = (l_{\text{Pl}})^{-3}$ is the Planck number density. At some later time t , the wormhole number density is $n(t) = \gamma n_{\text{Pl}} [R_{\text{Pl}}/R(t)]^3$. The volume of space filled by the interior of wormhole mouths (volume filling factor) is

$$\phi(t) = \frac{n(t)}{n_{\text{Pl}}} \left(\frac{l(t)}{l_{\text{Pl}}} \right)^3. \quad (16)$$

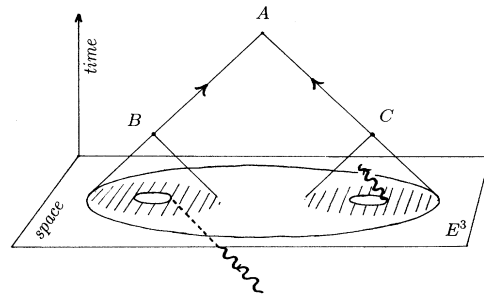


FIG. 1. A spacetime diagram representation of a Friedman cosmology with flat (E^3) spatial sections, with the past light cone for an observer at A . Radiation received at A from opposite directions was emitted from B and C . A wormhole connects events in the past light cones of the otherwise spatially separated points B and C . A photon is shown going down the wormhole in B 's past and exiting the wormhole in C 's past.

For comoving wormholes $\phi(t) = \gamma$, as expected, but let us concentrate on dynamic $p = \frac{1}{2}$ wormholes where $l = aR$, for which one finds $\phi(t) = \gamma[R(t)/R_P]^3$. Now, for example, the average particle in the cosmic soup will have traversed at least one wormhole by time t if $\gamma > [R(t)/R_P]^3$. For example, by the grand-unification time, $t_{\text{GUT}} \sim 10^{-34}$ sec, a $\gamma \geq 10^{-14}$ allows thermalization. Thermalization at even the high ν end of the spectrum can (small wormholes only) eventually lead to thermal equilibrium at a later time for causally disconnected regions. This in turn could provide a wormhole solution to the horizon problem. It is important to introduce a word of caution at this point. A more realistic ensemble of wormholes would take both relative motion and unequal throat masses into account. The former can lead to (i) relative time differences between wormhole mouths (second paper cited in [19] and [20]), even assuming the chronology protection conjecture, and the latter can lead to (ii) energy production/loss via wormhole mining [20]. We expect that effects (i) and (ii) will be reduced by averaging over wormhole contributions, but this would have to be checked by integrating over a more realistic initial wormhole ensemble.

It is rather natural to have an inflationary era in the early Universe. There may also be a cosmic wormhole era in the early Universe. However, to compete with inflation, a wormhole cosmology needs to address a list of other cosmological problems including flatness, monopoles (this may be solved directly in certain particle physics models), and density fluctuations. All these need further study.

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