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Thermalization Constraints and Spectral Distortions for Massive Unstable Relic Particles

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Spectral distortions to the cosmic background radiation due to massive unstable relic particles which decayed during the thermalization epoch are calculated. We impose stringent constraints on such particles from recent observations. Early energy injection of this type generates characteristic spectral distortions which could potentially serve to measure the baryon density of the Universe.

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The cosmic background radiation (CBR) provides a unique probe of the early Universe. In particular, spectral distortions constrain energy injection at the epoch of thermalization of the blackbody spectrum, at redshift $z=10^6$–10^7. Relic unstable massive particles that decay during this era or at subsequent epochs inevitably generate spectral distortions. The observational limits on possible spectral distortions are becoming increasingly constraining, but still allow the possibility of substantial early energy injection at epochs of astrophysical interest.

Observational signatures of spectral distortion have occasionally been claimed and interpreted in terms of massive particle decays. Although the recent data show no firm evidence of any spectral distortion at high frequency, there is a tantalizing, but statistically not significant, indication of distortion at low frequency. In fact, distortions due to early particle decays would be expected to peak at such low frequencies. In view of the recent high precision spectral measurements by cosmic background explorer far infrared absolute spectrophotometer (COBE FIRAS) [1], that now confirm a Planck spectrum to within 0.03% near the blackbody peak, it is accordingly timely to provide a rigorous prediction of the spectral distortion arising from early particle decays.

Earlier discussions employed approximations to describe the thermalization process [2–4]. Furthermore, several recent treatments have omitted a significant factor of 4π in the equation for thermalization by bremsstrahlung [4–6]. We have undertaken a numerical analysis of thermalization, incorporating both bremsstrahlung and double Compton scattering processes. Our principal results are the derivation of improved limits on massive relic particle decay, and predictions of the spectral distortion from such an event. Should these predicted distortions be found in the CBR, they will provide a potentially unique probe of the baryon density Ω_B. However, any marginal indications of distortions observed at low frequencies, if confirmed, *cannot* be adequately explained by massive unstable relic particles.

Spectral distortions in the CBR are thermalized by the joint action of elastic Compton scattering, double Compton scattering, and bremsstrahlung. Elastic Compton scattering is governed by the Kompaneets equation

$$\left(\frac{\partial n}{\partial t}\right)_K = n_e \sigma_T c \left(\frac{kT_e}{m_e c^2}\right) \frac{1}{x_e^2} \frac{\partial}{\partial x_e} \left[x_e^4 \left(\frac{\partial n}{\partial x_e} + n + n^2 \right) \right], \tag{1}$$

where $n(x_e,t)$ is the photon occupation number, n_e is the electron number density, σ_T is the Thomson cross section, T_e is the electron temperature, and $x_e = h\nu/kT_e$ is the dimensionless photon frequency. The electron temperature is given by [7,8]

$$T_e = \frac{1}{4}\frac{h}{k}\frac{\int \nu^4 n(n+1)d\nu}{\int \nu^3 n\, d\nu}. \tag{2}$$

Since elastic Compton scattering conserves photon number, photon-creating processes must also be considered in the thermalization process. To lowest order, these are bremsstrahlung and double Compton scattering. The kinetic equation for bremsstrahlung can be expressed as

$$\left(\frac{\partial n}{\partial t}\right)_{\rm br}=(n_e\sigma_T c)Q\frac{g(x_e)}{e^{x_e}}\frac{1}{x_e^3}[1-n(e^{x_e}-1)]\,, \quad (3)$$

where

$$Q=\frac{4\pi}{(2\pi)^{7/2}}\left(\frac{kT_e}{m_e c^2}\right)^{-1/2}\alpha\sum n_i Z_i^2\left(\frac{hc}{kT_e}\right)^3 .$$

Here n_i is number density of ions with atomic number Z_i, and α is the fine structure constant. The Gaunt factor $g(x_e)$ is approximated by

$$g(x_e)\simeq\begin{cases}\ln(2.25/x_e), & x_e\le 0.37,\\ \pi/\sqrt{3}, & x_e>0.37.\end{cases} \quad (4)$$

Some confusion has arisen in the recent literature [4,6] due to an omission of a factor of 4π in Lightman's equation for bremsstrahlung [5]. We correct for this factor here. Finally double Compton scattering can be described by [5]

$$\left(\frac{\partial n}{\partial t}\right)_{\rm dC}=n_e\sigma_T c\frac{4\alpha}{3\pi}\left(\frac{kT_e}{m_e c^2}\right)^2\frac{1}{x_e^3}[1-n(e^{x_e}-1)]\times\int dx_e x_e^4(1+n)n\,. \quad (5)$$

The full kinetic equation to lowest order is therefore

$$\left(\frac{\partial n}{\partial t}\right)=\left(\frac{\partial n}{\partial t}\right)_K+\left(\frac{\partial n}{\partial t}\right)_{\rm dC}+\left(\frac{\partial n}{\partial t}\right)_{\rm br}. \quad (6)$$

In general, the Wien tail of the spectrum may be described by a Bose-Einstein distribution with a chemical potential μ that decreases in magnitude with time.

We present numerical solutions to Eq. (6). Further details and results may be found in Ref. [9]. It is instructive to compare these numerical results with analytic approximations. An analytical treatment employing double Compton scattering and bremsstrahlung as source terms can predict the evolution of a *small* distortion to the spectrum [9,10]. We will use the constraints derived from these approximations for comparison purposes.

Stingent constraints can be placed on the epoch at which a given amount of energy can be injected by demanding consistency with observations of the CBR which require $\mu_0<3.3\times10^{-4}$ [1]. If the energy injection arises from the decay of a massive particle, we may translate this into a constraint on the mass m_X, lifetime τ_X, and branching ratio f for decay to photons of such a species. In the regime where the elastic Compton scattering time scale and particle lifetime are small compared with the photon creation time scale, the specific nature of the energy injection is irrelevant since, at high redshifts, it is rapidly processed into a μ distortion. Thus, the fractional energy and number density of the photons injected alone determine the spectral distortions [9]. For the case of massive relic particles, the number density of photons injected is negligible compared with that in the background. Therefore, the spectral distortions are determined by the integral of the fractional contributions to the energy ϵ of the CBR per comoving volume during the decay. Assuming that the comoving number density of species X decays exponentially in time with lifetime τ, we obtain

$$\frac{\delta\epsilon}{\epsilon}=\frac{m_X c^2}{kT(t_{\rm eff})}\left(\frac{n_X}{n_\gamma}\right)f\,, \quad (7)$$

where $T(t)$ is the CBR temperature and (n_X/n_γ) is the ratio of the number densities before decay. The functional form of Eq. (7) is identical to the case in which all particles decayed in a time $t_{\rm eff}=[\Gamma(1-\beta)]^{1/\beta}\tau$ for a time temperature relation of $T\propto t^{-\beta}$. Here Γ is the usual gamma function.

Let us first consider the case of a low $\Omega_B h^2$ universe as implied by nucleosynthesis where double Compton scattering dominates the thermalization process. For small energy injection, we find that the chemical potential today is given by

$$\mu_0\simeq 8.01\times10^2\left(\frac{\tau_X}{1\ {\rm s}}\right)^{1/2}\exp[-(\tau_{\rm dC}/\tau_X)^{5/4}]\times\left(\frac{m_X}{1\ {\rm GeV}}\right)fn_X/n_\gamma<3.3\times10^{-4}\,, \quad (8)$$

where

$$\tau_{\rm dC}=1.46\times10^8\Theta_{2.7}^{-12/5}(\Omega_B h^2)^{4/5}(1-Y_p/2)^{4/5}\ {\rm s}$$

with $\Theta_{2.7}=T_0/(2.7\ {\rm K})$, Y_p the primordial mass density in helium, and $h=H_0/(100\ {\rm km\,s^{-1}\,Mpc^{-1}})$. We have assumed here that we are in the radiation dominated epoch where $T\propto t^{-1/2}$.

If $\Omega_B h^2\gtrsim0.1$, bremsstrahlung plays an important role in the thermalization process. If bremsstrahlung dominates, this constraint becomes

$$\mu_0=8.01\times10^2\left(\frac{\tau_X}{1\ {\rm s}}\right)^{1/2}\exp[-(\tau_{\rm br}/\tau_X)^{5/8}]\times\left(\frac{m_X}{1\ {\rm GeV}}\right)fn_X/n_\gamma<3.3\times10^{-4}\,, \quad (9)$$

where

$$\tau_{\rm br}\simeq7.7\times10^9\Theta_{2.7}^{-36/5}(\Omega_B h^2)^{12/5}(1-Y_p/2)^{8/5}\ {\rm s}\,.$$

The weaker of the two constraints, Eqs. (8) and (9), is the relevant one to consider for intermediate cases.

These analytic formulas are only valid for small injection of energy $\delta\epsilon/\epsilon\ll1$. We therefore expect deviations from these predictions when particles decay near the epoch at which an arbitrary injection of energy can be thermalized. Large distortions are thermalized less rapidly than the analytic approximations above would imply. Figure 1 displays the results of numerical integration for

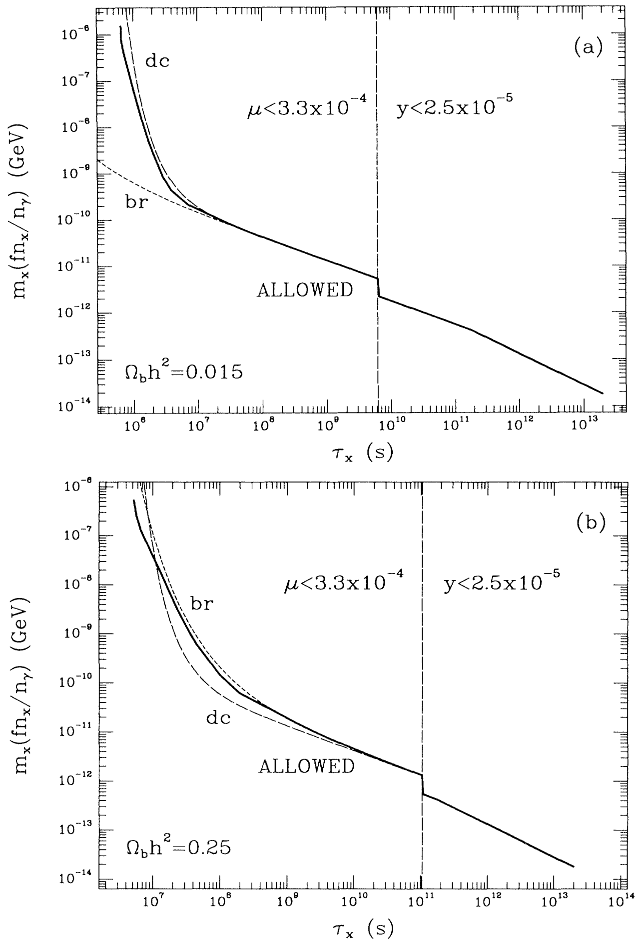

FIG. 1. Mass-lifetime-branching-ratio constraints for an (a) $\Omega_B h^2=0.015$ and (b) $\Omega_B h^2=0.25$ universe. Analytic predictions for thermalization under double Compton scattering (long dash) and bremsstrahlung (dash) are shown.

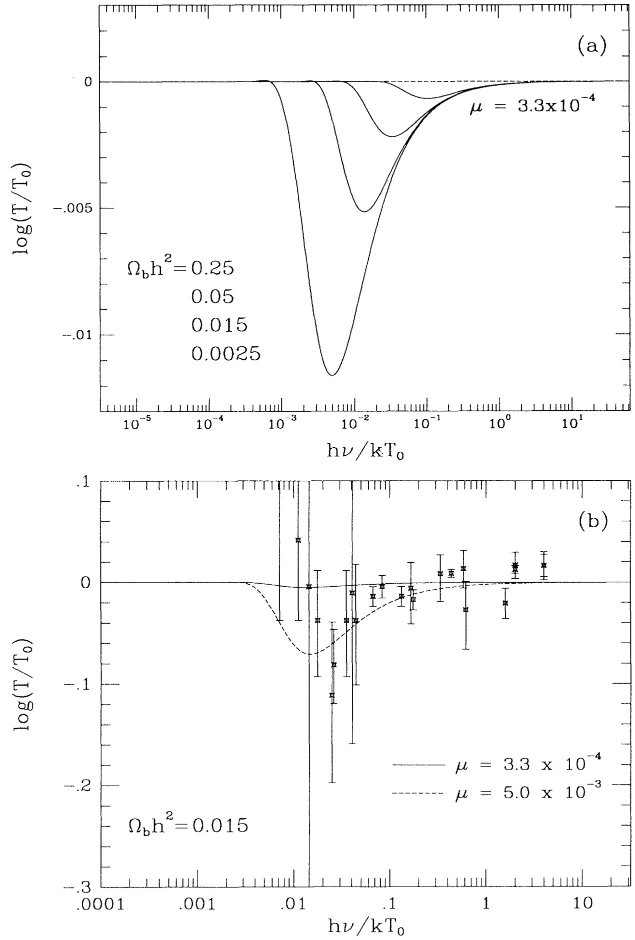

FIG. 2. (a) Spectral distortions due to the presence of $\mu=3.3\times10^{-4}$ in the Wien tail in an $\Omega_B h^2=0.25$, 0.05, 0.015, 0.0025 universe (in order of increasing distortions). (b) Observational data compared with expected spectral distortions with $\mu=5.0\times10^{-3}$ and 3.3×10^{-4}. We use $T_0=2.726$ [1] for normalization and the point $h\nu/kT_0=0.007$, $\log_{10}(T/T_0)=0.13$ falls beyond the limits of the graph.

(a) $\Omega_B h^2=0.015$ and (b) $\Omega_B h^2=0.25$. In both cases, particles with a short lifetime that decay during the critical epoch for thermalization are more stringently constrained than analytic predictions, also plotted, would suggest. For late decays with $\tau_X \gtrsim 4\times10^{11}\Omega_B h^2$ s, elastic Compton scattering can no longer establish a Bose-Einstein spectrum. Instead, the spectrum can be described by the Compton y parameter which is related to the energy release by $\delta\epsilon/\epsilon=4y$ [11]. We also plot the constraints implied by the most current value of $y<2.5\times10^{-5}$ [1] in Fig. 1.

Energy injection during the thermalization epoch also leaves a characteristic signature on the spectrum. Rayleigh-Jeans distortions show deviations from the pure Bose-Einstein form since double Compton scattering and bremsstrahlung can return the spectrum to blackbody. Analytic approximations, involving a frequency-dependent chemical potential and the partial suppression of Rayleigh-Jeans distortions by photon-creating processes can be found in Ref. [2]. However, even if there should be a positive detection of such distortions, we will not be able to obtain detailed information on the injection mechanism. The thermalization process erases all the specifics of the injection. It leaves a spectrum that is described only by the chemical potential of the Wien tail and the location of the peak distortion in the Rayleigh-Jeans region. The competition between elastic Compton scattering and the photon creating processes defines the location of the peak. Furthermore, the balance between these thermalization processes is fixed almost entirely by $\Omega_B h^2$ alone. This universality is useful in its own right. Figure 2(a) displays the expected spectral distortions for a fixed Wien tail distortion of $\mu=3.3\times10^{-4}$ for various choices of $\Omega_B h^2$. The difference between cosmologically interesting choices for $\Omega_B h^2$ is potentially measurable.

Observations of the Rayleigh-Jeans regime to date do not have such sensitivity. However, they do show marginally significant indications of a systematically low effective temperature. Figure 2(b) plots the observational data. Original references for the data may be found in Ref. [9]. These distortions are suggestive of those obtained from early massive particle decays (dotted lines). However, the implied distortion at high frequencies would then be inconsistent with the FIRAS result. If we were to demand consistency with the FIRAS $\mu < 3.3\times 10^{-4}$ result, the corresponding Rayleigh-Jeans distortions (solid line) would be too low to explain any "observed" distortions. In fact, early energy injection of any form can explain such distortions and be consistent with FIRAS only if the energy injection is accompanied by an extremely specific injection of photons $\delta n_\gamma / n_\gamma = \frac{3}{4}\,\delta\epsilon/\epsilon$ [9]. Therefore, nearly all early injection mechanisms are ruled out as explanations of any Rayleigh-Jeans distortions at a level of 0.1 K.

In summary, we have improved the constraints on massive unstable particles decaying during or after the thermalization epoch by employing a detailed numerical study of the thermalization process itself. We have also studied the expected spectral distortions from such decays. We find that they are sensitive primarily to the baryon density of the universe and may eventually be used to measure this fundamental cosmological parameter. However, the marginal indications of Rayleigh-Jeans distortions observed today cannot be explained by such a scenario.

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